Dynamics activity in HIRLAM

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Outlook

- Vertical discretization using finite elements
- Elimination of the extension zone from the grid-point representation
  - Biperiodization
  - Relaxation to the nesting model
  - Application of relaxation and biperiodization in spectral space
- Conservation of the dry air mass

Vertical discretization using finite elements (F.E.)

- In the hydrostatic version the only vertical operator is the integral
- In the non-hydrostatic version both the integral and the derivative are needed
  - This introduces some constraints when arriving at a Helmholtz equation
  - These constraints are not fulfilled by the F.E. operators

Construction of a vertical operator

$$F = \frac{df}{d\eta}$$

Derivative operator

$$F(\eta) \sim \sum_{i=1}^{M} F_i E^i(\eta)$$

$$f(\eta) \sim \sum_{i=1}^{N} f_i e^i(\eta)$$

Approximate functions as linear combinations of basis functions

$$\sum_{i=1}^{M} F_i E^i(\eta) \approx \sum_{j=1}^{N} f_j \frac{d}{d\eta} e^j(\eta)$$

Galerkin procedure

Scalarly multiply by a set of test functions

$$\sum_{i=1}^{M} F_i \underbrace{\int_0^1 E^i(\eta) T_k(\eta) d\eta}_{A_k^i \text{ (mass matrix)}} = \sum_{j=1}^{N} f_j \underbrace{\int_0^1 \frac{d}{d\eta} e^j(\eta) T_k(\eta) d\eta}_{B_k^j \text{ (operator matrix)}} \qquad \forall k \in (1-K)$$

Approximation error: orthogonal to space spanned by test functions *T*

$$\sum_{i=1}^{M} F_i A_k^i = \sum_{j=1}^{N} f_j B_k^j \Rightarrow \tilde{F}\mathbf{A} = \tilde{f}\mathbf{B}$$

K equations
M unknowns

Galerkin procedure (cont)

$\tilde{f}$ is the set of coefficients for the representation of function $f(\eta)$

If we are given the values $f(\eta_j)$ at a set of values of η (full level values)

$$f(\eta_j) = \sum_{i=1}^{M} f_i e^i(\eta_j) \equiv \tilde{f}\mathbf{P}$$

$$\tilde{f} = f(\eta_j)\mathbf{P}^{-1}$$

$\mathbf{P}^{-1}$ is the projection matrix to the space spanned by the basis functions *e*

Galerkin procedure (cont)

From the vector of values $\tilde{F}$

We can get the values of the function at full levels

$$F(\eta_l) = \sum_{j=1}^{N} F_j E^j(\eta_l) \equiv \tilde{F}\mathbf{S}$$

Where $\mathbf{S}$ Is the inverse projection matrix from the space spanned by the basis *E*

$$F(\eta_j) = \tilde{F}\mathbf{S} = \tilde{f}\mathbf{B}\mathbf{A}^{-1}\mathbf{S} = f(\eta_j)\mathbf{P}^{-1}\mathbf{B}\mathbf{A}^{-1}\mathbf{S} \equiv f(\eta_j)\mathbf{M}$$

Vertical operators (cont)

- Matrix **M** applied to the set of full-level values of field *f* *gives the set of full-level values of its derivative*
- *Similarly we can compute the matrix for the integral operator:* **N**
- The order of accuracy of both **M** and **N,** using cubic basis functions can be shown to be 8
- **M** and **N** are NOT the inverse of each other

Equations

$$\frac{d\mathbf{V}}{dt} + \frac{RT}{p}\nabla_{\eta} p + \frac{1}{m}\frac{\partial p}{\partial \eta}\nabla_{\eta}\phi = \varsigma$$

$$\gamma\frac{dw}{dt} + g\left(1 - \frac{1}{m}\frac{\partial p}{\partial \eta}\right) = \gamma\Omega$$

$$\frac{\partial m}{\partial t} + \nabla_{\eta}(m\mathbf{V}) + \frac{\partial}{\partial \eta}(m\dot{\eta}) = 0$$

$$\frac{dT}{dt} - \frac{RT}{C_p}\frac{1}{p}\frac{dp}{dt} = \frac{Q}{C_p}$$

$$\frac{dp}{dt} + \frac{C_p}{C_v} p D_3 = \frac{Qp}{C_v T}$$

$$\frac{d\phi}{dt} = gw$$

$$\frac{\partial \phi}{\partial \pi} = -m\frac{RT}{p}$$

Pressure departure and Vertical divergence

$$P = \frac{p - \pi}{\pi}$$

$$d = -g\,\frac{\rho}{m}\frac{\partial w}{\partial \eta}$$

The corresponding equations are

$$\frac{dP}{dt} = (1+P)\left(\frac{1}{p}\frac{dp}{dt} - \frac{1}{\pi}\frac{d\pi}{dt}\right) = -(1+P)\left(\frac{C_p}{C_v}D_3 + \frac{\dot{\pi}}{\pi}\right) + (1+P)\frac{Q}{C_v T}$$

$$\frac{d\mathrm{d}}{dt} = \mathrm{d}\,\frac{1}{p}\frac{dp}{dt} - \mathrm{d}\,\frac{1}{T}\frac{dT}{dt} - \mathrm{d}\,\frac{1}{m}\frac{dm}{dt} - g\,\frac{p}{mRT}\frac{d}{dt}\left(\frac{\partial w}{\partial \eta}\right)$$

Helmholtz equation

Eliminating from the discretized set of equations (with some constraints to be fulfilled by the operators) all the variables except the vertical divergence, we obtain a Helmholtz equation:

$$\left[1-(\Delta t)^2 c_*^2\left(m_*^2\nabla^2+\frac{\mathbf{L}^*}{rH_*^2}\right)-(\Delta t)^4\frac{N_*^2c_*^2}{r}m_*^2\nabla^2T^*\right]\mathrm{d}=r.h.s.$$

Which can be solved very easily in spectral space
In a projection on vertical eigenvectors

Choices to apply VFE in the NH version

- Choose a set of equations using only one vertical operator
  - Change the set of forecast fields
  - Change the vertical coordinate to one based on height instead of mass
- Solve a set of two coupled equations instead of a single Helmholtz equation

Change of the vertical coordinate to a height-based hybrid one

- Use of a time-independent coordinate eliminates the X-term if we use a covariant formulation.
- Only derivatives are used in the vertical (no integrals) which simplifies the *constraints* to arrive at a single Helmholtz equation
- The coordinate is still a hybrid coordinate. The data flow is maintained.

Change the vertical coordinate

- Juan Simarro has tested this option.
- Any vertical discretization, either finite differences or finite elements of accuracy order greater than 4 becomes unstable

Note: In general higher accuracy leads to lower stability

Solve a coupled system of equations
(Jozef Vivoda & Petra Smolikova)

- In order to arrive at a single Helmholtz equation, the following constraint (C1) has to be fulfilled

$$A_1 \equiv G^* S^* - S^* - G^* + N^* = 0$$

Where

$$\left(G^* \psi\right)_l \equiv \int_\eta^1 \frac{m^*}{\pi^*} \psi d\eta$$

$$\left(S^* \psi\right)_l \equiv \frac{1}{\pi_l^*} \int_0^{\eta_l} m^* \psi d\eta$$

$$\left(N^* \psi\right)_l \equiv \left(S^* \psi\right)_{L+1}$$

As this constraint is not fulfilled with the finite-elements integral operator, we cannot arrive at a single Helmholtz equation

Solve a coupled system of equations (cont)

Instead, we arrive at a coupled system involving both
The horizontal and the vertical divergences

$$\begin{pmatrix} \mathbb{E} & -\mathbb{F} \\ -\mathbb{B} & \mathbb{A}+\mathbb{C} \end{pmatrix} \begin{pmatrix} d \\ D \end{pmatrix} = \begin{pmatrix} d^\bullet \\ D^\bullet \end{pmatrix} .$$

where

$$\begin{aligned}
\mathbb{A} &= \left(1 - \delta t^2 c^2 \triangle\right), \\
\mathbb{B} &= \delta t^2 \triangle \left(-RT^* \mathcal{G}^* + c^2\right), \\
\mathbb{C} &= \delta t^2 \triangle R T^* A_1, \\
\mathbb{E} &= \left(1 - \delta t^2 c^2 \frac{\mathcal{L}^*}{rH^2}\right), \\
\mathbb{F} &= \delta t^2 \frac{\mathcal{L}^*}{rH^2} \left(-RT^* \mathcal{S}^* + c^2\right).
\end{aligned}$$

$$\mathcal{L}^* \psi = \frac{1}{m^*} \frac{\partial}{\partial \eta} \left(\frac{\pi^{*2}}{m^*}\right) \frac{\partial \psi}{\partial \eta} + \left(\frac{\pi^*}{m^*}\right)^2 \frac{\partial^2 \psi}{\partial \eta^2}$$

Solve a coupled system of equations (cont)

- The system of equations is twice as large as in the hydrostatic case
- An iterative procedure has been adopted for solving the system
- This method is being implemented in both HARMONIE and IFS

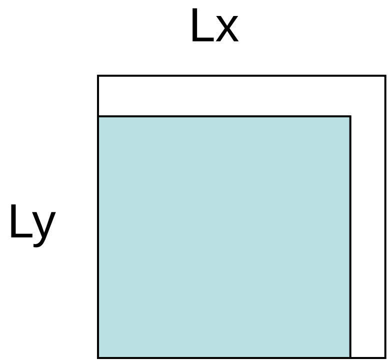
Elimination of the extension zone

- Spherical harmonics are not an appropriate basis for a limited-area domain
- The model equations are solved on a plane projection with Cartesian x-y coordinates
- Double Fourier functions are used as the basis for spectral discretization
- Fields should be periodic in both x and y
- An extension zone is used to biperiodize the fields

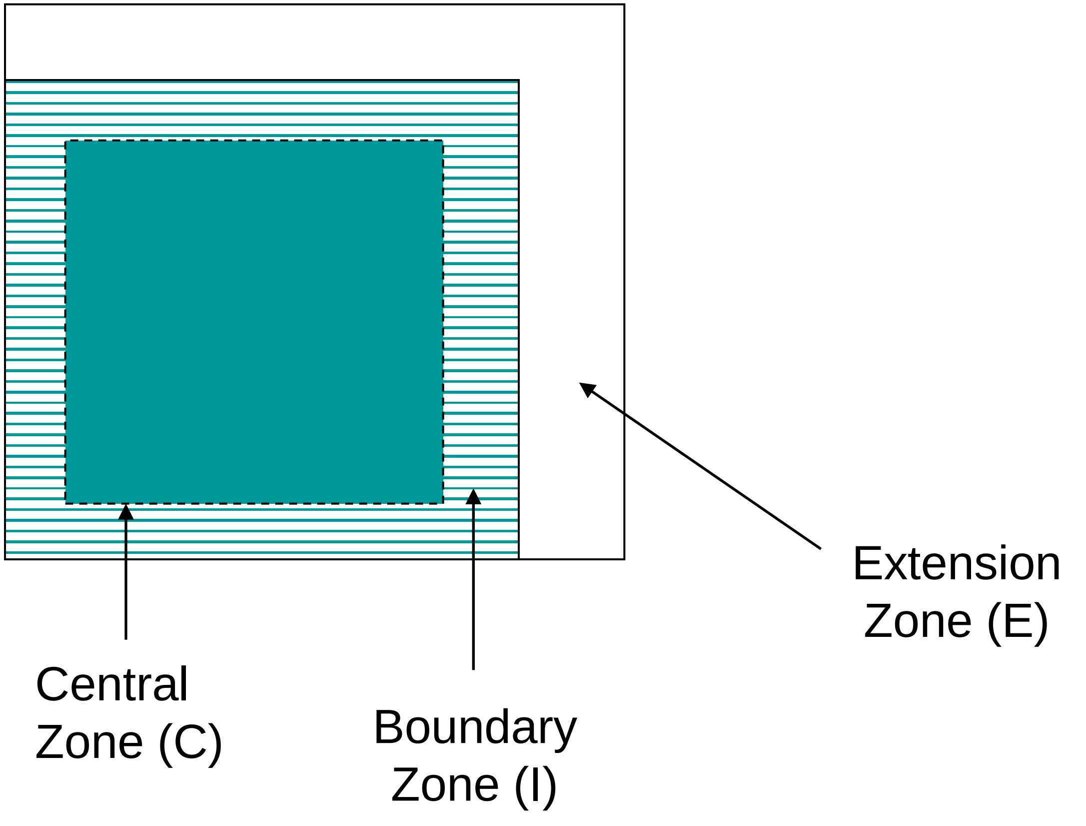
Biperiodization of fields

$$F(x,y) \approx \sum_{i=-I}^{I} \sum_{j=-J}^{J} f_k^l e^{ikx/L_x} e^{jly/L_y}$$

Periodic in x (period Lx) and in y (period Ly)



Boundary conditions



Boundary conditions

Gabor Radnoti 1995

Semi-implicit solution procedure:

$$(I - \Delta t \mathscr{L})\Psi_{t+\Delta t} = \underbrace{\Psi_{t+\Delta t(\exp)} + \Delta t \mathscr{L} \left(\Psi_{t-\Delta t} - 2\Psi_t\right)}_{\tilde{\Psi}}$$

Coupling to a nesting model (LS)

$$\Psi^C = (1-\alpha)\cdot\Psi^l + \alpha\cdot\Psi^{LS}$$

α =1 at the whole of E.
α =0 at the whole of C
Smoothly changing at I

Implementation:

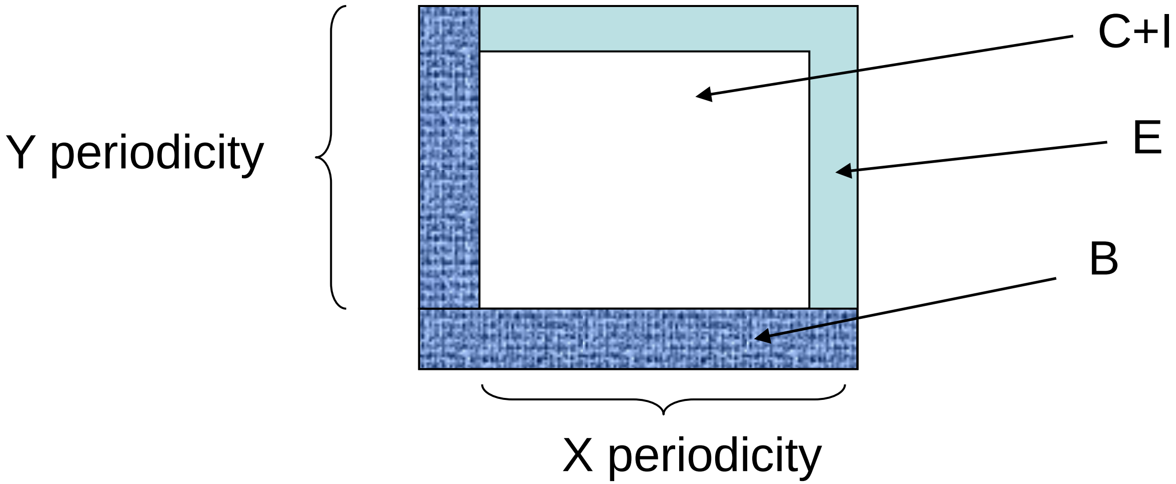
$$(I - \Delta t \mathscr{L})\Psi_{t+\Delta t} = (1-\alpha)\Psi^l + \alpha(I - \Delta t \mathscr{L})\Psi^{LS}_{t+\Delta t}$$

Boundary cond. (cont)

$\Psi^{LS}_{t+\Delta t}$ Are values derived from the nesting model.

Their values at the right border of E should join smoothly with their values at the left border of I

They can be computed by means of smoothed splines
Or by Boyd's linear combination of the values at E and at B



Increasing the width of E and eliminating it from the grid-point

- In data assimilation the influence of an observation covers an area around the observation position
- Due to the periodicity of fields, an observation close to the right border of the inner domain can affect the fields on the left border.
- That can be eliminated by increasing the width of the extension zone

Increasing the width of E (cont)

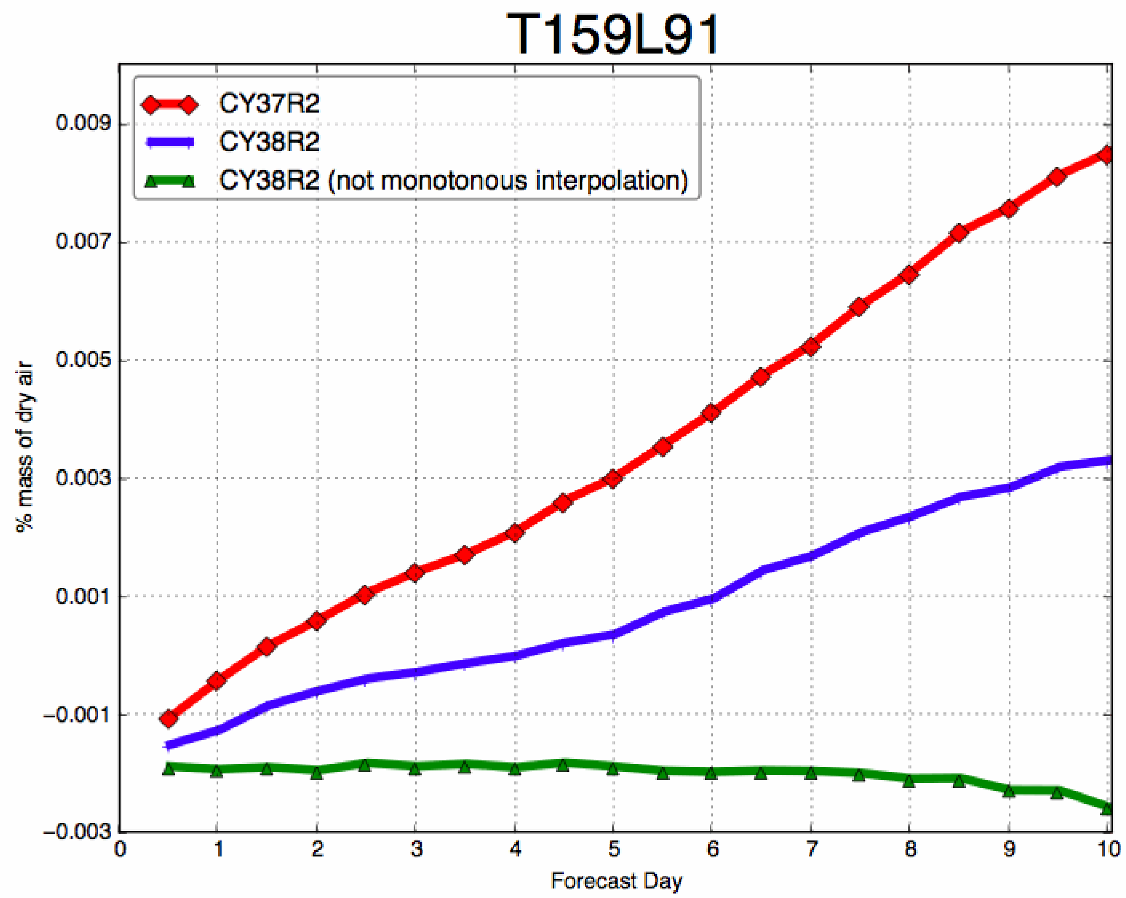
- If the points in the extension zone are present in the grid-point representation
  - The cost of running the model increases if we increase the width of E
  - Due to the clipping of the semi-Lagrangian trajectories to the C+I area, the interpolation points could fall outside the semi-Lagrangian buffer, producing floating-point errors or segmentation faults
- Elimination of the extension zone from the grid-point representation
  - Application of the boundary conditions and biperiodization in spectral space

Conservation of mass

- Semi-Lagrangian advection is not designed to conserve mass
- Mass conserving semi-Lagrangian schemes are much more expensive than the standard semi-Lagrangian
- Conserving schemes based on finite-volume are subject to CFL limit of stability
- The chosen option in HIRLAM is to approach conservation through improvements in accuracy

Conservation of mass

Total change in dry air mass using the IFS global model (Tomas Morales)



Conservation of mass (cont)

- The improvement achieved by the introduction of CY38R2 comes from the elimination of aliasing on the vorticity over orography introduced by Nils Wedi
- The use of quasi-monotonicity on the high-order (2D) interpolation for the continuity equation is not needed, because the interpolated field is not close to zero (always of the order of 11.)
- The elimination of aliasing will be coded for the limited-area version and the default for the interpolation in the continuity equation will be made non quasimonotone

Elimination of aliasing in the contribution to vorticity from the pressure gradient term N. Wedi (2013)

- De-aliasing in IFS: By subtracting the difference between a specially filtered and the unfiltered pressure gradient term at every time-step the stationary noise patterns can be removed at a cost of approx. 5% at T1279

Thank you very much

Questions?