About a consistent description of sub grid processes:

In what respect is the set of commonly used parameterization schemes inconsistent?

How can we achieve consistency?

Outline of the talk:

- Reasons of inconsistency are related to the principal characteristics of parameterizations
  - Parameterizations in terms of variable fields …
    - on the local scale
      - substituting source terms in primitive budget equations
    - on the scale of the numerical grid
      - substituting statistical moments in filtered budget equations
      - ➢ Closure strategies and the need of process separation
  - Inconsistency in current parameterization packages due to missing interaction

Cloud-microphysics ⟺ Radiation transport

SGS Turbulence ⟺ SGS Circulations

- Towards a consistent set of parameterizations schemes via scale separation
  - Separated turbulence including interaction with non-turbulent SGS processes (circulations)
    - Horizontal shear eddies
    - SSO wakes
    - Convection

The primitive equations:

advection flux density

molecular flux density

source term

$$\partial_t(\rho\phi_i)+\nabla\cdot\Big(\boxed{\rho\phi_i\,\underline{\mathbf{v}}}\;\underbrace{+\,\underline{\mathbf{e}}^{\phi_i}\Big)=\boxed{Q^{\phi_i}}}_{\text{linear or non-linear}}$$

$$\phi_i\in\{1,\overbrace{u,v,w}^{v_1,v_2,v_3},\overbrace{c_{p_d}T,q_k}^{\text{scalar variables}}\}\qquad p=\rho R_d\overbrace{\left[1+\left(\frac{R_v}{R_d}-1\right)q_v-q_c\right]T}^{T_v}$$

linear or **non-linear** functions in all model variables (including spatial derivatives) $\underline{\phi},p$

dependent on a list of general valid parameters $\underline{\alpha}$

simplified for efficiency reasons using effective parameters

local parameterizations:

- molecular flux densities
- phase changes sources (cloud microphysics)
- radiation flux convergence

Numerical scheme solves filtered equations:

– Filter may be a resolution dependent moving volume average

– Filter removes SGS variability

$\bar{\varsigma}$: filtered (mean) variable with fluctuation $\varsigma'=\varsigma-\bar{\varsigma}$

$\hat{\varsigma}=\dfrac{\overline{\rho\varsigma}}{\bar{\rho}}$: density weighted mean with fluctuation $\varsigma''=\varsigma-\hat{\varsigma}$

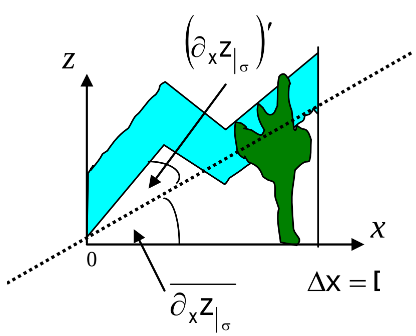
Non-linearity causes generation of statistical moments:

– Non-commutability of filter and (e.g.) multiplication or spatial differentiation

$$\overline{\rho\phi_i\,\underline{\mathbf{v}}}=\bar{\rho}\,\hat{\phi}_i\,\hat{\underline{\mathbf{v}}}+\underbrace{\overline{\rho\phi_i''\underline{\mathbf{v}}''}}_{\text{SGS covariance:}}\qquad\overline{\partial_j\varsigma}=\bar{\partial}_j\bar{\varsigma}+\underbrace{\overline{\partial_j'\varsigma'}}_{\text{roughness layer terms:}}$$

SGS covariance:

roughness layer terms:



contribution by SGS slopes of model layers or non-atmospheric intersections

The filtered model equations:

SGS mixing

SGS conversion

$$\partial_t\left(\bar\rho\hat\phi_i\right)+\bar\nabla\cdot\Big(\overbrace{\bar\rho\hat\phi_i\underline{\hat{\mathbf{v}}}}^{\text{GS flux density}}+\underbrace{\overline{\rho\phi_i''\underline{\mathbf{v}''}}}_{\text{SGS flux density}}\overbrace{-k^{\phi_i}\nabla\hat\phi_i}^{\text{molecular flux density}}\Big)+\underbrace{\overline{\nabla'\cdot\left(\rho\phi_i\underline{\mathbf{v}}-k^{\phi_i}\nabla\phi_i\right)'}}_{\text{roughness layer modification of transport}}=\overbrace{Q^{\phi_i}_{\underline{\lambda}}\left(\underline{\hat\phi},\bar p\right)}^{\text{GS source term}}+\underbrace{\boxed{\overline{Q^{\phi_i}_{\underline{\lambda}}{}'\left(\underline{\phi''},p'\right)}}}_{\text{SGS source term}}$$

non-linear function

SGS source term including roughness layer effects (form drag)

$$\overline{Q^{v_i}_{\underline{\lambda}}{}'\left(\underline{\phi''},p'\right)}=-\overline{\partial_i'p'}$$ form drag

$$\overline{Q^{q_k}_{\underline{\lambda}}{}'\left(\underline{\phi''},p'\right)}$$ SGS contribution by cloud microphysics

$$\overline{Q^{T}_{\underline{\lambda}}{}'\left(\underline{\phi''},p'\right)}$$ SGS contribution by cloud microphysics and radiation

functions in various covariance terms of scalar variables

$$\bar p=\bar\rho R_d\left[1+\left(\frac{R_v}{R_d}-1\right)\overline{q_v}-\overline{q_c}\right]\bar T+R_d\left[1+\left(\frac{R_v}{R_d}-1\right)\overbrace{\overline{\rho q_v''T''}-\overline{\rho q_c''T''}}\right]$$

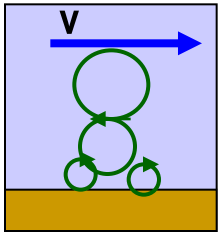
Parameterizations in terms of grid scale (GS) variables :

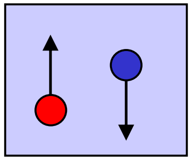
- **Further information** (**assumptions**) about these **additional covariance terms** has to be introduced:

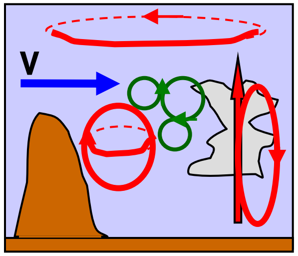
functions in the GS model variables $\overline{\rho}, \underline{\hat{\phi}}, \overline{p}$
dependent on a list of additional parameters $\underline{\beta}$

GS parameterizations due to
– ***SGS variability***

- **Closure assumptions** are **additional constraints** that **can't be general valid**
  - ➢ distinguish **different SGS flow structures** more or less according to their **length scales coherence**
  - ➢ Each with **significant mixing potential** (waves so far excluded) and **specific closure assumptions**







SGS Turbulence: **isotropic**, **normal distributed**, only **one** characteristic **length scale** at each grid point, forced by **shear and buoyancy**

SGS Circulation: **non isotropic**, arbitrarily **skewed** and **coherent** structures of **several length scales,** supplied by **various pressure forces**

such as:

Convection:	large vertical scales of coherence, full microphysics, forced by buoyancy feed back
Kata- and anabatic density circulations:	direct thermal circulation forced by lateral cooling or heating by sloped surfaces of the earth; dominated by SGS surface structures like SSO
Horizontal shear eddies:	produced by strong horizontal shear e.g. at frontal zones; dominated by horizontal grid scale
Wake eddies:	produced by blocking at SGS surface structures (form drag forces)
Breaking gravity wave eddies:	belong to wave length of instable gravity waves of arbitrary scales

Closure strategies:

- Describing the covariance terms within **different frameworks** all based on first principals

- Introduction of **closure assumptions** by application of a related **truncation procedure**

- Finding a **flow structure separation** according to the validity of closure assumptions

- Setting up a **consistently separated set of parameterization schemes** being to some extend **general valid**

- **Two different frameworks** available:

  – **Higher order closure (HOC):** Using **budget equations** for needed **statistical moments** (that always contain new ones, even such of higher orders) and **truncating the order of considered moments**

    ➢ **Second order closure:** fits very well to **turbulence**

  – **Conditional domain closure (CDC):** Using **budget equations** for **conditional averages of model variables** (e.g. according to **classes of vertical velocity**) and building the needed covariance terms by the related **truncated statistics**

    ➢ **Mass flux closure** (bi- or tri-modal distribution): fits very well to **convection**

2-nd order budgets:

$\phi, \psi \cdots \in \{\phi_1, \phi_2, \cdots\}$ **prognostic model variables**

influenced by pressure force, microphysics and radiation

$$\partial_t\left(\overline{\rho\phi''\psi''}\right) = -\overline{\nabla}\cdot\left(\overline{\rho\phi''\psi''}\,\hat{\underline{\mathbf{v}}} + \overline{\rho\phi''\psi''\underline{\mathbf{v}}''}\right) - \left(\overline{\rho\psi''\underline{\mathbf{v}}''}\cdot\overline{\nabla}\hat{\phi} + \overline{\rho\phi''\underline{\mathbf{v}}''}\cdot\overline{\nabla}\hat{\psi}\right) + \left(\overline{\mathbf{e}^{\psi}\cdot\nabla\phi''} + \overline{\mathbf{e}^{\phi}\cdot\nabla\psi''}\right) + \boxed{\left(\overline{\phi''Q^{\psi}} + \overline{\psi''Q^{\phi}}\right)}$$

GS flux density **SGS flux density** **shear production** **dissipation sink** **source term correlation**

$$+\cdots$$

to be parameterized

laminar transport

roughness layer modification of transport

Conditional domain budgets:

$$\overline{\zeta}\big|_G(\mathbf{r},t) := \frac{1}{|G(\mathbf{r},t)|}\int_{\mathbf{s}\in G(\mathbf{r},t)} \zeta(\mathbf{s},t)\, d^3s$$

conditional average (representing e.g. the convective updraft or downdraft

G_+ G_-

- budget for a conditional averaged property:

$$\partial_t\left(a\bar{\rho}\hat{\phi}\right) + \overline{\nabla}\cdot\left(\overline{a\rho\phi\underline{\mathbf{v}}^{\phi}}\right) = a\cdot\left(Q^{\phi}_{sur} + \boxed{\overline{Q^{\phi}}}\right)$$

$$Q^{\phi}_{sur} := -\frac{1}{|G|}\int_{\mathbf{s}\in B} \rho\phi\cdot\left(\mathbf{v}^{\phi} - \partial_t \mathbf{s}\right)\cdot\mathbf{n}\, d^2s$$

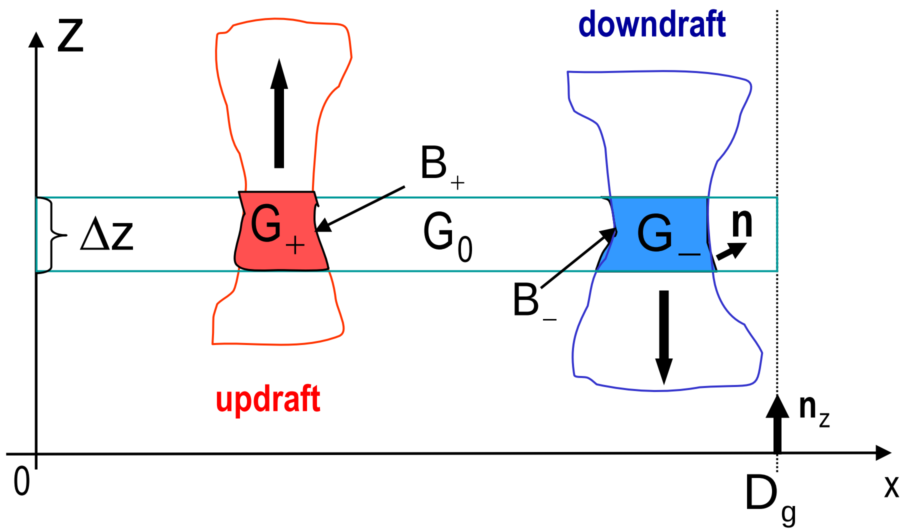
inflow via the inner boundary surface

volume fraction of the related subdomain

- continuity equation:

$$\partial_t \ln a = \frac{1}{|G|}\int_{\mathbf{s}\in B} \partial_t \mathbf{s}\cdot\mathbf{n}\, d^2s = \frac{|B|}{|G|}\,\overline{\partial_t s_n}\big|_B$$

Inner boundary surface of the subdomain



Equations to be solved under simplifying assumptions

– stationarity, same horizontal advection for each subdomain, …

Interactions to be considered:

I.

Increased set of diagnostic or prognostic variables

Mass fraction and number concentration of
- cloud constituents,
- precipitation and
- passive tracers

Cloud-microphysics	Radiation transport

Local parameterizations: Parameterizations of source terms

II. ⟹ integrated in

GS parameterizations: Parameterizations of SGS processes

III. Separation ⟹ Turbulence ⟺ (interaction) Circulations

STIC

UTCS :

- all within a single HOC framework without separation
- ➢ Only feasible, if circulations are in common with turbulence approximations: CONTRADICTION!!

Current realization:

I. Some coupling between local parameterizations is missing

– Radiation does not consider all cloud- and precipitation constituents

II. Some SGS contributions of source terms in <u>1-st order budgets</u> as well as in the <u>budgets for SGS motions</u> are only considered partly or inconsistently (for radiation not at all)

SGS contributions of cloud microphysics	in budgets of SGS motions	in 1-st order budgets (directly)
due to turbulence (2-nd order equations)	only statistical saturation adjustment by using conservative variables	not at all
due to convection (mass flux equations)	specific formulation including precipitation	by convective source term tendencies (e.g. convective precipitation)

III. We apply parameterizations of effects on 1-st order budgets due to <u>different processes</u> (turbulence, convection, SSO wakes) <u>without using a clear separation procedure</u>

– Grid scale parameterizations are so far formulated as if they are independent form each other (e.g.: turbulence does not "feel" that convection is present and vice versa)

➢ **Problems with incomplete description: double counting, non realizability, unrealistic or contradicting results**

Principle of scale separation (in order to solve problem III.):

❖ **Should provide the missing interaction between turbulence and circulations automatically**

- **Assume that turbulence approximations can be assigned to all horizontal scales not smaller than a maximal turbulent length scale L_p (mainly dependent on the distance from the surface of the earth)**

- **Spectral separation** by
  - considering **budgets** with respect to the **separation scale** $L = min\{L_p, D_g\}$
  - **averaging** these budgets along the **whole control volume** (**double averaging**)

➢ **1-st order budgets with SGS contributions form turbulence and circulations**

$$\overline{\rho\phi\psi} = \bar{\rho}\hat{\phi}\hat{\psi} + \overline{\overline{\rho\phi''\psi''\big|_L}} + \overline{\bar{\rho}\big|_L \hat{\phi}\big|_L{}'' \hat{\psi}\big|_L{}''}$$

$\big|_L$: with respect to the separation scale L

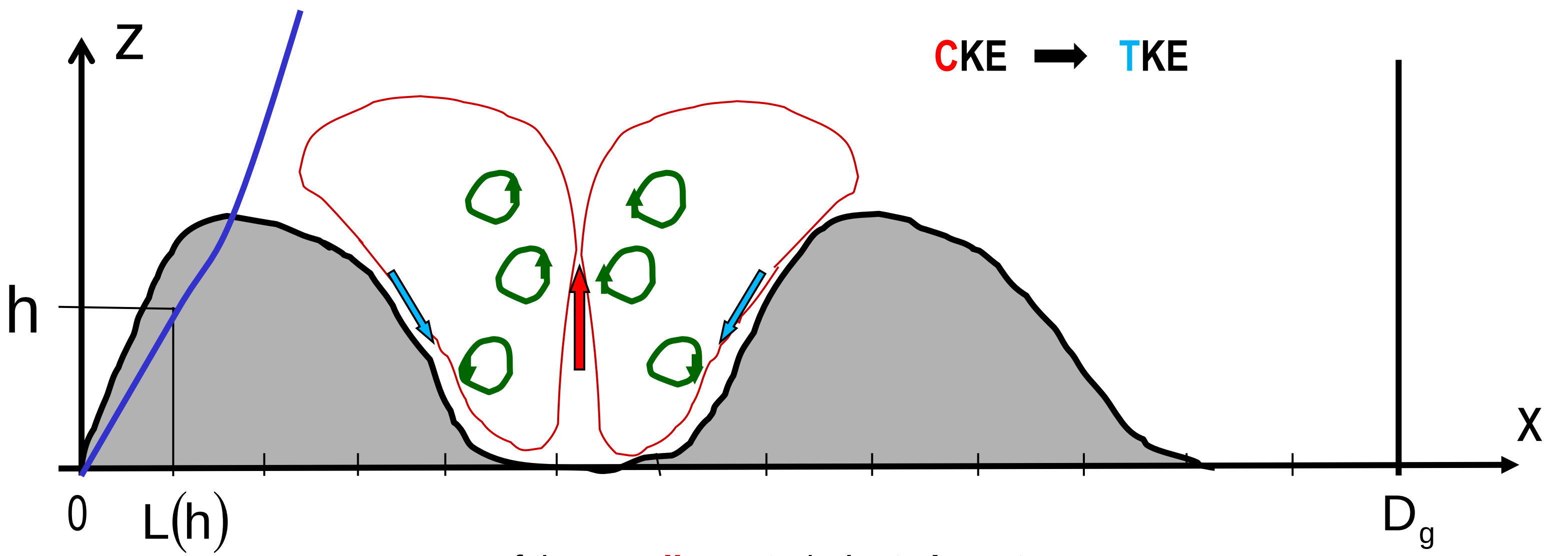
➢ **Two sets of 2-nd order equations containing additional scale interaction terms**

one set for pure turbulence and another for pure circulations

⇕

Mass flux equations describing initial conditions and lateral mixing of cells using properties of turbulence

Additional circulation terms in the turbulent 2-nd order budgets:



average of the **non linear** turbulent **shear terms**

$$D_t\left(\overline{\rho\overline{\phi''\psi''}|_L}\right) = \cdots \left[\underbrace{\boxed{-\overline{\rho\overline{\phi''\mathbf{v}''}|_L \cdot \left(\overline{\nabla}\hat{\psi}\right)|_L}}} \quad + \quad \underbrace{\boxed{-\overline{\rho\overline{\psi''\mathbf{v}''}|_L \cdot \left(\overline{\nabla}\hat{\phi}\right)|_L}}} \right] + \cdots$$

$|_L$: with respect to the separation scale L

$$\underbrace{-\overline{\rho\overline{\phi''\mathbf{v}''}|_L} \cdot \overline{\nabla}\hat{\psi}}_{\substack{\textbf{turbulent}\\ \textbf{shear term}}} \quad \underbrace{\boxed{-\overline{\rho\overline{\phi''\mathbf{v}''}|_L{}' \cdot \left(\overline{\nabla}\hat{\psi}\right)|_L{}'} - \overline{\rho\overline{\psi''\mathbf{v}''}|_L{}' \cdot \left(\overline{\nabla}\hat{\phi}\right)|_L{}'}}}_{\substack{Q_C^{\phi\psi}\\ \textbf{circulation}\\ \textbf{shear term}}} \quad \underbrace{-\overline{\rho\overline{\psi''\mathbf{v}''}|_L} \cdot \overline{\nabla}\hat{\phi}}_{\substack{\textbf{turbulent}\\ \textbf{shear term}}}$$

Separated TKE equation

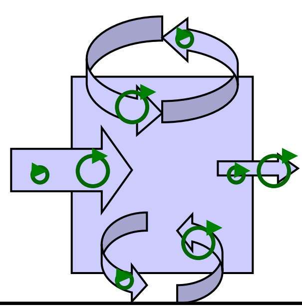
- **Semi-parameterized (neglecting laminar transport and roughness layer modification of transport)**

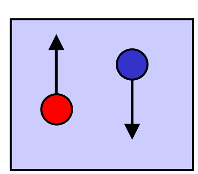
$$2 \cdot \mathrm{TKE} := q_{|L}{}^2 := \frac{1}{\overline{\rho}} \sum_{i=1}^{3} \overline{\rho v_i''^2}\Big|_L$$

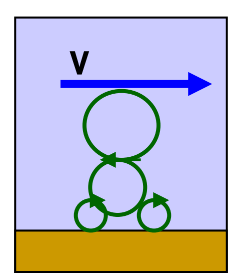
$|_L$: with respect to the separation scale L

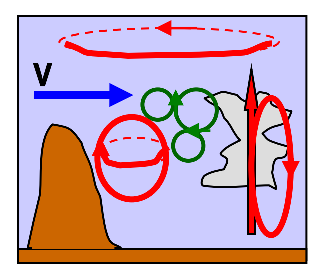
$$\partial_t \left(\frac{1}{2} \overline{\rho} \cdot q_{|L}{}^2 \right) = \frac{1}{2} \overline{\nabla} \cdot \left(\begin{array}{c} \overline{\rho}\, q^2_{|L}\, \underline{\hat{\mathbf{v}}} \\ + \sum_{i=1}^{3} \overline{\left(\rho v_i''^2 \underline{\mathbf{v}''} \right)}_{|L} \end{array} \right) + \frac{g}{\hat{\theta}_v} \overline{\overline{\rho \theta_v'' w''}\Big|_L} + \left[- \sum_{i=1}^{3} \overline{\rho v_i'' \underline{\mathbf{v}''}}\Big|_L \cdot \overline{\nabla} \hat{v}_i \right] + \left[- \sum_{i=1}^{3} \overline{\overline{\rho v_i'' \mathbf{v}''}\Big|_L{}' \cdot \left(\overline{\nabla} \hat{v}_i \right)_{|L}{}'} \right] + \left[- \overline{\rho} \frac{q_{|L}{}^3}{\alpha^{MM} \ell} \right]$$

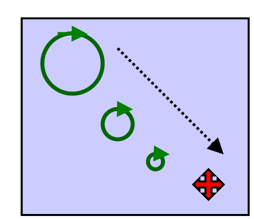
time tendency	transport (advection diffusion)	buoyancy production	shear production by the mean flow	shear production by sub grid scale circulations	eddy-dissipation rate (**EDR**)
		labil: > 0 neutral: $= 0$ stabil: < 0	$\boxed{\geq 0}$	$\boxed{\geq 0}$	$\boxed{< 0}$











TKE-production by separated horizontal shear modes:

- **Separated horizontal shear production term:**

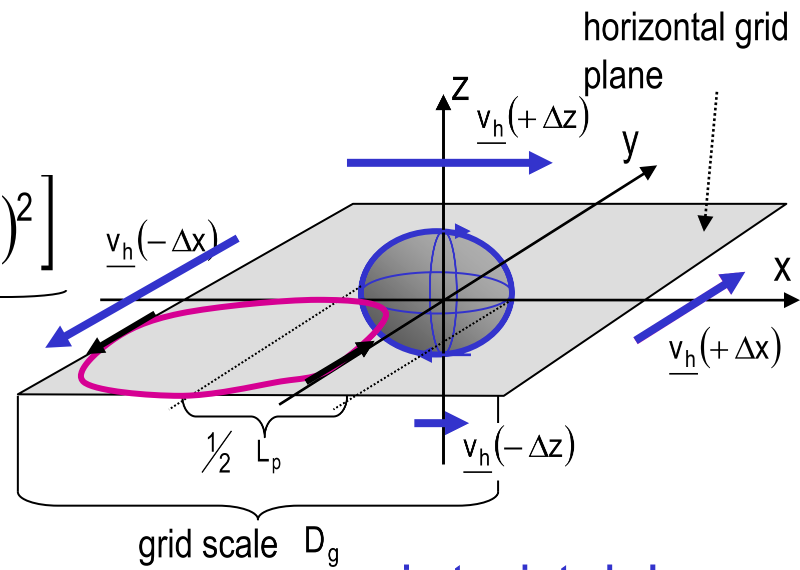
$$Q^{v\cdot v}_{C\_SHS} := q_H \cdot \underbrace{\beta_H D_g}_{} \cdot \underbrace{\left[\left(\partial_1 \bar{v}_2 + \partial_2 \bar{v}_1\right)^2 + 2\left(\partial_1 \bar{v}_1\right)^2 + \left(\partial_2 \bar{v}_2\right)^2 \right]}_{=:\, F_H^M}$$

separated horizontal shear

effective mixing length of diffusion by horizontal shear eddies

velocity scale of the separated horizontal shear mode

$\beta_H < 1$ **scaling parameter**



isotropic turbulence

horizontal shear eddy

- **Equilibrium of production and scale transfer towards turbulence:**

$$q_H \cdot \beta_H D_g \cdot F_H^M = \frac{q_H^3}{\alpha_H D_g}$$

$\alpha_H < 1$ **scaling parameter**

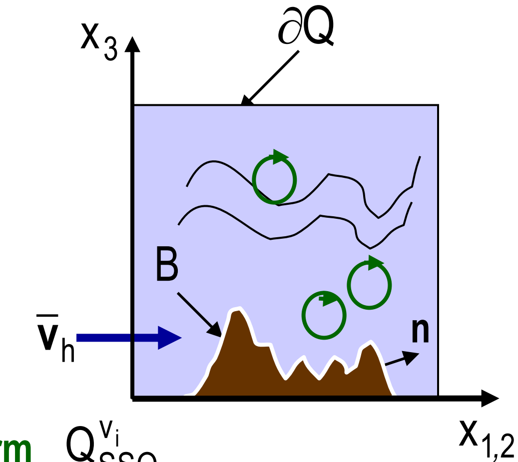
$$\Longrightarrow Q^{v\cdot v}_{C\_SHS} = q_H \beta_H D_g \cdot F_H^M = \underbrace{\alpha_H^{\frac{1}{2}} \beta_H^{\frac{3}{2}}}_{=:\, \alpha_S^2} D_g^2 \cdot F_H^{M\,\frac{3}{2}}$$

additional TKE source term

$=: \alpha_S^2$ ....**effective scaling parameter**

- **Already used for EDR forecast ;** **to be tuned and verified for operational use**

<u>TKE-production by separated wake modes due to SSO:</u>



- **SSO-term in filtered <u>momentum budget</u>**:

$$\partial_t\left(\bar{\rho}\,\hat{v}_i\right) = -\bar{\nabla}\cdot\left[\bar{\rho}\left(\hat{v}_i\hat{\mathbf{v}} + \overline{\rho v_i''\underline{\mathbf{v}''}} - \upsilon\nabla\hat{v}_i\right)\right] - \bar{\rho}g \quad \underbrace{-\,\partial_i\bar{p} \;\; -\,\overline{\partial_i' p'}}_{\text{blocking term}} \quad Q_{SSO}^{v_i}$$

blocking term $Q_{SSO}^{v_i}$

currently Lott und Miller (1997)

- **Pressure term in <u>kinetic energy budget</u>**:

$$-\overline{\mathbf{v}\cdot\nabla p} \;=\; -\bar{\nabla}\cdot\left(\bar{p}\hat{\mathbf{v}}\right) \qquad \overbrace{\qquad\qquad\qquad + \bar{p}\,\bar{\nabla}\cdot\hat{\mathbf{v}}}^{\text{from inner energy}} \qquad \boxed{-\,\hat{\mathbf{v}}\cdot\overline{\nabla' p'}}$$

$$-\bar{\nabla}\cdot\overline{\mathbf{v}p'} \qquad -\overline{\mathbf{v}''}\cdot\bar{\nabla}\bar{p} \qquad +\overline{p'\nabla\cdot\mathbf{v}} \qquad \boxed{+\,\hat{\mathbf{v}}\cdot\overline{\nabla' p'}}$$

$$-\overline{\nabla'\cdot\left(\mathbf{v}p\right)'} \qquad\qquad\qquad\qquad + \bar{p}\,\overline{\nabla'\cdot\mathbf{v}'}$$

pressure transport | buoyancy production | expansion production | **wake source**

sources of **mean** kinetic energy MKE $Q_{C\_SSO}^{\mathbf{v}\cdot\mathbf{v}}$

sources of **sub** grid scale kinetic energy SKE being a **scale transfer term** towards TKE

- **Contribution taken form SSO scheme :** **already operational**

TKE-Production by thermal circulations:

- **Circulation scale 2-nd order budgets with proper approximations valid for thermals**:

→ **circulation scale temperature variance ~ circulation scale buoyant heat flux ≈ TKE source term**

separated thermals

vertical velocity scale of circulation

virtual potential temperature of ascending air

$$\overline{\overline{\rho}|_L \, \hat{\theta}_v|_L''^2} \propto Q^{v\cdot v}_{C\_STH} \approx \frac{g}{\hat{\theta}_v} \cdot \overline{\overline{\rho}|_L \, \hat{w}|_L'' \, \hat{\theta}_v|_L''} \approx \frac{g}{\hat{\theta}_v} \cdot \overline{\rho} \, w_C \cdot \left(\theta_v^+ - \theta_v^-\right) \geq 0$$

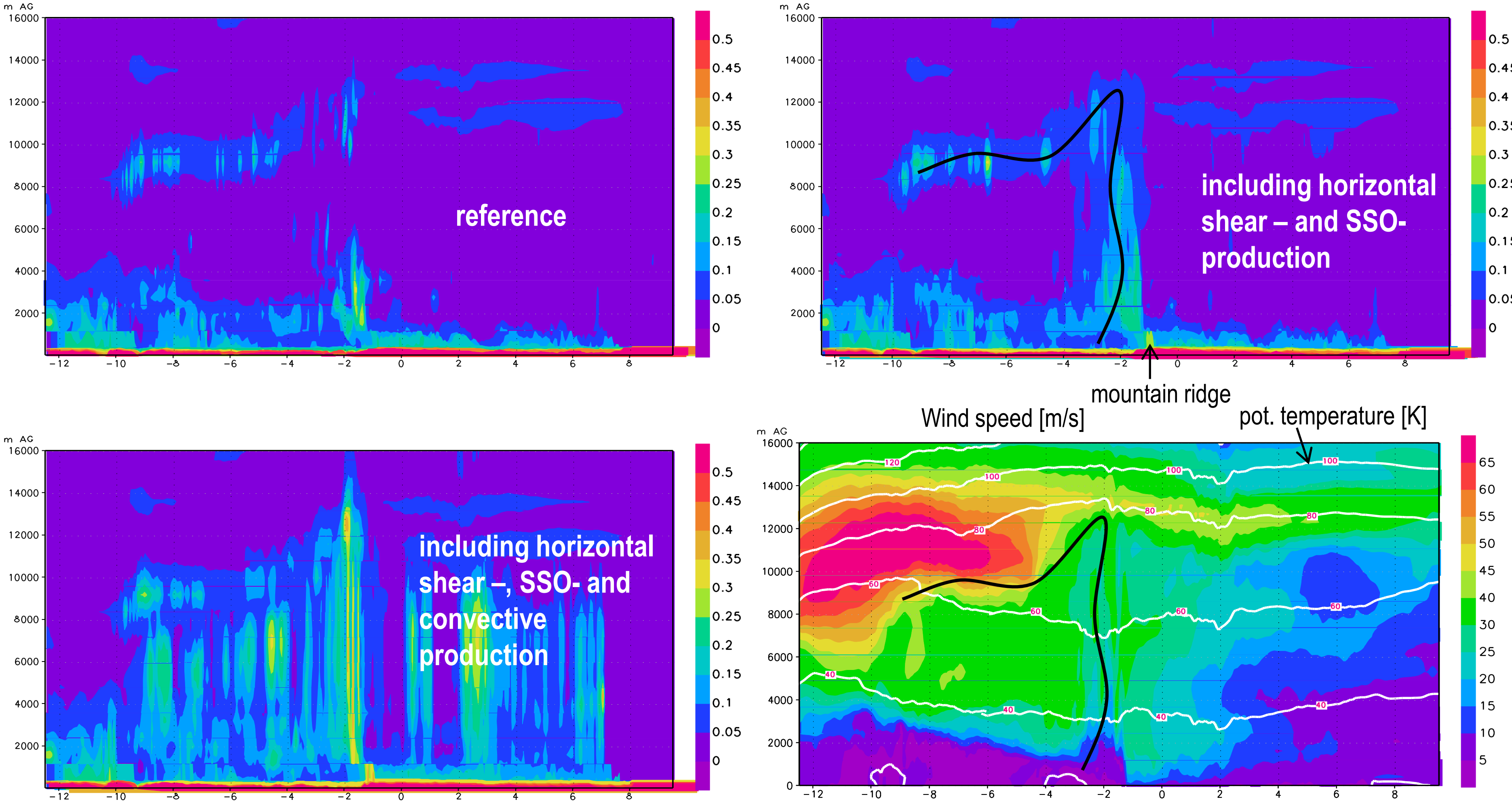
virtual potential temperature of descending air

buoyant production of sub grid scale kinetic energy can be derived directly form current **mass flux convection scheme**

- Two contributions:

  - **one taken form convection scheme: already used for EDR forecast ; to be verified**
  - **one being a <u>crude estimate</u> of surface induced density flows: active since years; to be revised**

pow_1/3 (eddy dissipation rate (EDR) [m^2/s^3])



COSMO-US: cross section across frontal line and Appalachian mountains

st_time=00z01may2010 pr_hour=18hr – 19hr

Conclusion:

- **Main sources of inconsistencies: Missing interaction**
  - **I. among local parameterizations**
  - **II. between local and grid scale parameterizations**
  - **III. among grid scale parameterizations**

- **A prerequisite for consistency of closure schemes: scale separation**
  - Provides a **consistent overlap** between flow structures, for which **incompatible closure assumptions** are valid
  - **Separation of turbulence** by a sub-filter only smoothing "turbulence" provides variance equations really valid for turbulence
  - They automatically contain **shear production terms by non-turbulent sub-gird processes** (**scale transfer terms**)
    - **Turbulent fluxes** remain in **flux gradient form**, those by **non-turbulent flow structures do not**.

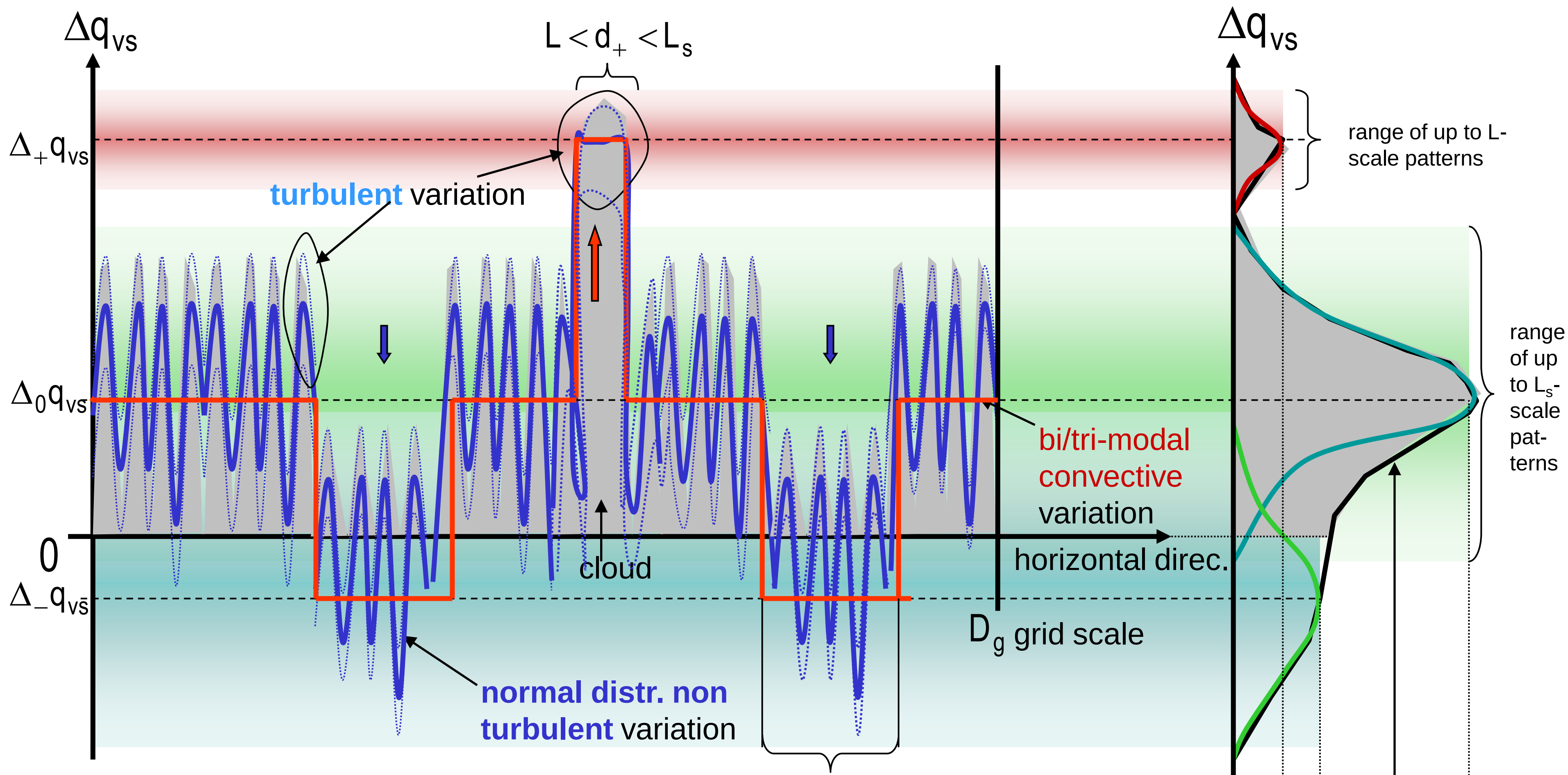
- **Already (partly) implemented TKE-production by scale transfer from kinetic energy of …**

- **wakes generated** by surface inhomogeneity (from SSO-blocking scheme)		already operational
- **thermal circulation** by surface inhomogeneity (due to differential heating/cooling)		only crude approximation
- **horizontal eddies** generated by horizontal shear (e.g. at frontal zones)	**already used for EDR forecast**	not yet verified
- **Convection circulation** (buoyant production from convection scheme)		not yet verified

- **Still missing are scale adaptive formulations of the circulation parameterizations!**

Thank you for attention

Non-turbulent (convective) modulation of normal distributed patterns in a statistical condensation scheme:



$\Delta q_{vs} = q_w - q_{vs}$: local over saturation

$\propto a_-$ $\propto a_0$ PDF

$L = min\{L_p, D_g\}$: separation scale for turbulence

$\propto a_+$

multimodal common PDF

L_s : horizontal scale of largest normal distr. patterns (turbulence, wakes, gravity waves, etc)