THE EFFECT OF STRATIFICATION ON THE ROUGHNESS LENGTH AN DISPLACEMENT HEIGHT

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Reference

S. S. Zilitinkevich, I. Mammarella, A. A. Baklanov, and S. M. Joffre, 2008: The effect of stratification on the roughness length and displacement height. *Boundary-Layer Meteorology.* In press.

Content

- Roughness length and displacement height:

$$u(z)=\frac{u_*}{k}\left[\ln\frac{z-d_{0u}}{z_{0u}}+\Psi_u\left(\frac{z}{L}\right)\right]$$

- No stability dependence of z_{0u} (and d_{0u}) in engineering fluid mechanics: neutral-stability z_0 = level, at which $u(z)$ plotted vs. $\ln z$ approaches zero; $z_0 \sim {}^1\!/_{25}$ of typical height of roughness elements, h_0

- Meteorology / oceanography: h_0 comparable with MO length $L=\frac{u_*^3}{-\beta F_{\theta s}}$

- Stability dependence of the actual roughness length, z_{0u}:

  $z_{0u} < z_0$ in stable stratification; $z_{0u} > z_0$ in unstable stratification

Surface layer and roughness length

Self similarity in the surface layer (SL) $5h_0 < z < 10^{-1}h$

Height-constant fluxes: $\tau \approx \tau|_{z=5h_0} \equiv u_*^2$

u_* and z serve as turbulent scales: $u_T \sim u_*,\ l_T \sim z$

Eddy viscosity $(k \approx 0.4)$ $K_M\ (\sim u_T l_T) = k u_* z$

Velocity gradient $\partial U / \partial z = \tau / K_M = u_* / kz$

Integration constant: $U = k^{-1} u_* \ln z + \text{constant} = k^{-1} u_* \ln(z / z_{0u})$

z_{0u} (redefined constant of integration) is “roughness length”

“Displacement height” d_{0u} $U = k^{-1} u_* \ln[(z - d_{u0}) / z_{u0}]$

Not applied to the roughness layer (RL) $0 < z < 5h_0$

Parameters controlling z_{0u}

Smooth surfaces: viscous layer → $z_{0u} \sim \nu / u_*$

Very rough surfaces: pressure forces depend on:

obstacle height h_0

velocity in the roughness layer $U_R \sim u_*$

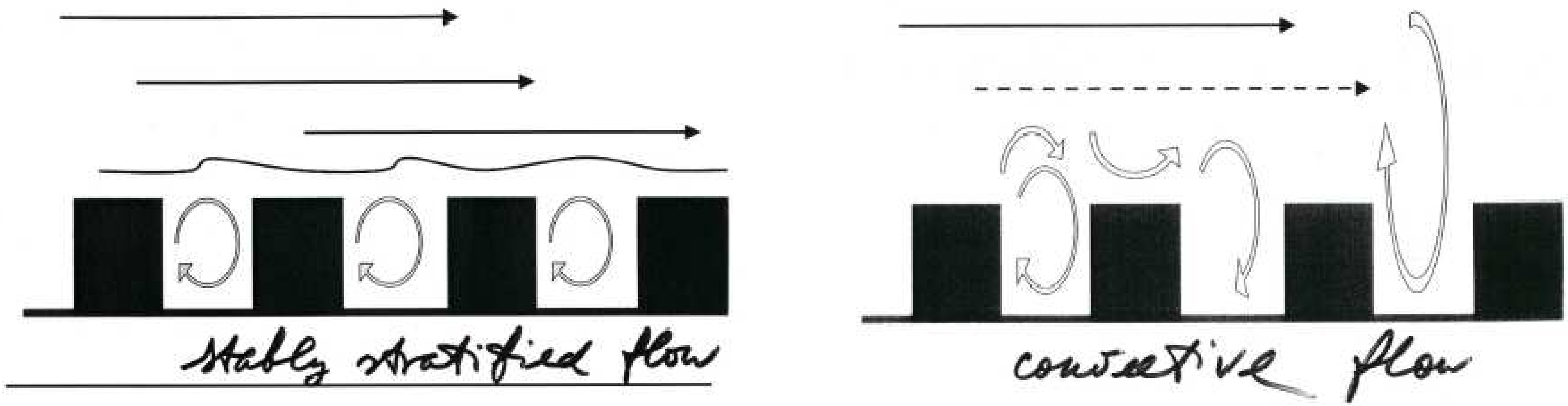
$z_{0u} = z_{0u}(h_0, u_*) \sim h_0$ (in sand roughness experiments $z_{0u} \approx \frac{1}{30} h_0$)

No dependence on u_*; surfaces characterised by z_{0u} = constant

Generally $z_{0u} = h_0 \, f_0(\mathrm{Re}_0)$ where $\mathrm{Re}_0 = u_* h_0 / \nu$

Stratification at M-O length $L = -u_*^3 F_b^{-1}$ comparable with h_0

Stability Dependence of Roughness Length



For urban and vegetation canopies with roughness-element heights (20-50 m) comparable with the Monin-Obukhov turbulent length scale, L, the surface resistance and roughness length depend on stratification

Background physics and effect of stratification

Physically z_{0u} = depth of a sub-layer within RL ($0 < z < 5h_0$) with 90% of the velocity drop from $U_R \sim u_*$ (approached at $z \sim h_0$)

From $\tau = K_{M(RL)} \partial U / \partial z$, $\tau \sim u_*^2$ and $\partial U / \partial z \sim U_R / z_{0u} \sim u_* / z_{0u}$

$$z_{0u} \sim K_{M(RL)} / u_*$$

$K_M(\mathrm{RL}) = K_M(h_0 + 0)$ from matching the RL and the surface-layer

Neutral: $K_M \sim u_* h_0 \Rightarrow$ classical formula $z_{0u} \sim h_0$

Stable: $K_M = k u_* z (1 + C_u z / L)^{-1} \sim u_* L \Rightarrow z_{0u} \sim L$

Unstable: $K_M = k u_* z + C_U^{-1} F_b^{1/3} z^{4/3} \sim F_b^{1/3} z^{4/3} \Rightarrow z_{0u} \sim h_0 (-h_0 / L)^{1/3}$

Recommended formulation

Neutral ⇔ stable

$$\frac{z_{0u}}{z_0} = \frac{1}{1 + C_{SS} h_0 / L}$$

Neutral ⇔ unstable

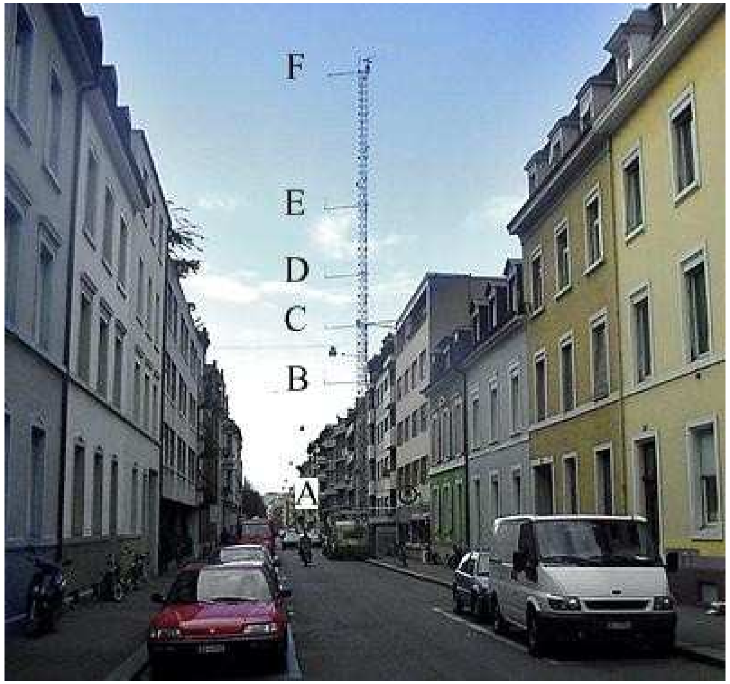
$$\frac{z_{0u}}{z_0} = 1 + C_{US} \left(\frac{h_0}{-L} \right)^{1/3}$$

Constants: $C_{SS} = 8.13 \pm 0.21$, $C_{US} = 1.24 \pm 0.05$

Experimental datasets

Sodankyla Meteorological Observatory, Boreal forest (FMI)

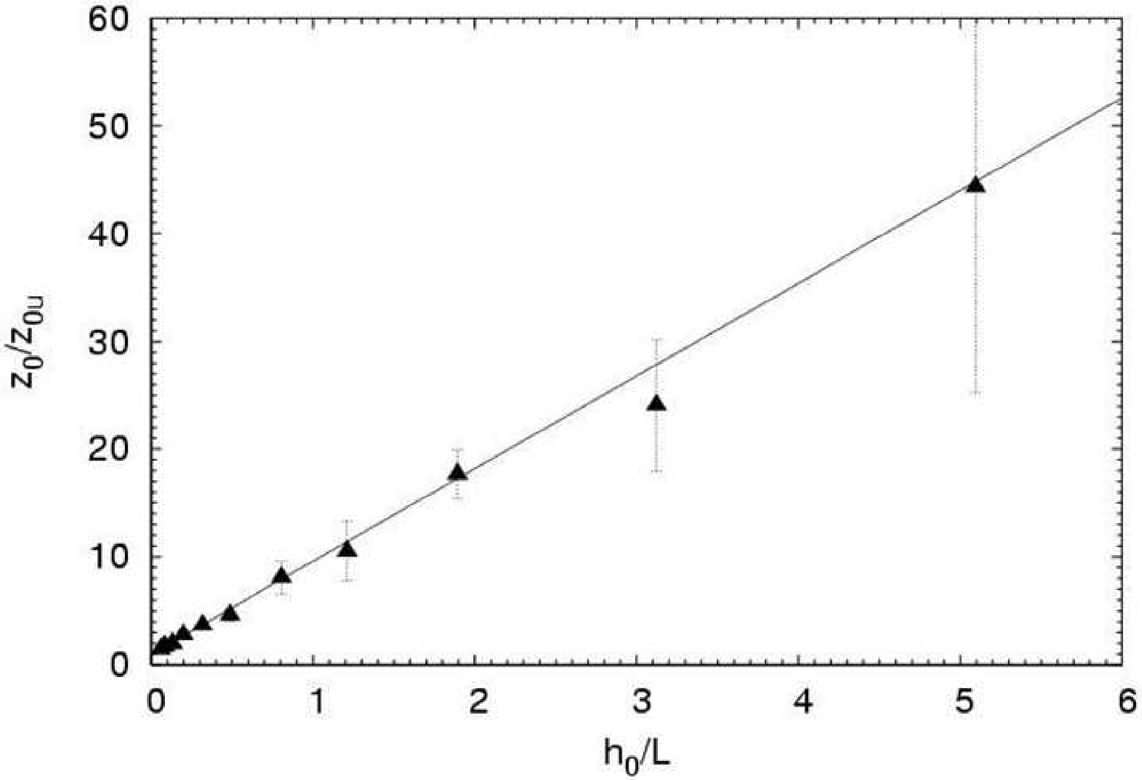
h ≈ 13 m, measurement levels 23, 25, 47 m



BUBBLE urban BL experiment, Basel, Sperrstrasse (Rotach et al., 2004)

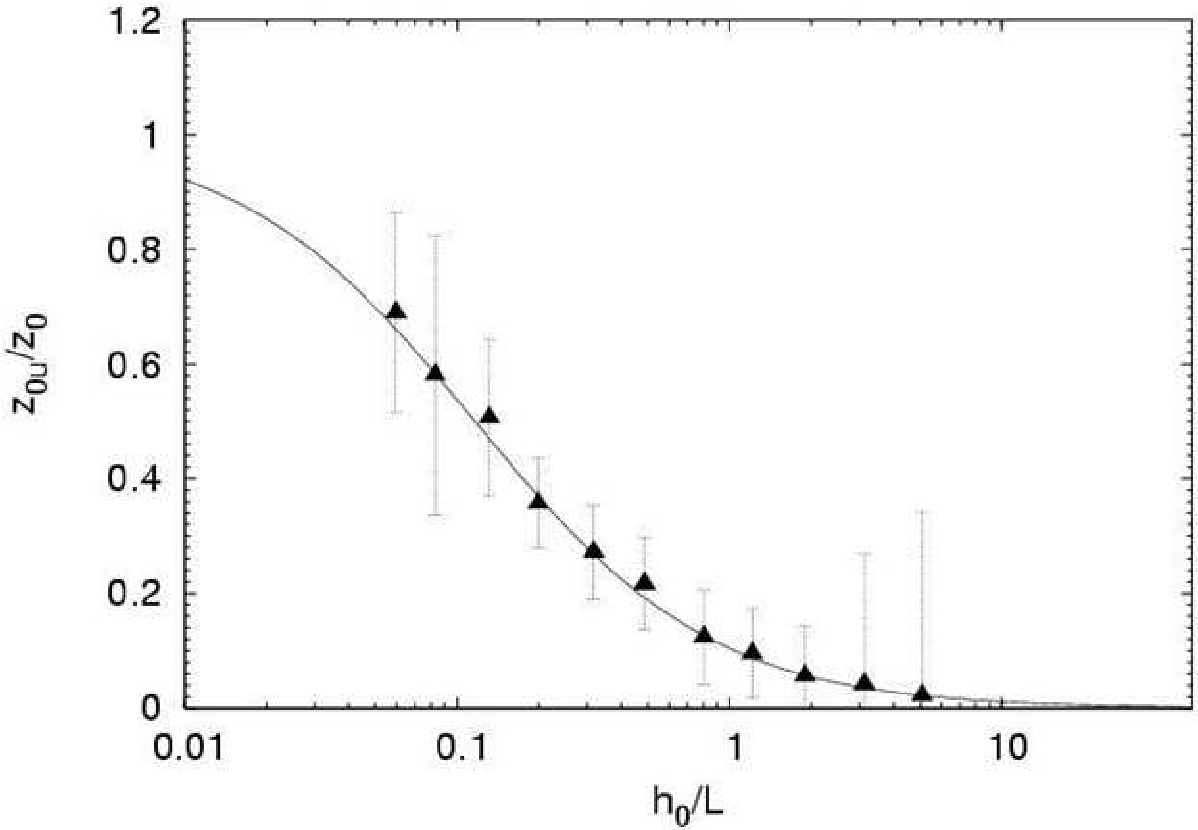
h ≈ 14.6 m, measurement levels 3.6, 11.3, 14.7, 17.9, 22.4, 31.7 m

Stable stratification



Bin-average values of z_0 / z_{0u} (neutral- over actual-roughness lengths) versus h_0/L in stable stratification for Boreal forest (h_0=13.5 m; z_0 =1.1±0.3 m). Bars are standard errors; the curve is $z_0 / z_{0u} = 1 + 8.13 h_0 / L$.

Stable stratification



Bin-average values of z_{0u}/z_0 (actual- over neutral-roughness lengths) versus h_0/L in stable stratification for boreal forest (h_0=13.5 m; z_0 =1.1±0.3 m). Bars are standard errors; the curve is $z_{0u}/z_0 = (1+8.13 h_0/L)^{-1}$.

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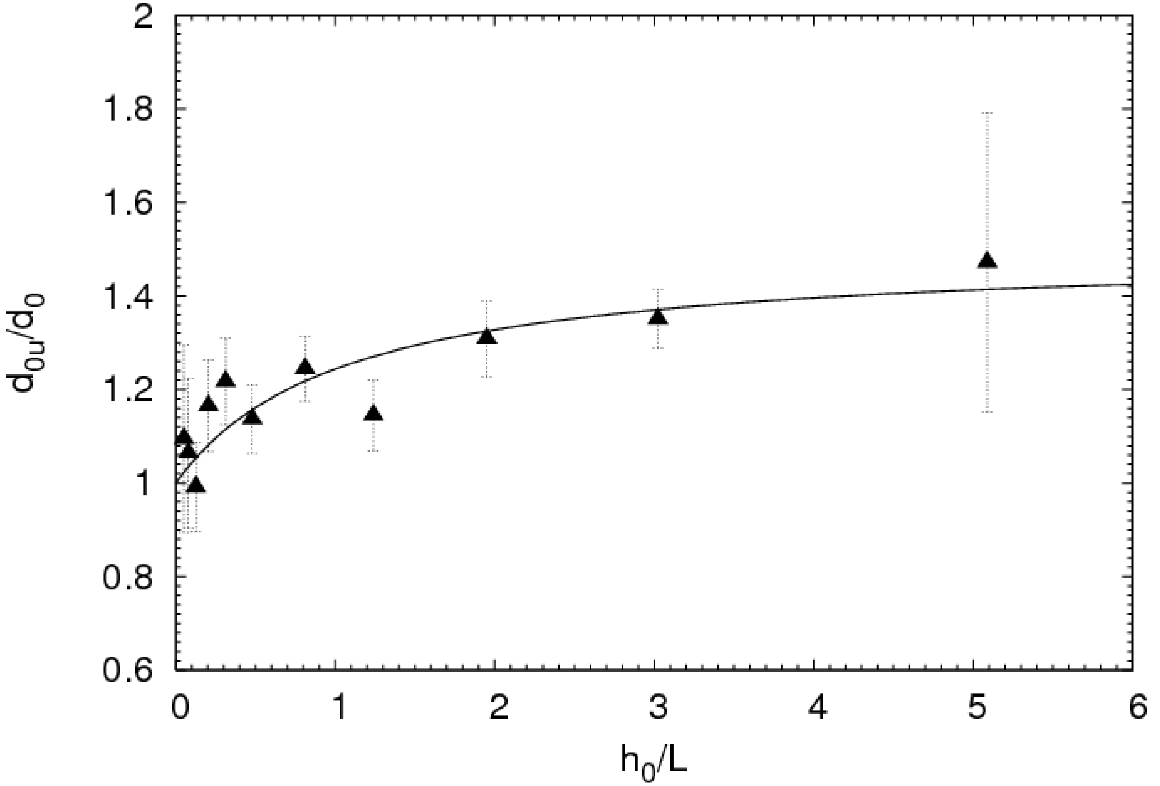
Stable stratification



Displacement height over its neutral-stability value in stable stratification. Boreal forest (h_0 = 15 m, d_0= 9.8 m).

The curve is $$d_{0u} / d_0 = 1 + 0.5(h_0 / L)(1.05 + h_0 / L)^{-1}$$

Unstable stratification

Convective eddies extend in the vertical causing $z_0 > z_{0u}$

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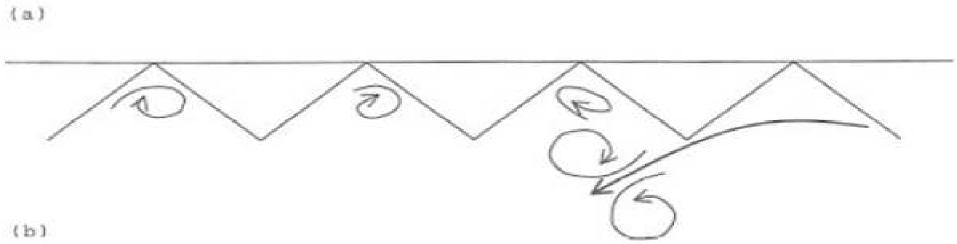
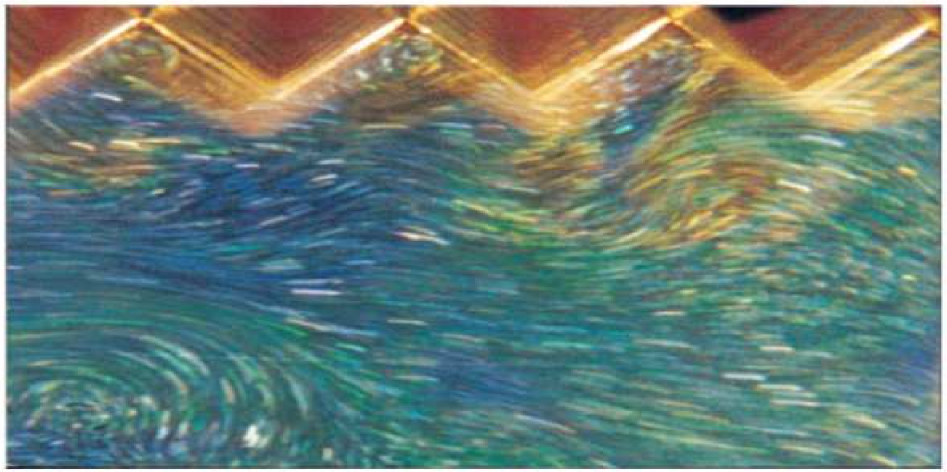
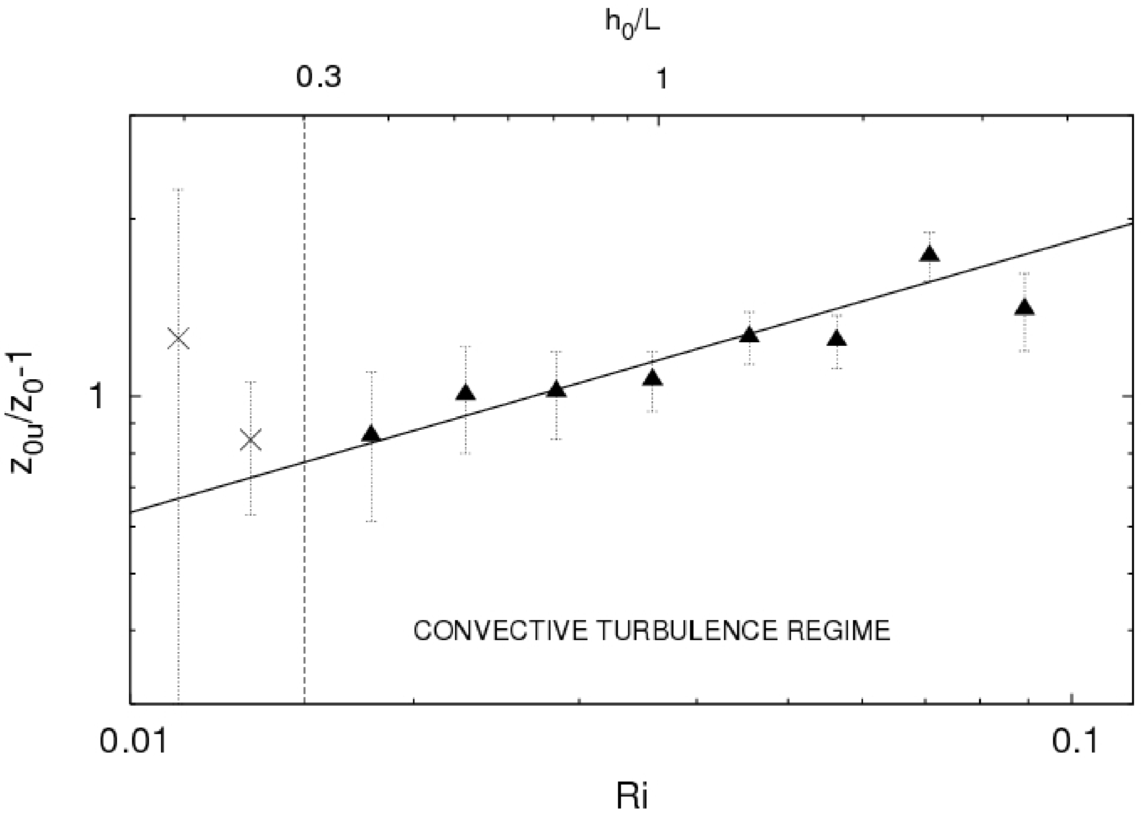
Y.-B. Du and P. Tong, Enhanced Heat Transport in Turbulent Convection over a Rough Surface



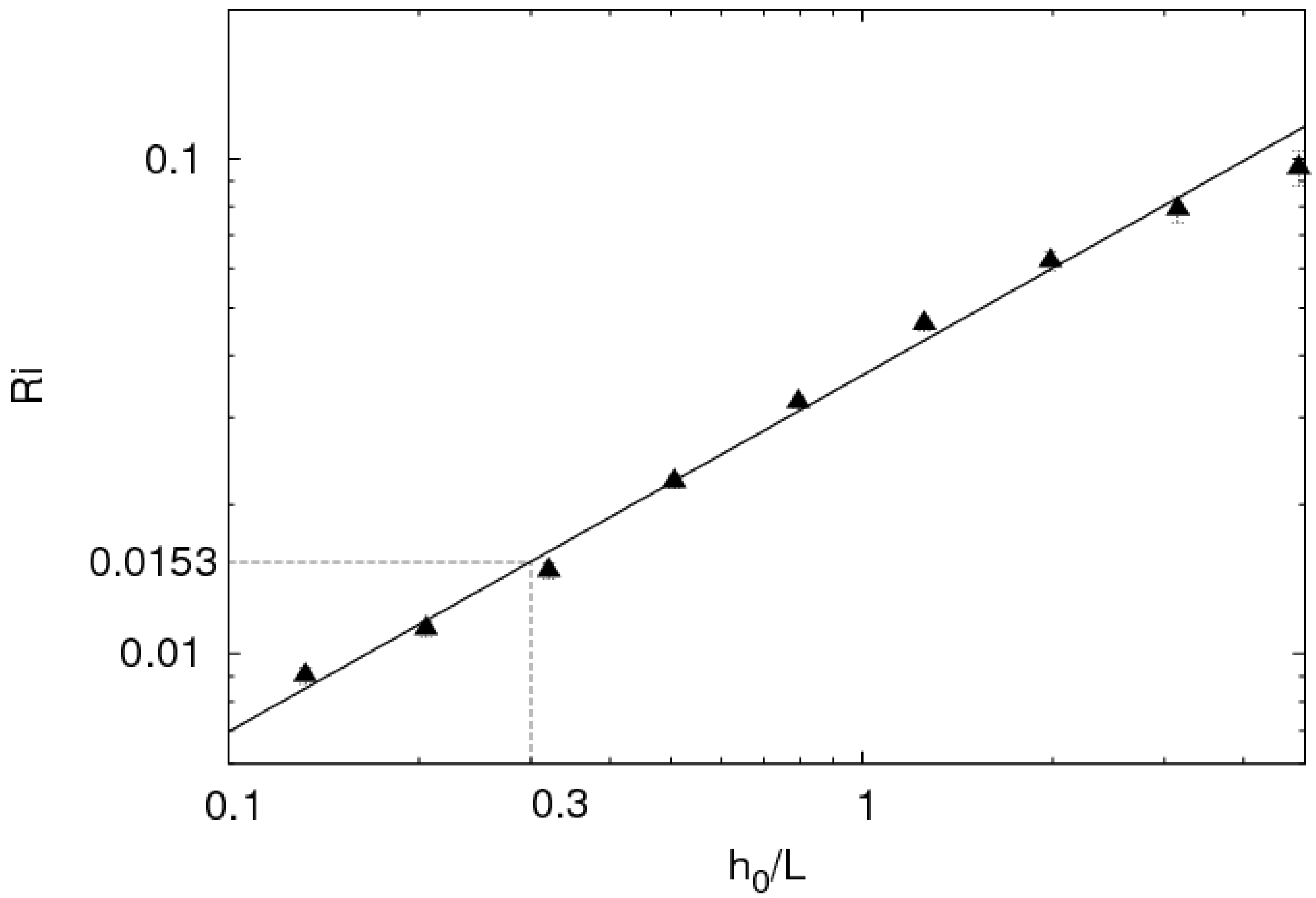


Unstable stratification



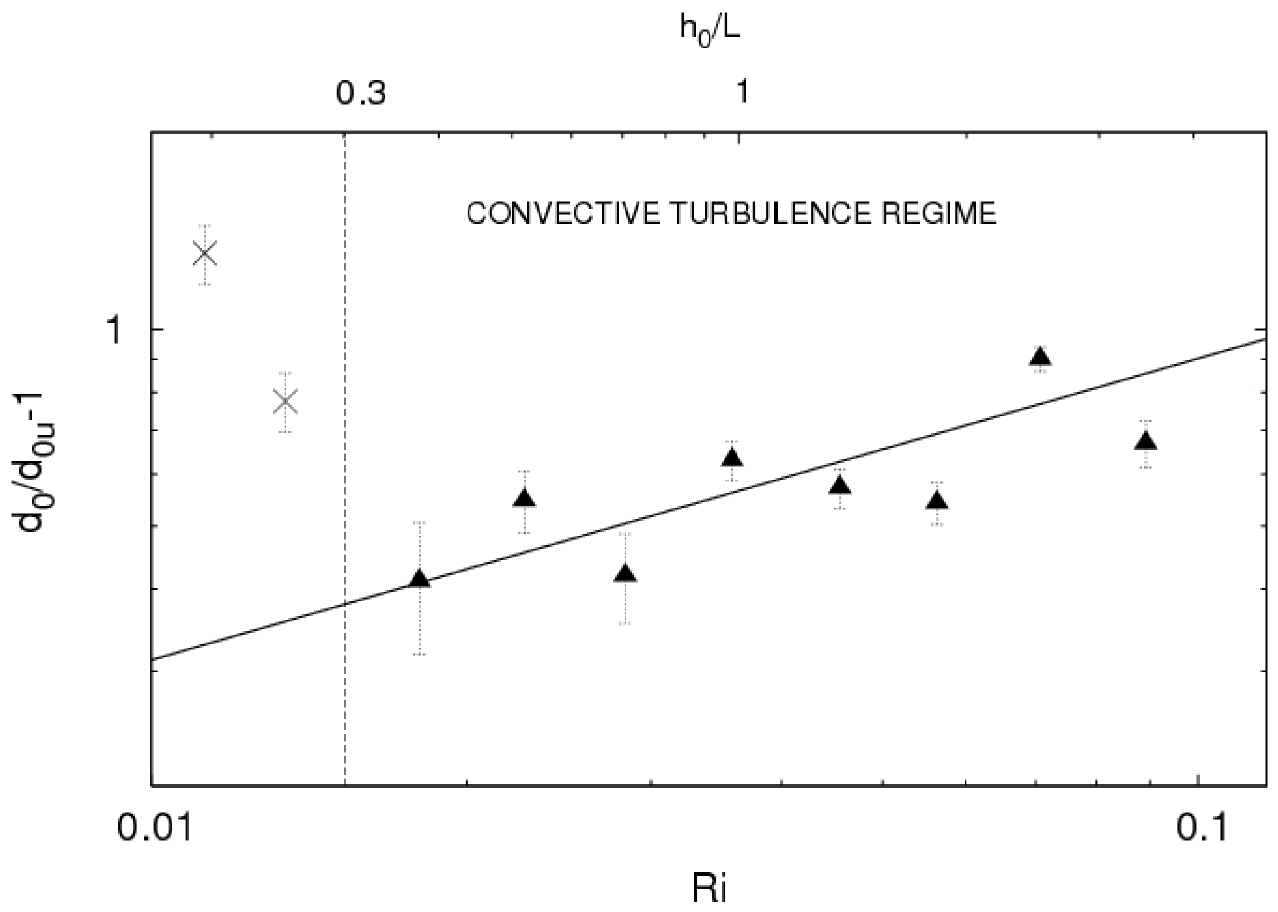
Unstable stratification, Basel, z_0/ z_{0u} vs. Ri = $(gh_0/\Theta_{32})(\Theta_{18}-\Theta_{32})/(U_{32})^2$
Building height =14.6 m, neutral roughness z_0 =1.2 m; BUBBLE, Rotach et al., 2005).
h_0/L through empirical dependence on Ri on (next figure)
The curve $(z_0/ z_{0u} = 1+5.31\mathrm{Ri}^{6/13})$ confirms theoretical $z_{0u}/z_0 = 1 + 1.15(h_0/\text{-}L)^{1/3}$

Unstable stratification



Empirical Ri = 0.0365 $(h_0/–L)^{13/18}$

Unstable stratification



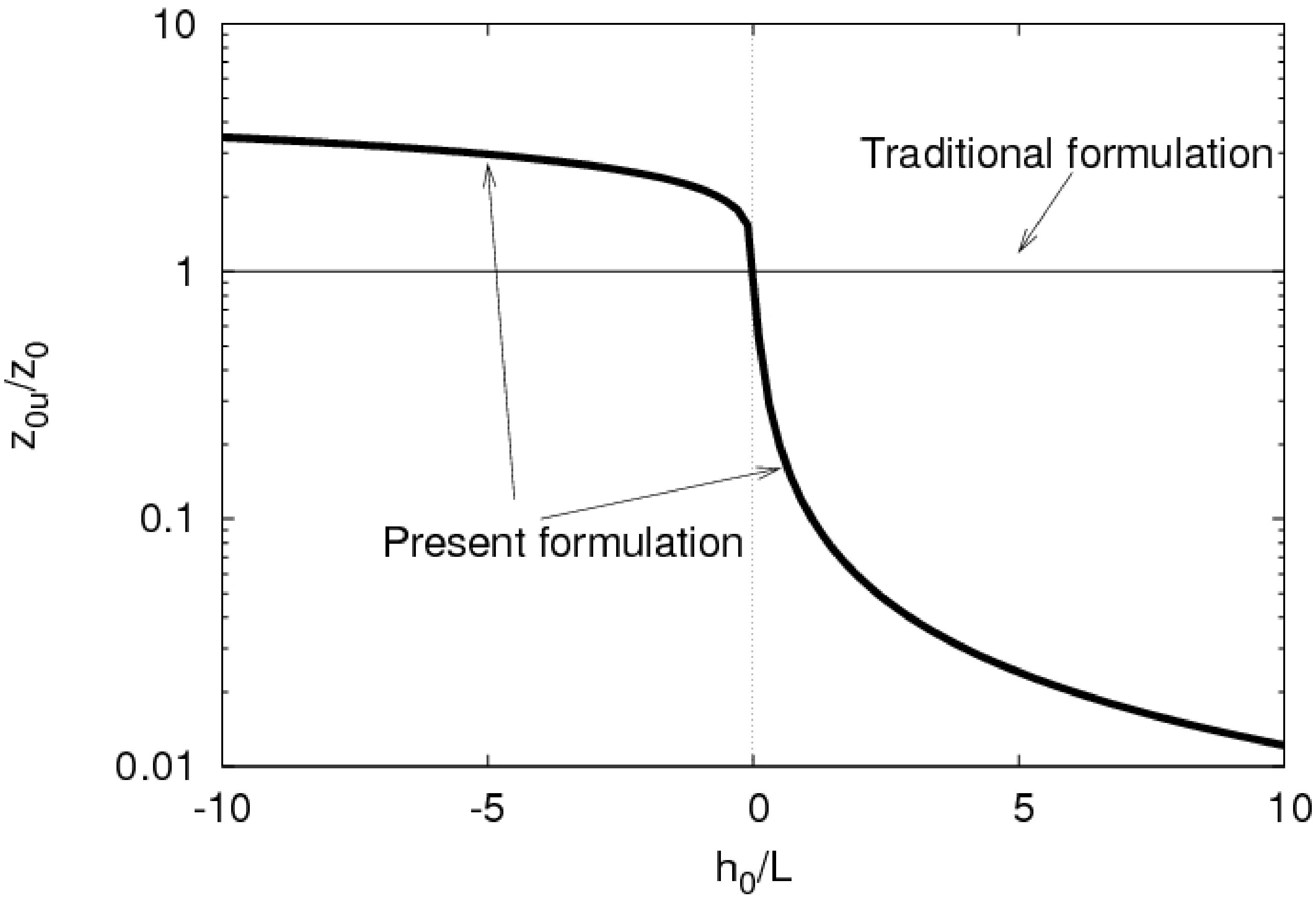
Displacement height in unstable stratification (Basel): $d_0 / d_{ou} - 1$ versus Ri

The line confirms theoretical dependence: $d_{0u} = \dfrac{d_0}{1 + C_{DC}(h_0 / -L)^{1/3}}$

STABILITY DEPENDENCE OF THE ROUGHNESS LENGTH

in the "meteorological interval" -10 < h_0/L <10 after new theory and experimental data

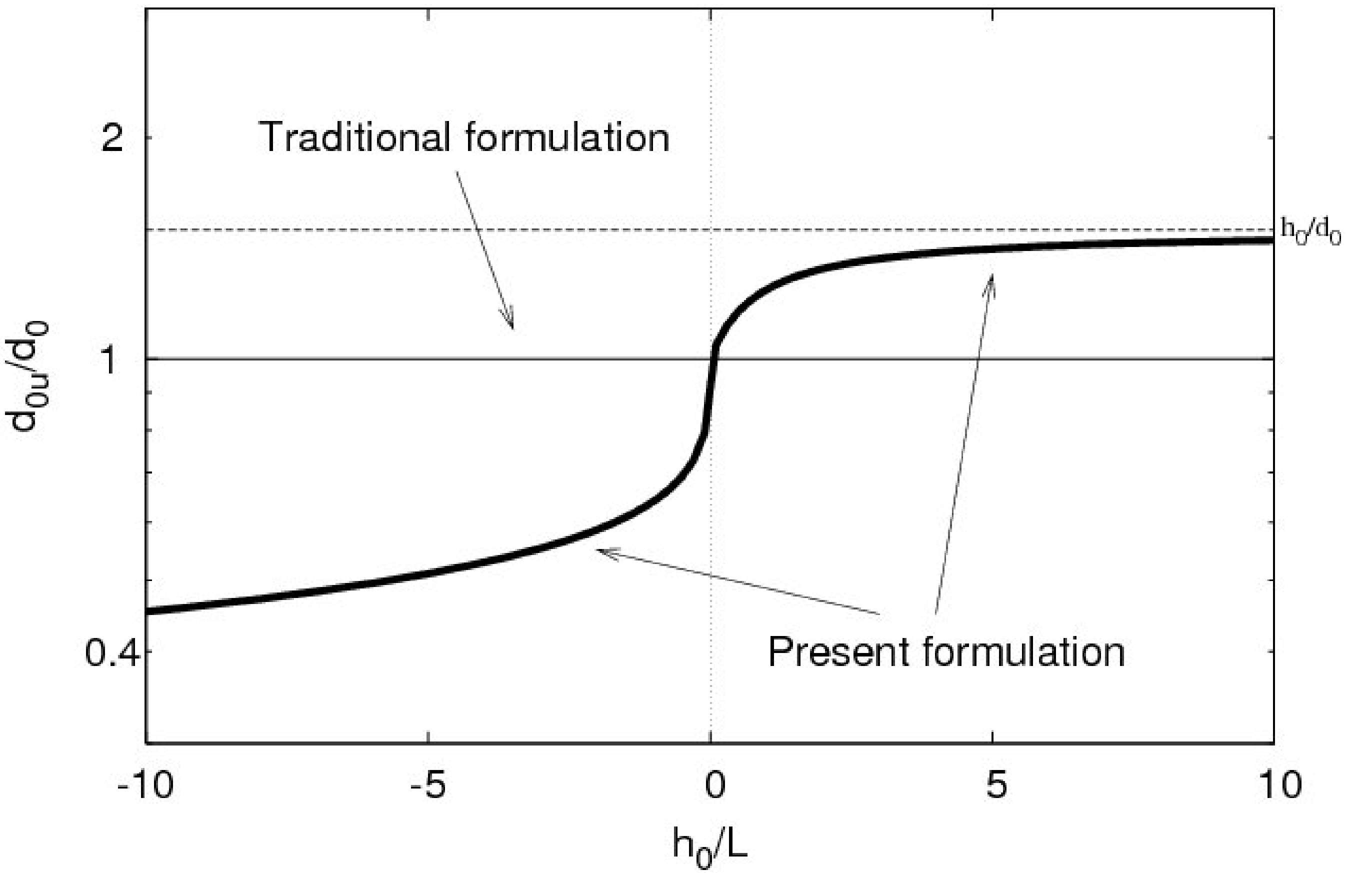
Solid line: z_{0u}/z_0 versus h_0/L **Thin line:** traditional formulation $z_{0u} = z_0$



STABILITY DEPENDENCE OF THE DISPLACEMENT HEIGHT

in the "meteorological interval" -10 < h_0/L <10 after new theory and experimental data

Solid line: d_{0u}/d_0 versus h_0/L **Dashed line:** the upper limit: $d_0 = h_0$



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Conclusions (Roughness length)

- **Traditional:** roughness length and displacement height fully characterised by geometric features of the surface

- **New:** essential dependence on hydrostatic stability especially strong in stable stratification

- **Applications:** to urban and terrestrial-ecosystem meteorology

- **Especially:** urban air pollution episodes in very stable stratification