Advances in LAM 3D-VAR formulation

Vincent Guidard and Claude Fischer
Météo-France, CNRM/GMAP,
42 avenue Coriolis, 31057 Toulouse, FRANCE
tel. +335 6107 8577, fax. +335 6107 8453
vincent.guidard@meteo.fr

1 Compactly supported background error statistics

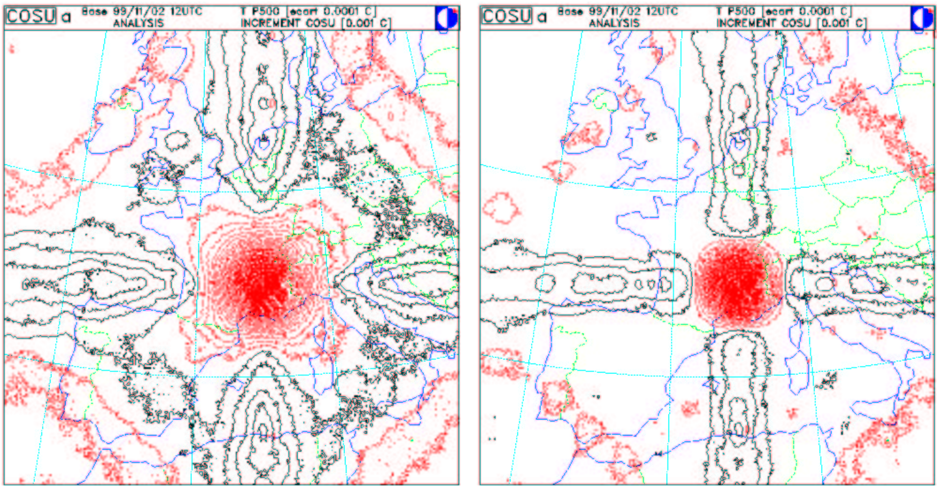
In the general background of studies in ALADIN 3D-VAR, a drawback of biperiodisation has arisen: an observation too close to the border of the domain generates an increment which affects the opposite side of the domain. Increasing the length of the extension zone could be a cure, but it would imply a computation and archiving overcost. To solve this problem, we have decided to control the lengthscale of the analysis increments thanks to the compactly supported formulation.

To obtain compactly supported (COSU) correlation functions, the initial gridpoint correlation function has to be multiplied by a mask function (chosen to be cosine-shape between d_1 and d_2, where d_1 and d_2 are tunable distances; the COSU function is expected to remain unmodified below d_1 and to be zero beyond d_2) :
$q_{\text{COSU}}^{(i,j)}(x,y) = q^{(i,j)}(x,y) \times mask\left(\sqrt{(x-i)^2+(y-j)^2}\right)$.
According to Gaspari and Cohn (1999), this mask should be applied to the square root of the gridpoint correlations. As the background error statistics have a spectral formulation in ALADIN, here is the method proposed by Loïk Berre to implement the COSU approach:

1. convert the power spectrum into modal variances;
2. fill a 2D spectral array from the 1D square root of the modal variances;
3. inverse bi-Fourier transform - mask the gridpoint structure - direct bi-Fourier transform;
4. collect isotropically and square to obtain modified modal variances;
5. convert the modal variances into power spectrum.

In a global overview, since the autocorrelations are compactly supported, the values of the power spectrum for the 3 first total wavenumbers are decreased, and there is quite no change of the power spectrum for large total wavenumbers (ranging from 40 to 140). In gridpoint space, the lengthscale is controlled, even though (small) non-zero values are observed, because of non totally symmetric steps (direct and inverse bi-Fourier transforms, fill in and isotropic collection). Therefore, this method has been implemented in ALADIN, with the use of the previous recipe and based on the SUJBCOSU routine already implemented in ARPEGE-IFS by François Bouttier to compactly support the horizontal autocorrelations.

The first experiments are performed using the univariate formulation of the background error statistics, in the framework of single observation experiments. The results are quite convincing (Fig. 1): the lengthscale is under control, but some noise still remains (a cross centered on the observation).



(a) With the original B

(b) With COSU horizontal correlations, $d_1 = 100$ km and $d_2 = 300$ km

Figure 1: Temperature at 500hPa analysis increment of a single observation of relative humidity at 500 hPa. Units : 0.001 K.

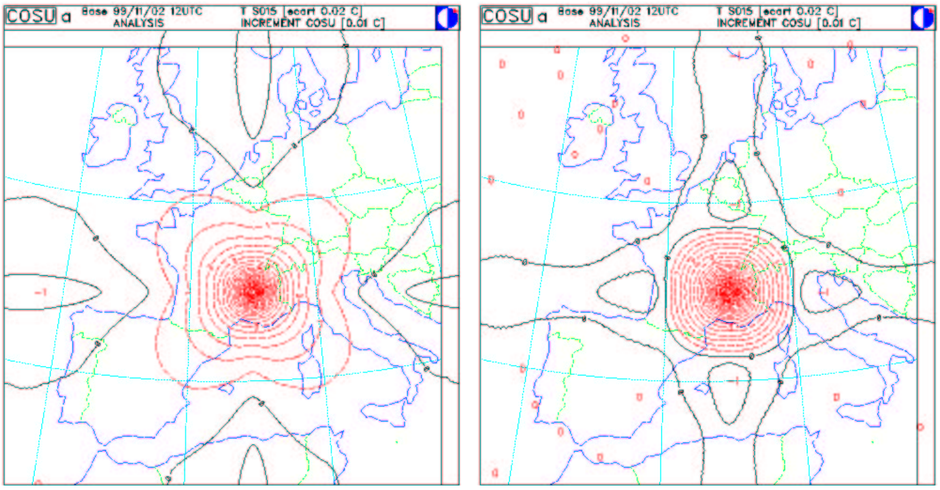
Then the single observation experiments are performed using the multivariate formulation, given by Berre (2000):

$$\begin{cases} \zeta & = & \zeta \\ \eta & = & \mathbf{MH}\zeta + \eta_u \\ (\mathbf{T}, P_s) & = & \mathbf{NH}\zeta + \mathbf{P}\eta_u + (\mathbf{T}, P_s)_u \\ \mathbf{q} & = & \mathbf{QH}\zeta + \mathbf{R}\eta_u + \mathbf{S}(\mathbf{T}, P_s)_u + \mathbf{q}_u \end{cases}$$

where ζ, η, $(\mathbf{T}, P_s)$ and $\mathbf{q}$ are the forecast errors for the vorticity, the divergence, the couple temperature and surface pressure, and the specific humidity; $*_u$ is the unbalanced par of the error $*$; $\mathbf{H}$ is the horizontal balance operator, and $\mathbf{M}$, $\mathbf{N}$, $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{R}$ and $\mathbf{S}$ are the vertical balance operators.

First, only the horizontal autocovariances are compactly supported. The results are "worse" than those obtained with the original statistics. An explanation can be: the main part of the increment is balanced, and only the autocovariances for ζ are compactly supported, not for $\mathbf{H}\zeta$.
Then the power spectrum of ζ is modified in order to have COSU $\mathbf{H}\zeta$. The results

are neutral in comparison to those obtained with the original statistics.
A more drastic solution is eventually used: using a compactly supported horizontal balance operator. This has cured the problem (Fig. 2). Through these experiments, it seems that the only way to control the lengthscale of the analysis increment in the multivariate approach is to use a COSU horizontal balance operator.



(a) With the original B

(b) With all COSU, $d_1 = 300$ km and $d_2 = 500$ km

Figure 2: Temperature on model level # 15 analysis increment of a single observation of temperature ate 500 hPa. Units : 1 mK.

The academic framework of single observation experiments is simple enough to manage to control the lengthscale of the analysis increments, one way or another. But, if a full set of observations (even only a band of observations) is used, the COSU approach has no impact, even if drastic measures are implemented. This may be due to a large-scale error in the background state that the ALADIN 3D-VAR, which is supposed to be a mesoscale analysis, tries to reduce. That is why another source for the large scale information has to be used in the 3D-VAR formulation.

2 Introduction of a large-scale cost-function

The analysis from the global model (x^{AA}) is chosen to be the large scale information. There are now three sources of information: the LAM background x^b (a short-range forecast), the observations y and the global analysis put to a LAM low resolution geometry $\mathcal{H}_1\left(x^{\mathrm{AA}}\right)$. Usually, the background errors and the observation errors are supposed to be uncorrelated, which leads to a block-diagonal matrix for the covariances between the sources of information (which is the condition to write the objective cost-function as the sum of two terms). As a new

source is added, this matrix is not block-diagonal anymore under this sole hypothesis. We have to assume that the LAM background errors and the global analysis errors are not correlated, in order to simply translate the introduction of this new source of information into a new term to add to the "classical" cost-function. This new term, measuring the distance of the LAM state to the large-scale information, can be written as follows:
$J_k(x) = \left(\mathcal{H}_1(x^{\text{AA}}) - \mathcal{H}_2(x)\right)^T V^{-1} \left(\mathcal{H}_1(x^{\text{AA}}) - \mathcal{H}_2(x)\right)$, where $\mathcal{H}_*$ are operators performing a change of geometry, V is the large-scale error covariances matrix.

To evaluate this new formulation of the 3D-VAR, an academic 1D model is used: a 1D Shallow-Water model. The global part was written by Ilian Gospodinov, and its LAM counterpart using a Davies coupling has been implemented by Piet Termonia. Both models are spectral. A gridpoint analysis is added to these forecast models. Three kinds of LAM analysis are available:
– a classical analysis, using x^b and the observations $\rightarrow$ BO analysis;
– an analysis using x^b and the large-scale information $\rightarrow$ BK analysis (which can be considered as a replacement for the DFI-blending technique);
– an analysis using the three sources of information $\rightarrow$ BOK (a replacement for the BLEND-VAR technique).

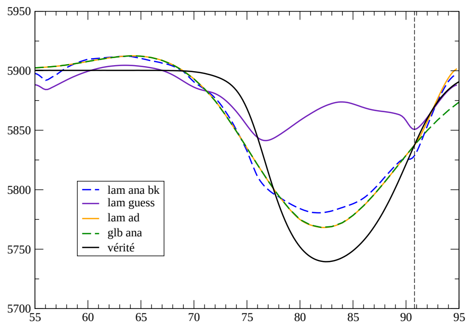


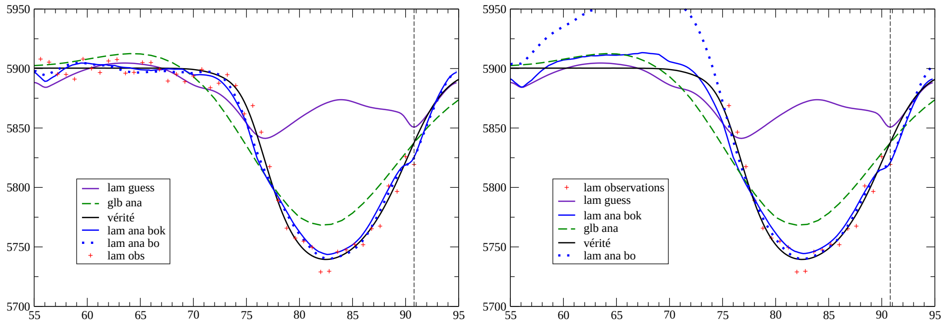
Figure 3: Comparison between the dynamical adaptation of the global analysis, the BK analysis, the global analysis, the LAM background and the truth for the geopotential.

The evaluation divides into two parts: first comparing the BK analysis to the dynamical adaptation of the global analysis, and then comparing the "classical" BO analysis to the BOK analysis.

There is quite no difference between BK and the dynamical adaptation (Fig. 3 is a typical example of what is obtained in that case). Statistical tests (Fisher and Student tests) computed for the bias and the rmse lead to the same conclusion. One can notice that the LAM background is of rather poor quality, which may be due to the Davies coupling formulation which is probably not suitable in this framework (1D Shallow-Water model).

When the observations are regularly spaced over all the domain, both BO and BOK analyses are very close to the truth (Fig. 4(a)) and the two analyses are indicernable. But if only a band of observations is used (which mimics an ALADIN analysis using only raw radiances over a part of its domain, for instance), the BO analysis generates an overshoot in the region with no observation, whereas the BOK analysis does not, thanks to the use of another source of information covering all the domain.

The use of a new source of information about the large scales has a quite neutral impact, except for some border-line cases, in this 1D Shallow-Water framework. This has to be evaluated in another 1D model (based on the Burger equa-



(a) Observations all over the domain (b) A band of observations

Figure 4: Comparison beetween the BO analysis, the BOK analysis, the global analysis, the observations, the LAM background and the truth for the geopotential.

tion), and then implemented and evaluated in ALADIN, after an ensemble evaluation of the statistics thanks to the works of Loïk Berre, Margarida Belo-Pereira and Simona Stefanescu.

References

Berre, L. (2000). Estimation of synoptic and mesoscale forecast error covariances in a limited-area model. *Mon. Wea. Rev. 128*, 644–667.

Gaspari, G. and S. Cohn (1999). Construction of correlation functions in two and three dimensions. *Quart. J. Roy. Meteor. Soc. 125*, 723–757.