Regional Cooperation for
Limited Area Modeling in Central Europe

LACE
nwp central europe

LACE – DEVELOPMENT IN DYNAMICS

Petra Smolíková (CHMI), Jozef Vivoda (SHMI), Radmila Brožková,
Ján Mašek (CHMI), Alexandra Craciun(NMA) and others
Hirlam coop.: Juan Simarro, Alvaro Subias (AEMET)

DHMZ SHMU OMSZ ZAMG ROMANIA ANM

Summary

1. **Finite element method in vertical discretization of NH model** (J.Vivoda, P.Smolíková)

   - based on hydrostatic version of VFE (being developped by A.Untch, M.Hortal)

   - cooperation with HIRLAM colleagues (J.Simarro, A.Subias)

2. **ENO technique in SL interpolations** (J.Mašek, A.Craciun)

3. **Scale adaptive horizontal diffusion - SLHD tuning for high resolutions** (P.Smolíková, R.Brožková)

Finite elements in vertical for NH

What we want to have:

Pure FE vertical discretisation in NH model with

- an **arbitrary** choice of the order of basis functions
- **stability** – similar as for FD
- **accuracy** – improved compared to FD

≠

What we have:

FE vertical discretisation in NH model with

- several **FD features** (transformations w <-> d, BC of vertical Laplacian)
- an **almost arbitrary** choice of the order of basis functions
- **stability** – similar as for FD in 2D and 3D (2.2km) tests
- **accuracy** – improved according to theoretical study for distinct FE operators, not confirmed in 2D and 3D tests

Eliminating FD features

- Stability reasons ⇒ **vertical divergence *d*** in spectral calculations
- Accuracy reasons ⇒ **vertical velocity *w*** in GP calculations
- **Transformations** (ones per time step) by vertical operators ***I,D***

  Derivative $\boldsymbol{D} : w \rightarrow d$ after GP calculations

  Integral $\boldsymbol{I} : d \rightarrow w$ after SP calculations

- To keep the steady state $\frac{\partial w}{\partial t} = 0$ we need **invertibility**

$$\boldsymbol{I.D}\, f = \boldsymbol{D.I}\, f = f$$

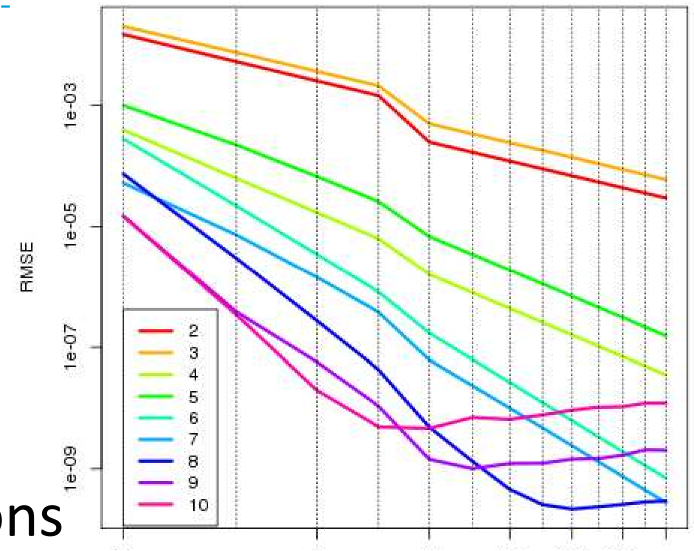
- Looking for operators in the space of B-splines of a given order was unsuccessful ⇒ FD version of ***I,D*** was kept limiting the accuracy of the whole FE scheme

Order of splines

- vertical operators applied on a smooth function $\sin(\pi\eta)^3 \cos(\pi\eta)$
- regular eta levels
- saturation for higher orders
- saturation for higher vertical resolutions

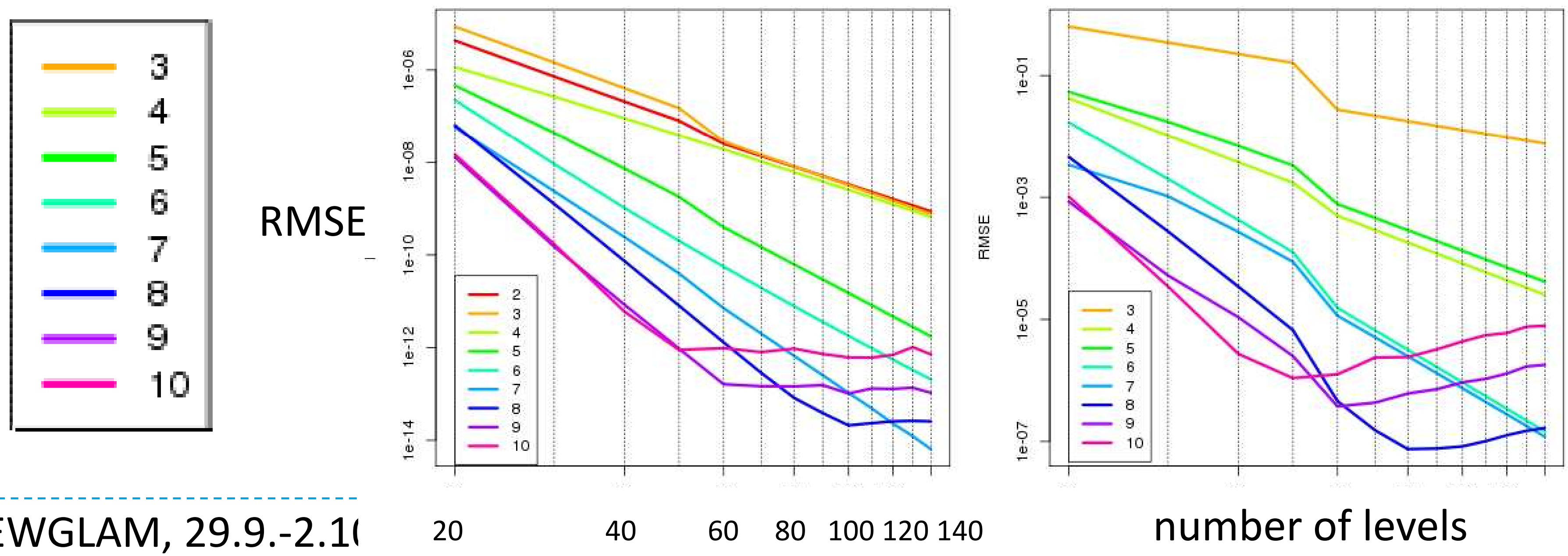
<= rounding error and high number of operations

DER



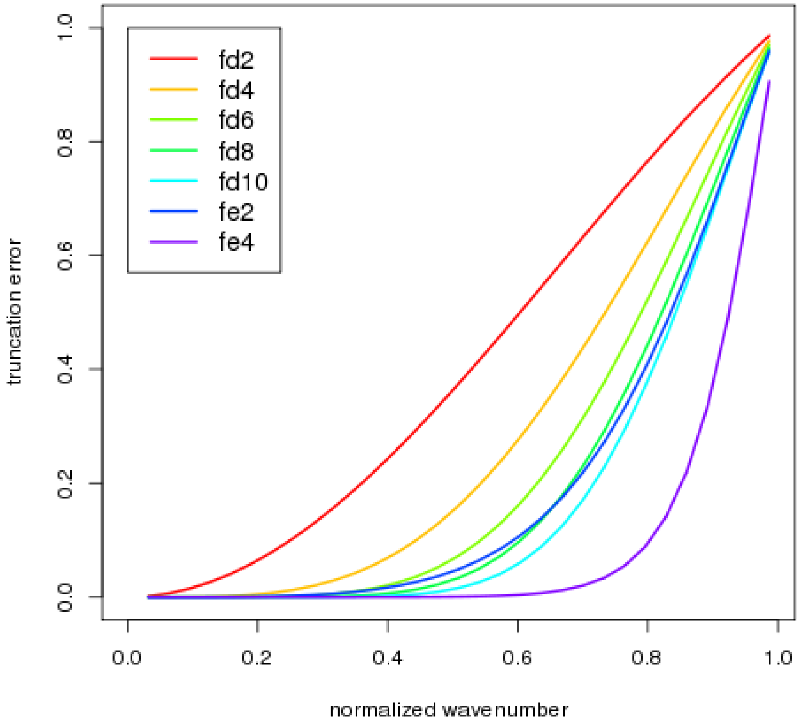
INTEGRAL

SEC.DER



Theoretical accuracy of vertical operators – truncation error

First derivative operator



Taylor series expansion (Staniforth, Wood):

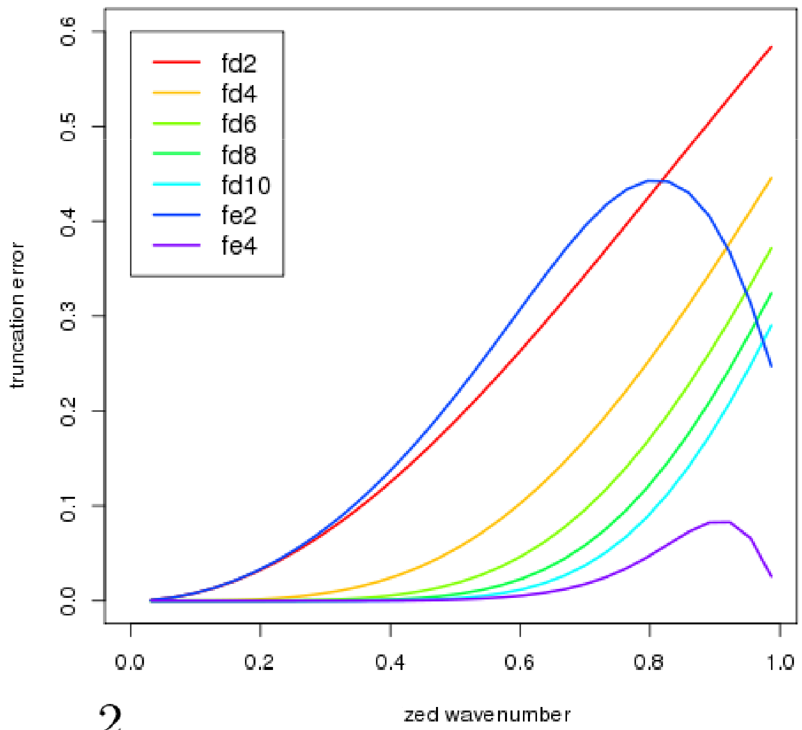
FD 4th order $\approx 1 - \frac{x^4}{30} + \mathcal{O}\left(x^6\right)$

FE linear spline $\approx 1 - \frac{x^4}{180} + \mathcal{O}\left(x^6\right)$

FE cubic spline $\approx 1 - \frac{x^8}{151200} + \mathcal{O}\left(x^{10}\right)$

Theoretical accuracy of vertical operators – truncation error

Second derivative operator



FE linear spline $\approx -1 - \frac{x^2}{12} + \mathcal{O}\left(x^4\right)$

FE cubic spline $\approx -1 - \frac{x^6}{30240} + \mathcal{O}\left(x^8\right)$

Theoretical accuracy of vertical operators – truncation error

Confirmed by application on a smooth function $\sin(\pi\eta)^3 \cos(\pi\eta)$ satisfying operator's boundary conditions

- 140 regular levels
- cubic splines (spline order=4)
- MAE for central part (without boundary effects)

Operator	MAE	Order	
First derivative	8.6228e-12	8.0202	≈ 8
First derivative h->f	1.0362e-05	4.0008	≈ 4
Second derivative	4.4721e-08	6.0648	≈ 6
Integral	4.2197e-16	9.1289	≈ 8

ENO technique in SL interp.

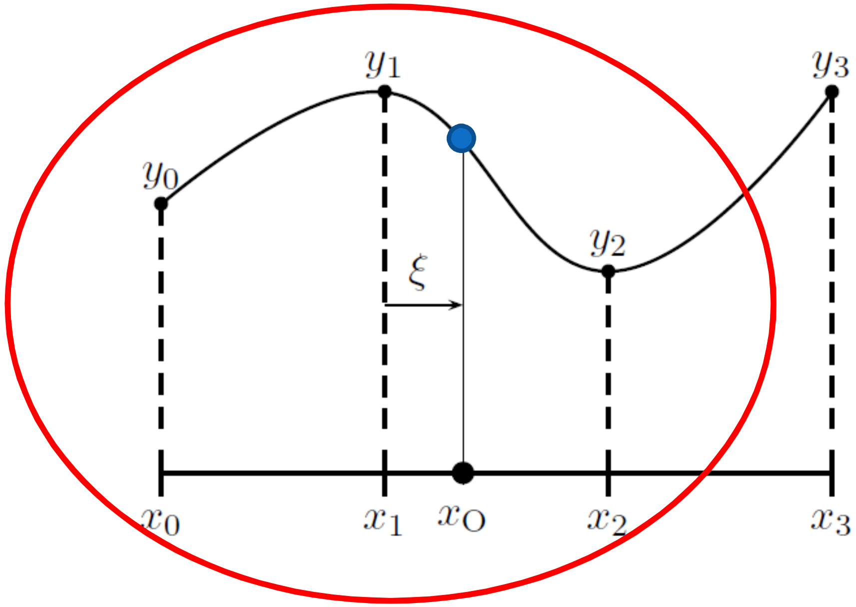
ENO (Essentially Non-Oscilatory)/WENO (Weighted ENO) techniques

Idea of Ján Mašek (inspired by literature):

- to explore alternative interpolators which are
- less overshooting than Lagrange polynomials
- more accurate than their quasi-monotonic versions

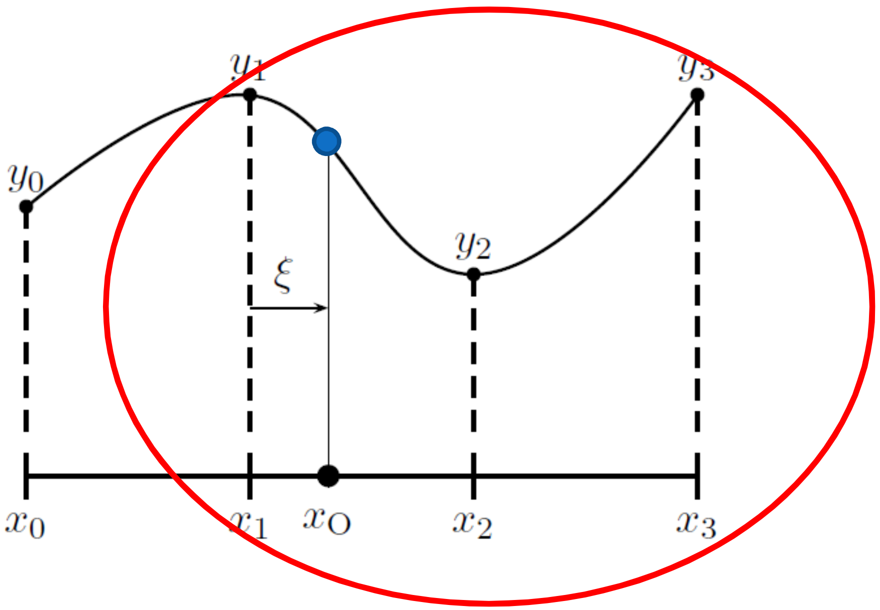
⇒ interpolation depends on the smoothness of the interpolated field

ENO technique in SL interp.



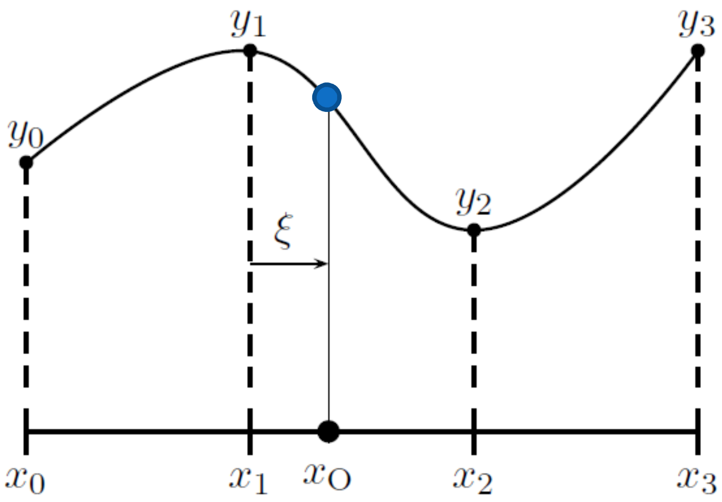
Second order interpolation scheme (quadratic) needs 3 points to find ●:
we may choose the first stencil

ENO technique in SL interp.



Second order interpolation scheme (quadratic) needs 3 points to find ●:
or the second stencil

ENO technique in SL interp.



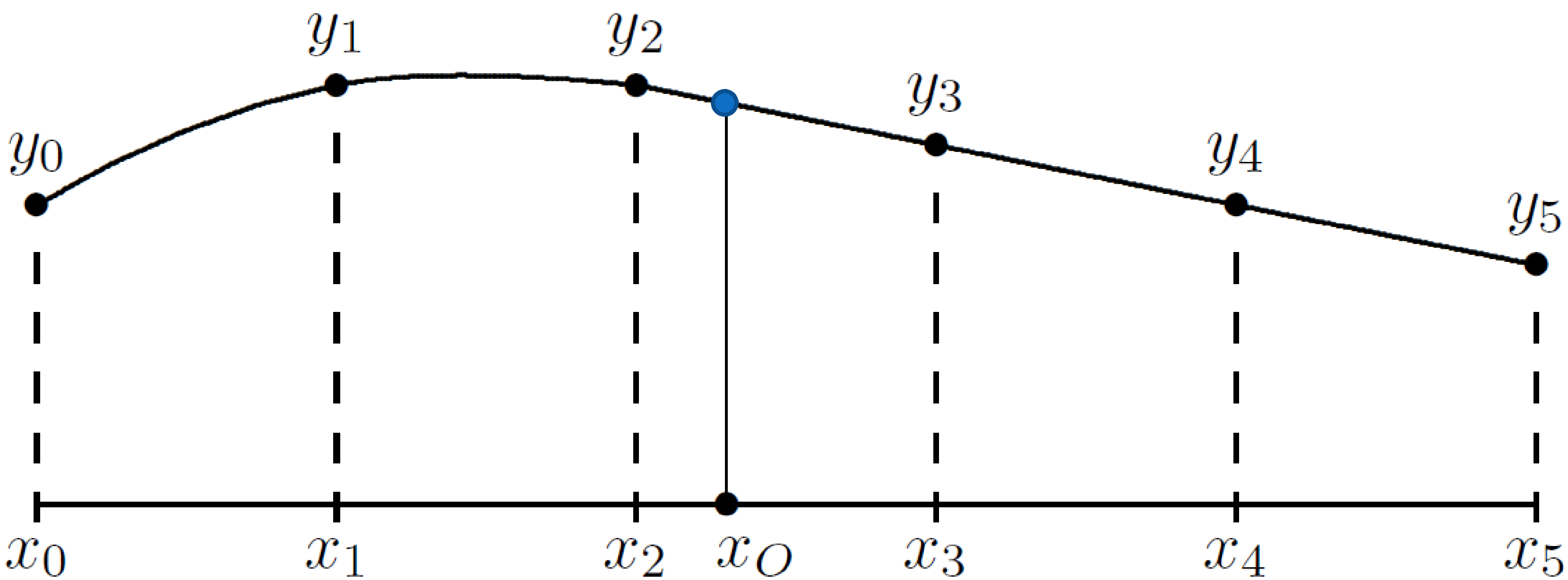
p_1, p_2 interpolated values on the first and second stencil

$$y = p_1 \cdot w_1 + p_2 \cdot w_2, \quad w_1 + w_2 = 1$$

ENO chooses the smoothest solution (w_1=0 or w_2=0)
WENO weighted combination based on smoothness
Linear/cubic p_1, p_2 interpolated with linear/cubic Lagrange polynomial, weights depend on smoothness

ENO technique in SL interp.

Third order interpolation scheme (cubic) needs 4 points to find ●:



ENO technique in SL interp.

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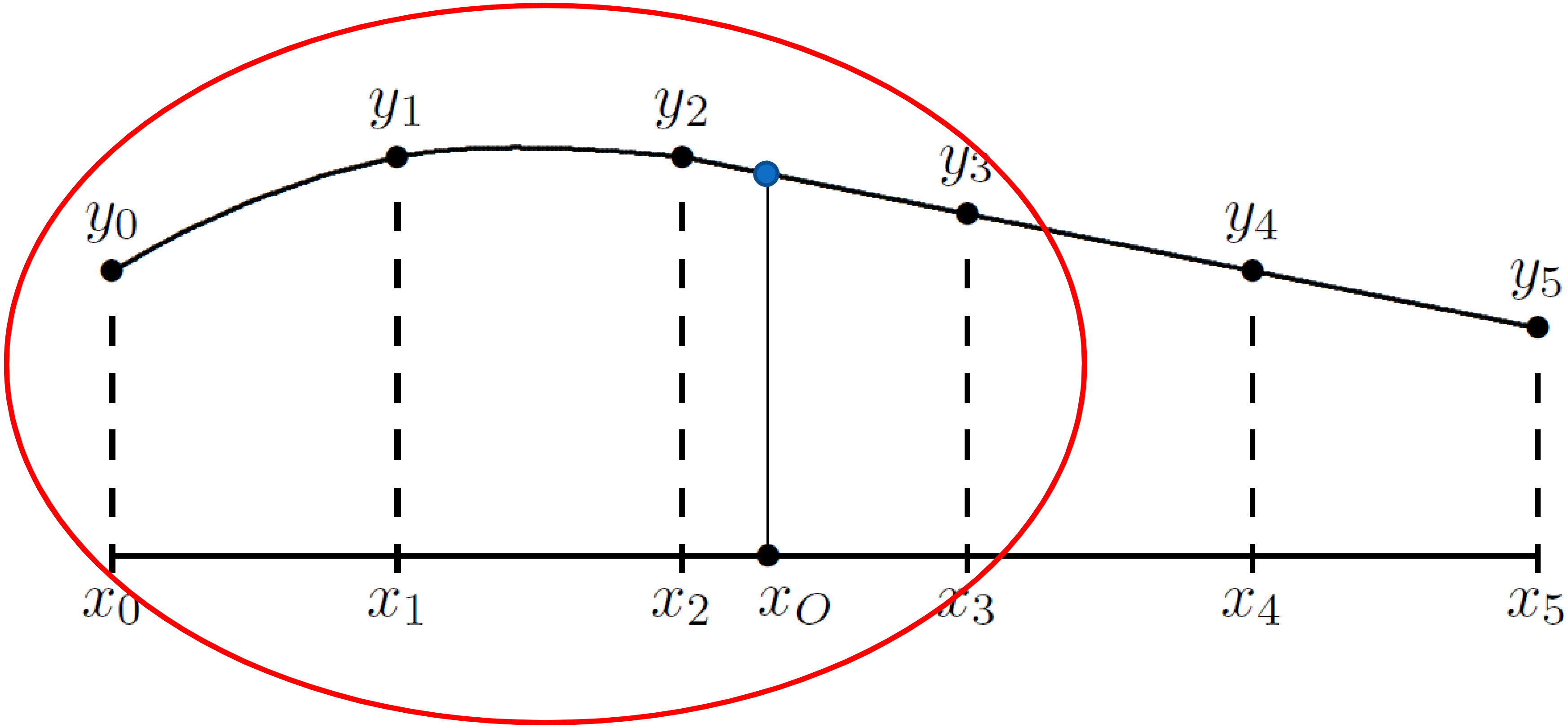

ENO technique in SL interp.

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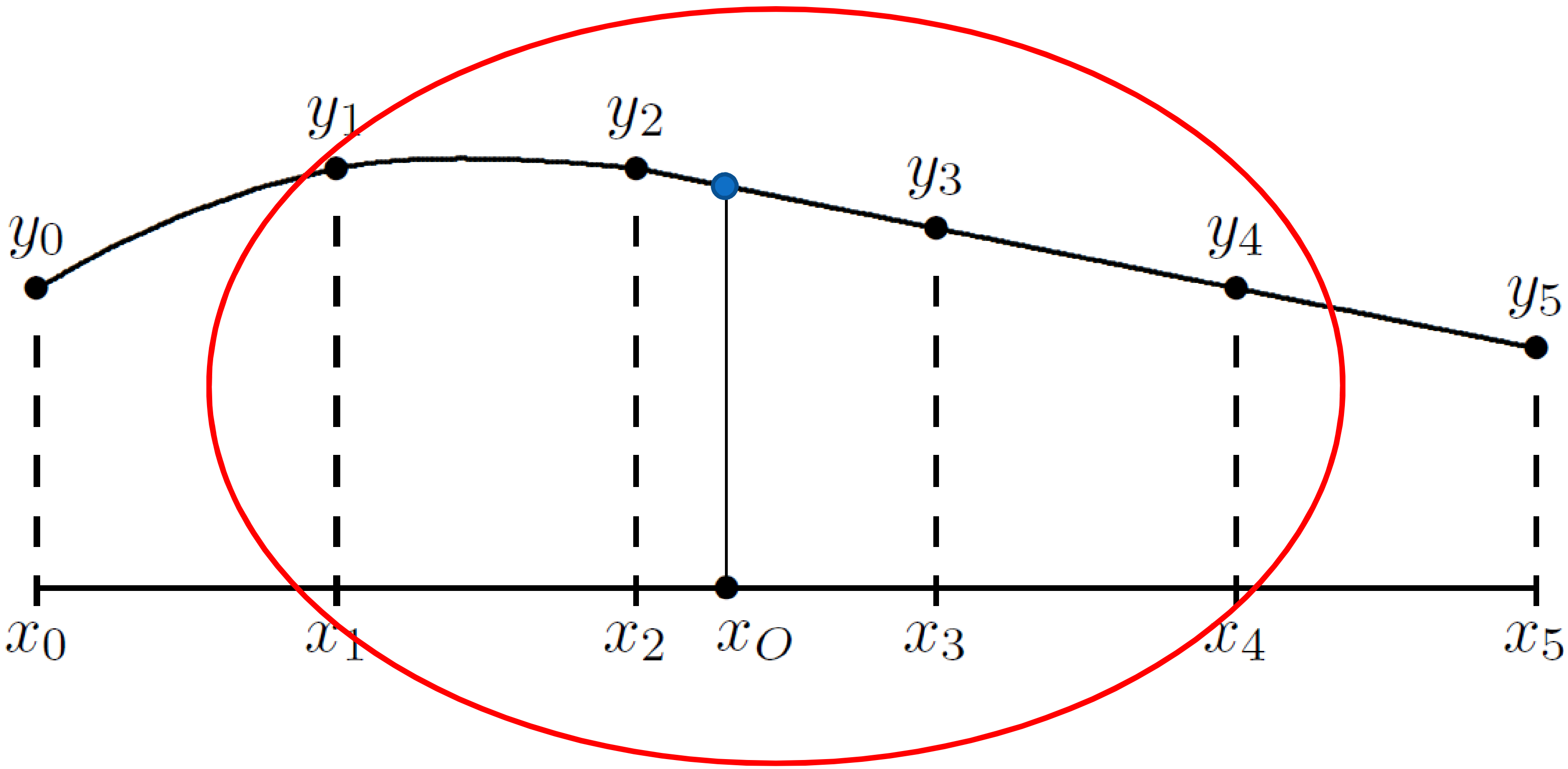

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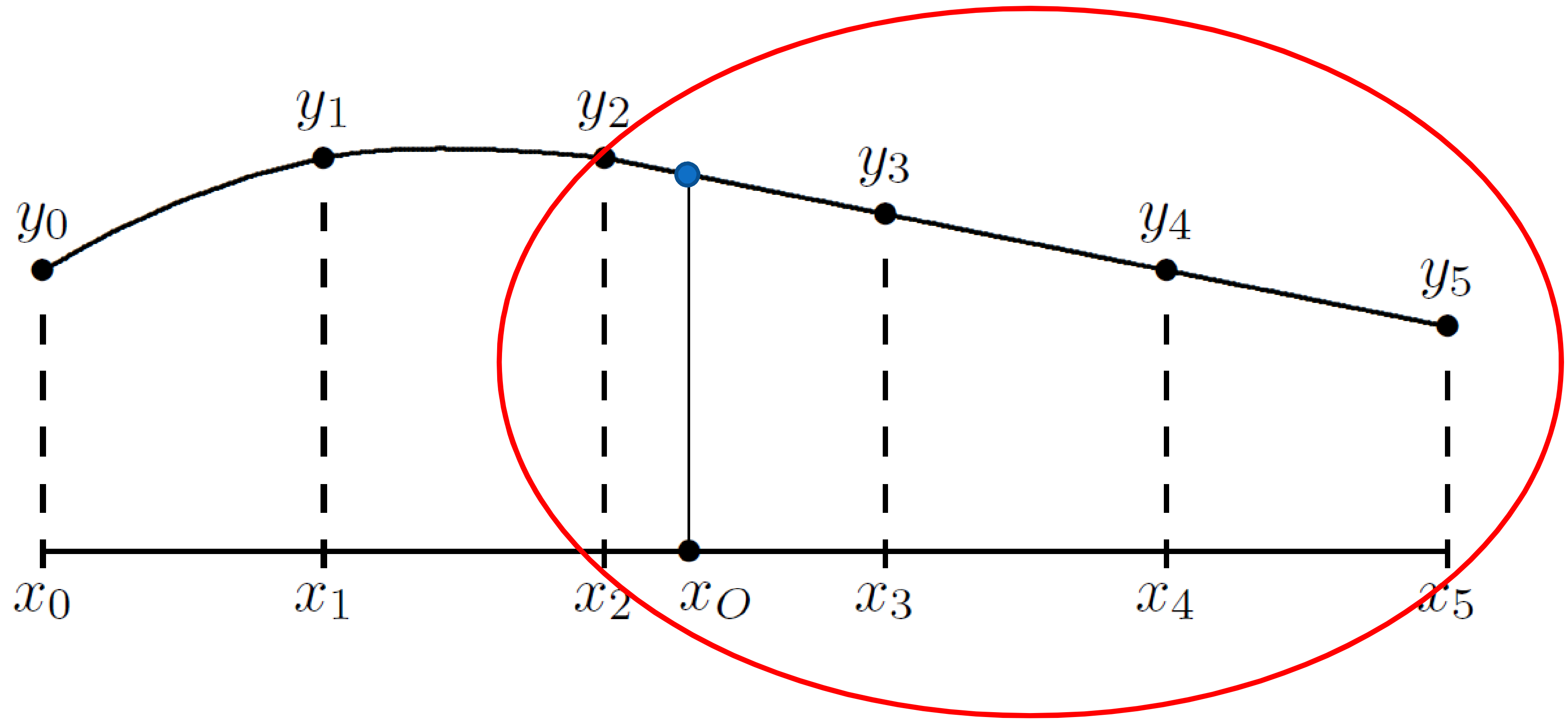

ENO technique in SL interp.

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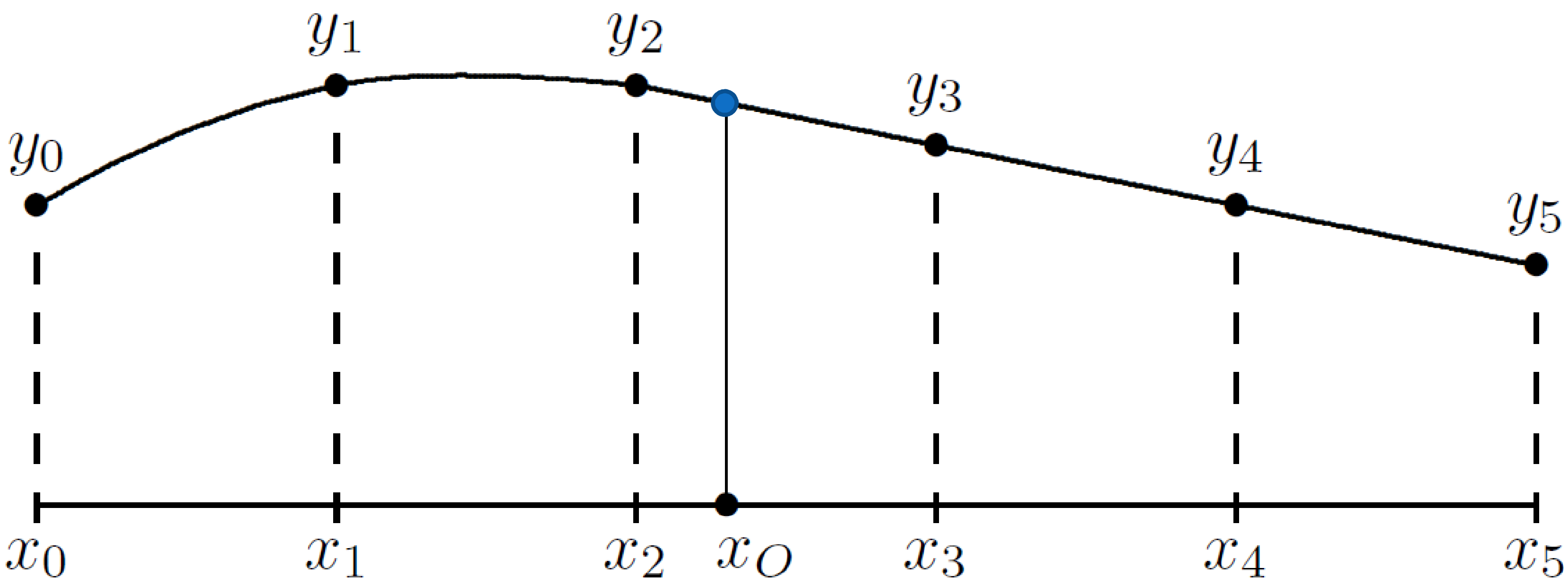
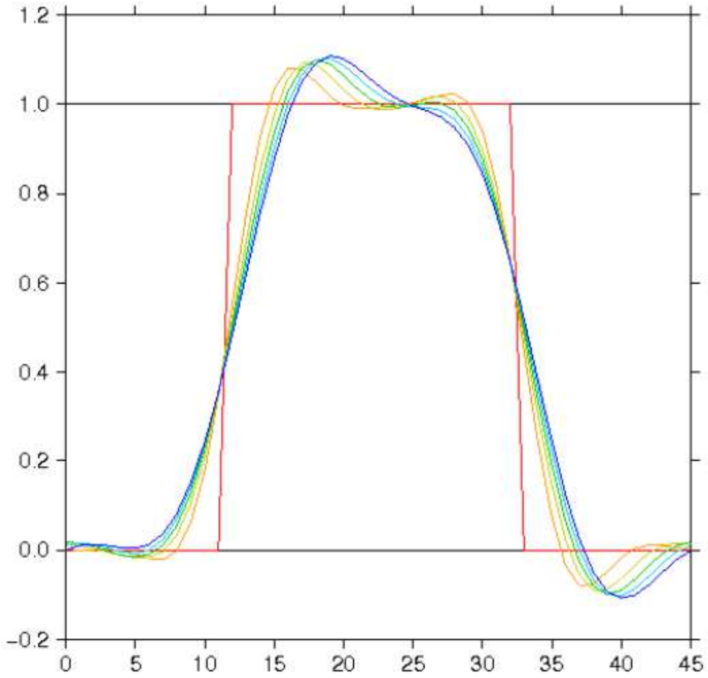


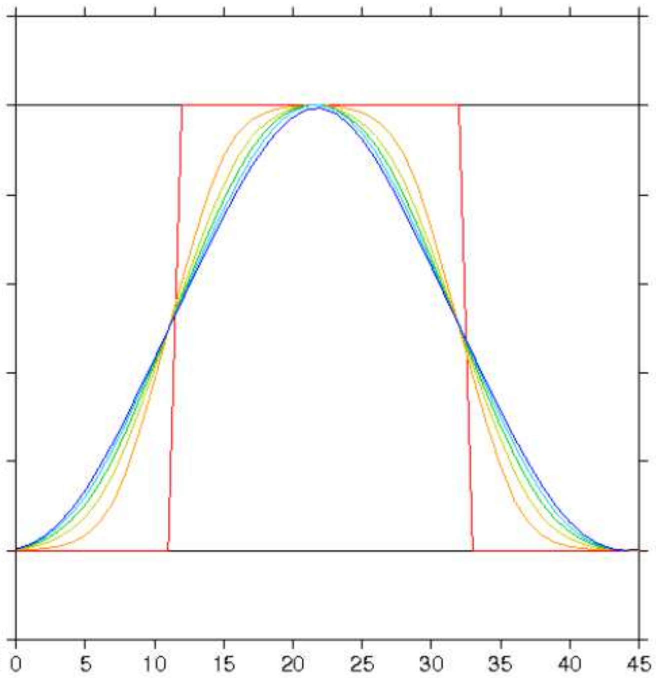
=> **6 points stencil** needed for ENO/WENO interpolations !!!

ENO technique in SL interp.

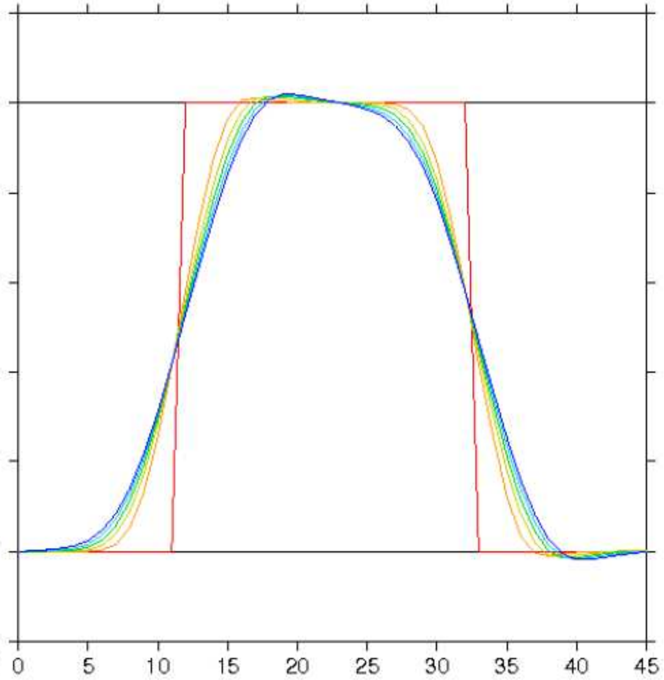
Toy model – 1D linear advection of rectangular pulse in a periodic domain (courtesy of Ján Mašek)



quadratic



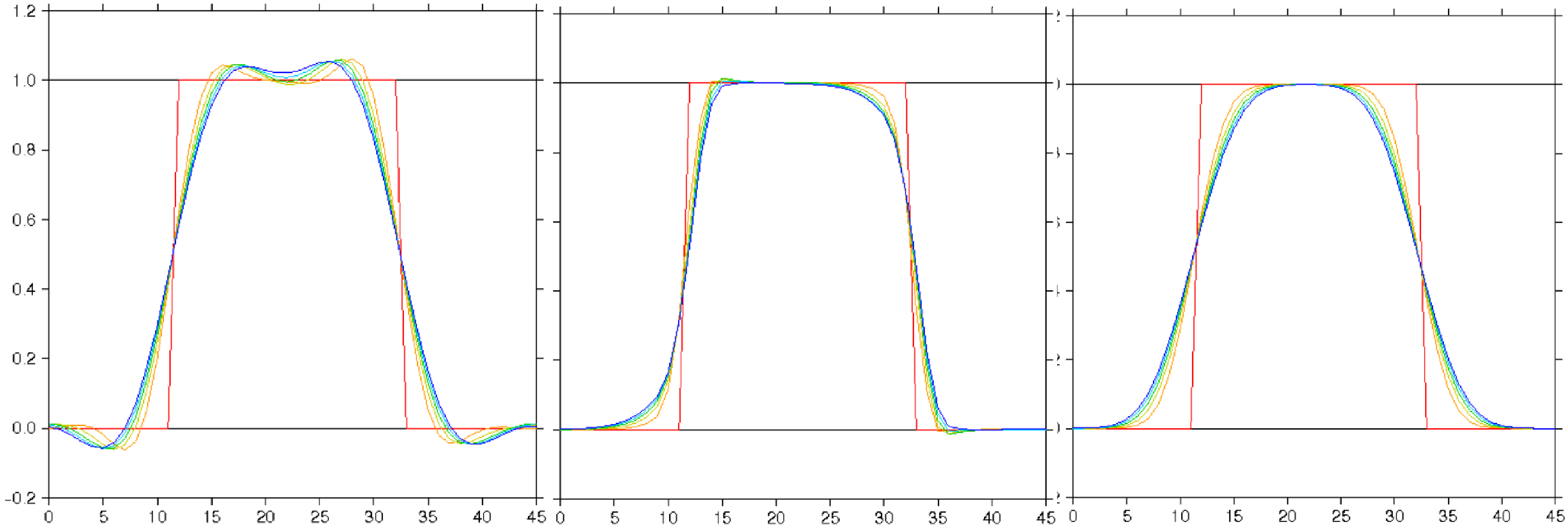
quadratic ENO



quadratic WENO

ENO technique in SL interp.

Toy model – 1D linear advection of rectangular pulse in a periodic domain (courtesy of Ján Mašek)



cubic | cubic ENO | linear/cubic

ENO technique in SL interp.

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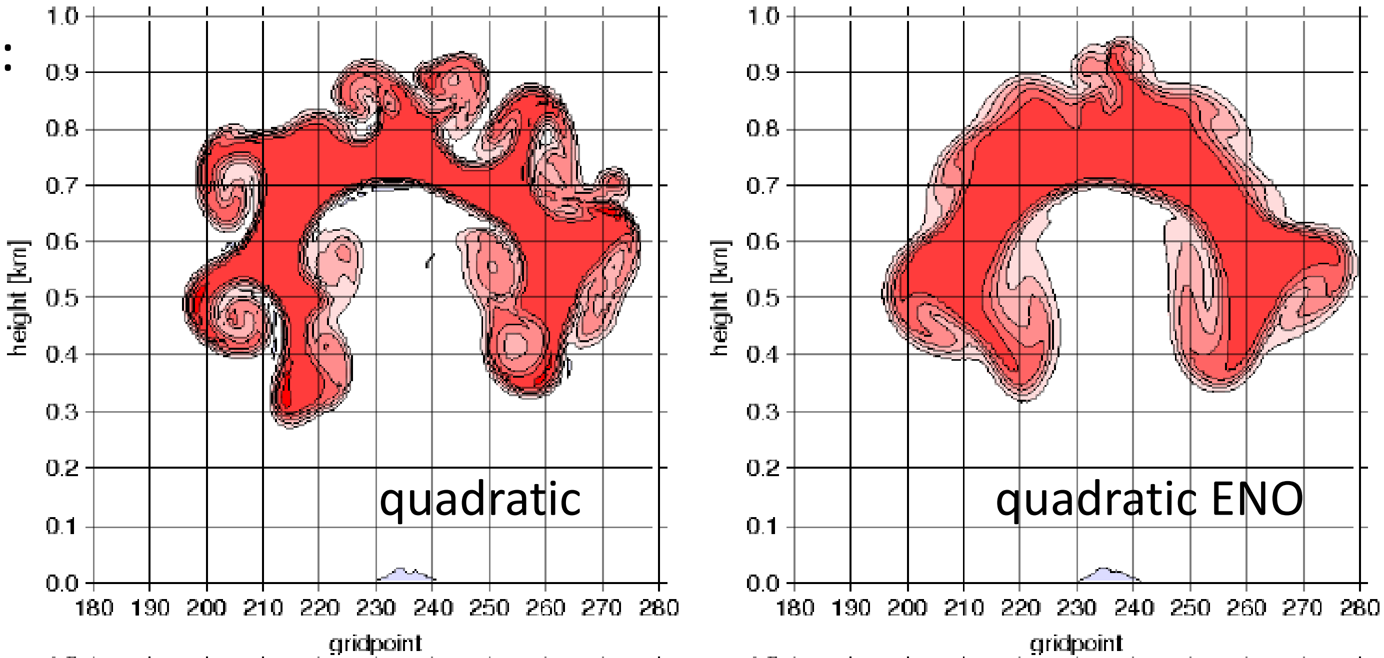
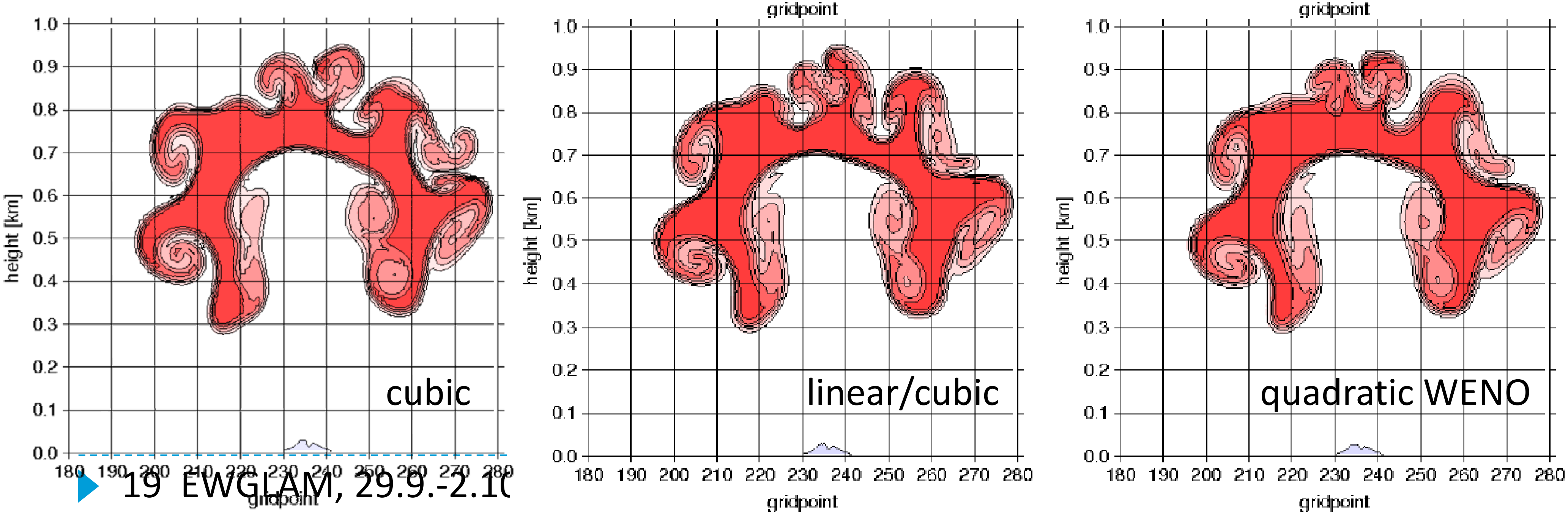
Robert's test in 2D model:
warm bubble (+0.5K)
in the field of potential temperature (300K)
advected with the wind speed 2m/s
(courtesy of A.Craciun)





ENO technique in SL interp.

Conclusions:

- Quadratic interpolator too smoothing to work well
- Cubic ENO/WENO technique promising, but technically and computationally demanding (number of cubic interpolations increased from 7 to 21 !!!)
- Combined linear/cubic interpolation may be easily tested and gives promising results – controlled damping depending on the interpolated field

=> 2 last points worth to be tested in 3D – planned for future work

SLHD tuning in ALARO 1km

Semi-Lagrangian horizontal diffusion
(implemented to ALADIN by Filip Váňa)

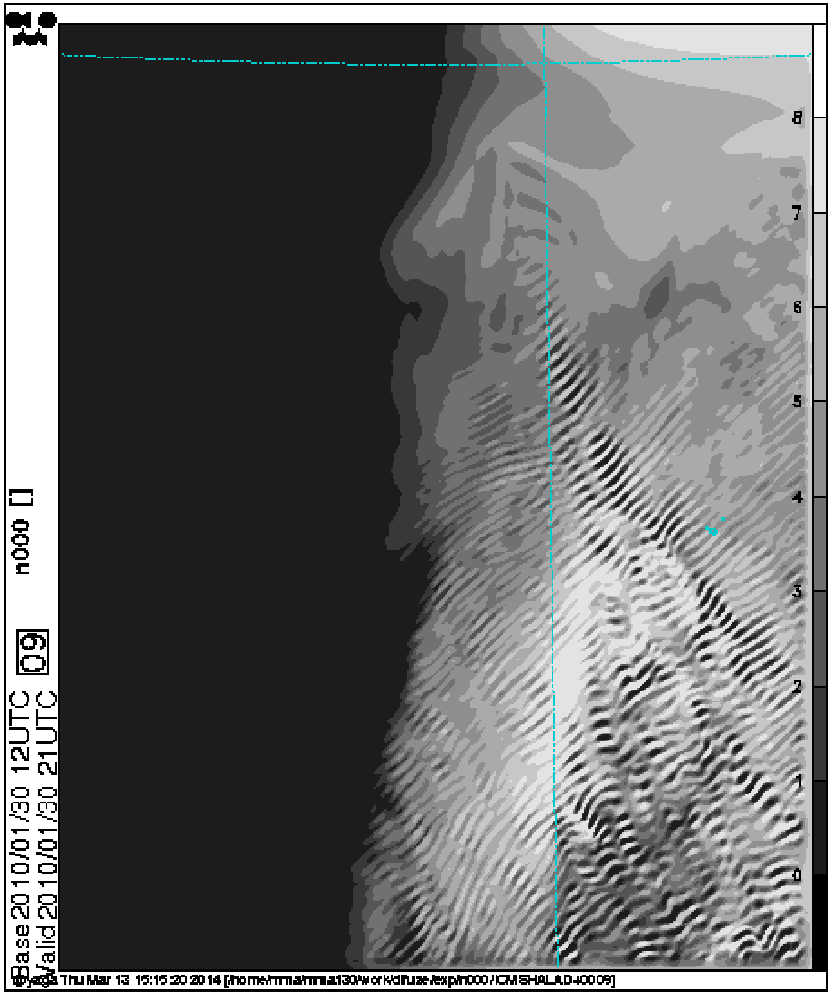
For the following purposes:

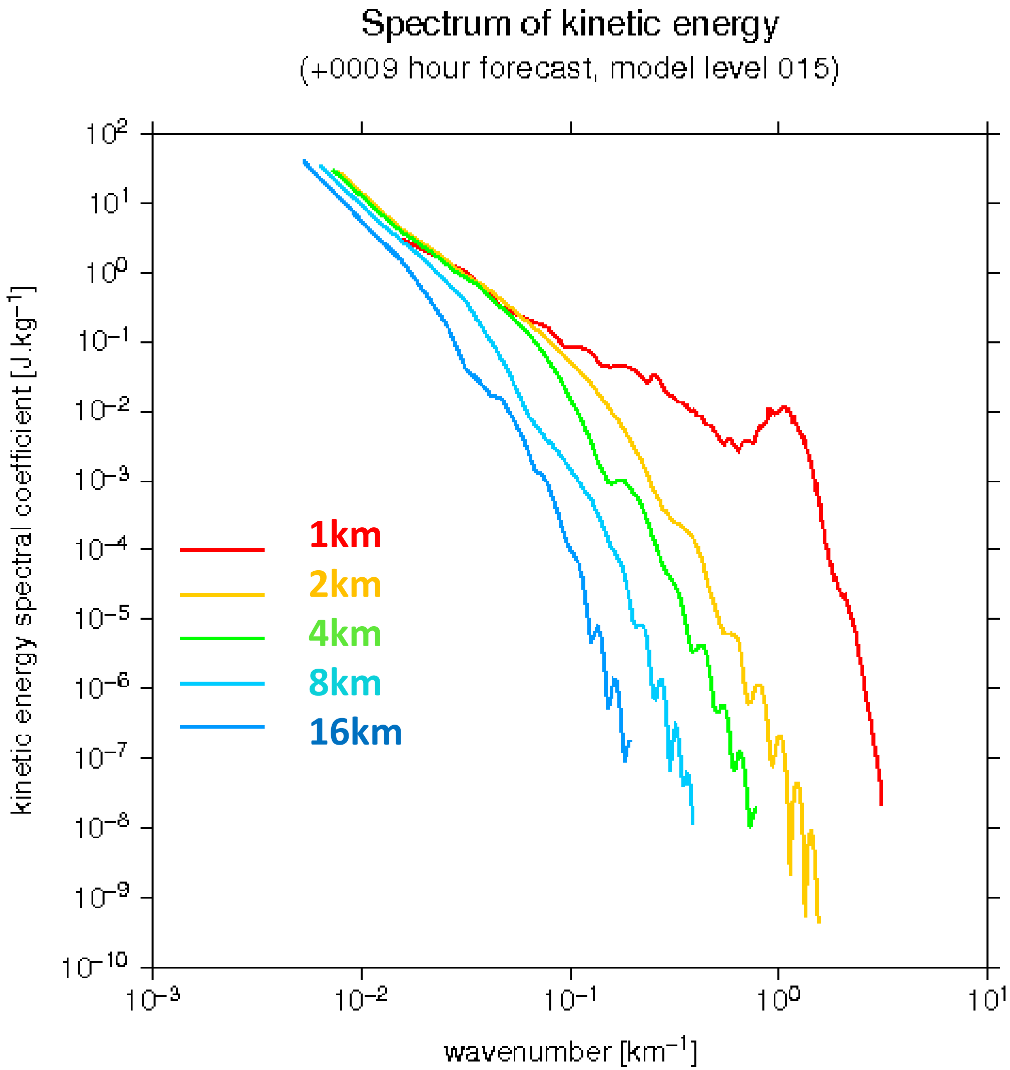
1) To represent the subgrid horizontal effect of **turbulence** and molecular dissipation
2) To **damp** the waves without predictive skills (to improve model scores)
3) To avoid the accumulation of energy at the end of the **model spectrum**

The diffusion coefficient for any diffused field is a function of **deformation** with several tunable parameters.

SLHD tuning in ALARO 1km

Grey zone experiment in the cascade of resolutions (R.Brožková)





SLHD tuning in ALARO 1km

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Gridpoint part of SLHD

LSLHD_X = .T.
SLHDA0 = 0.25
SLHDB = 4.
SLHDD00 = 6.5E-05
ZSLHDP1 = 1.7
ZSLHDP3 = 0.6

adaptation on resolution

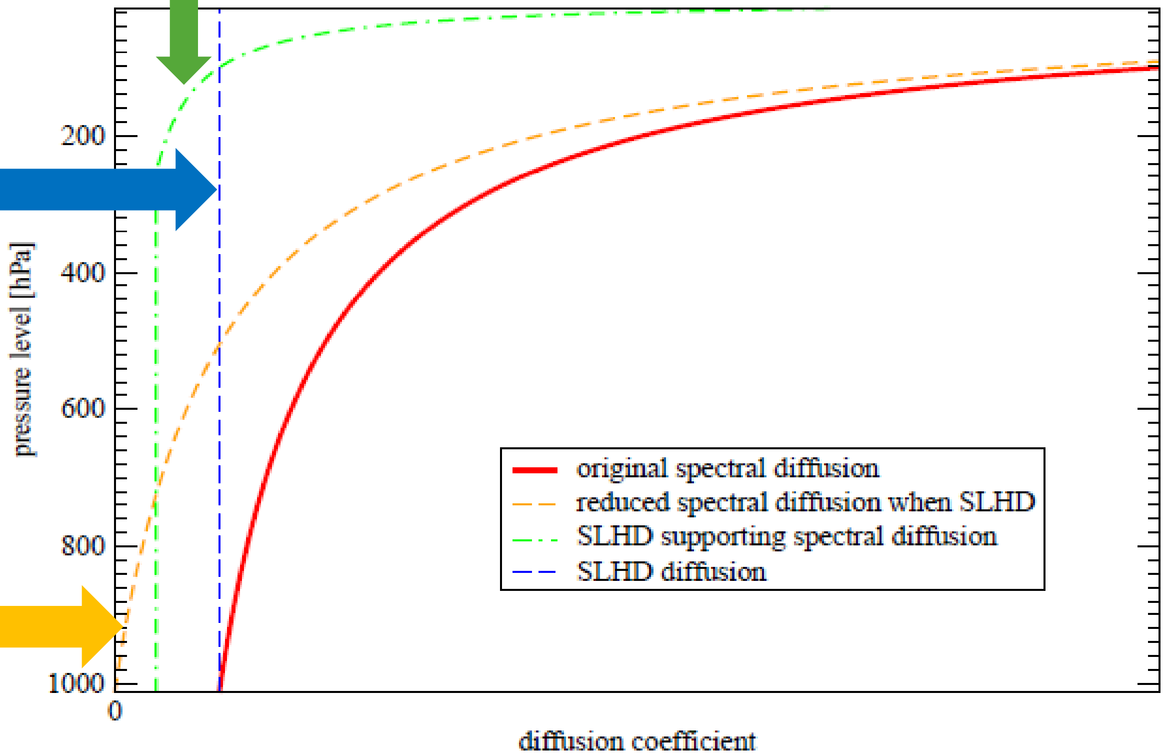
YX_NL%LSLHD = .T.
SLHDEPSH = 0.016
SLHDEPSV = 0.016
SLHDKMAX = 6.
SLHDKMIN = -0.6

Reduced spectral diffusion - enhanced damping with height

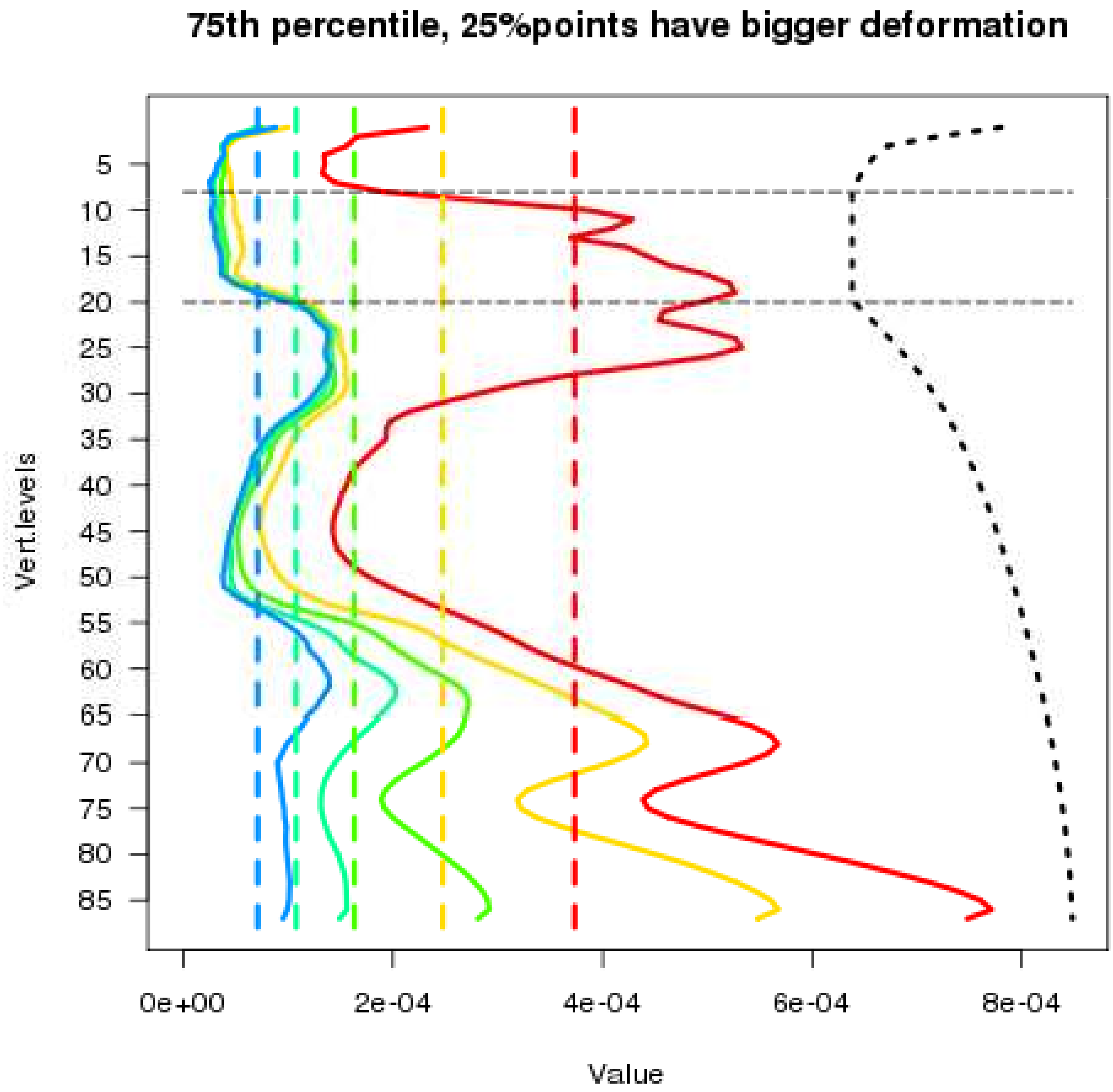
RRDXTAU=123.
REXPDH = 2.
SDRED = 1.
RDAMPX = 0.,...,1. for X=T,Q,VOR,DIV,VD,PD
SLEVDH = 0.1

Supporting spectral diffusion – to control impact of orography

REXPDHS = 6.
RDAMPXS = 10.
SLEVDHS = 1.



SLHD tuning in ALARO 1km

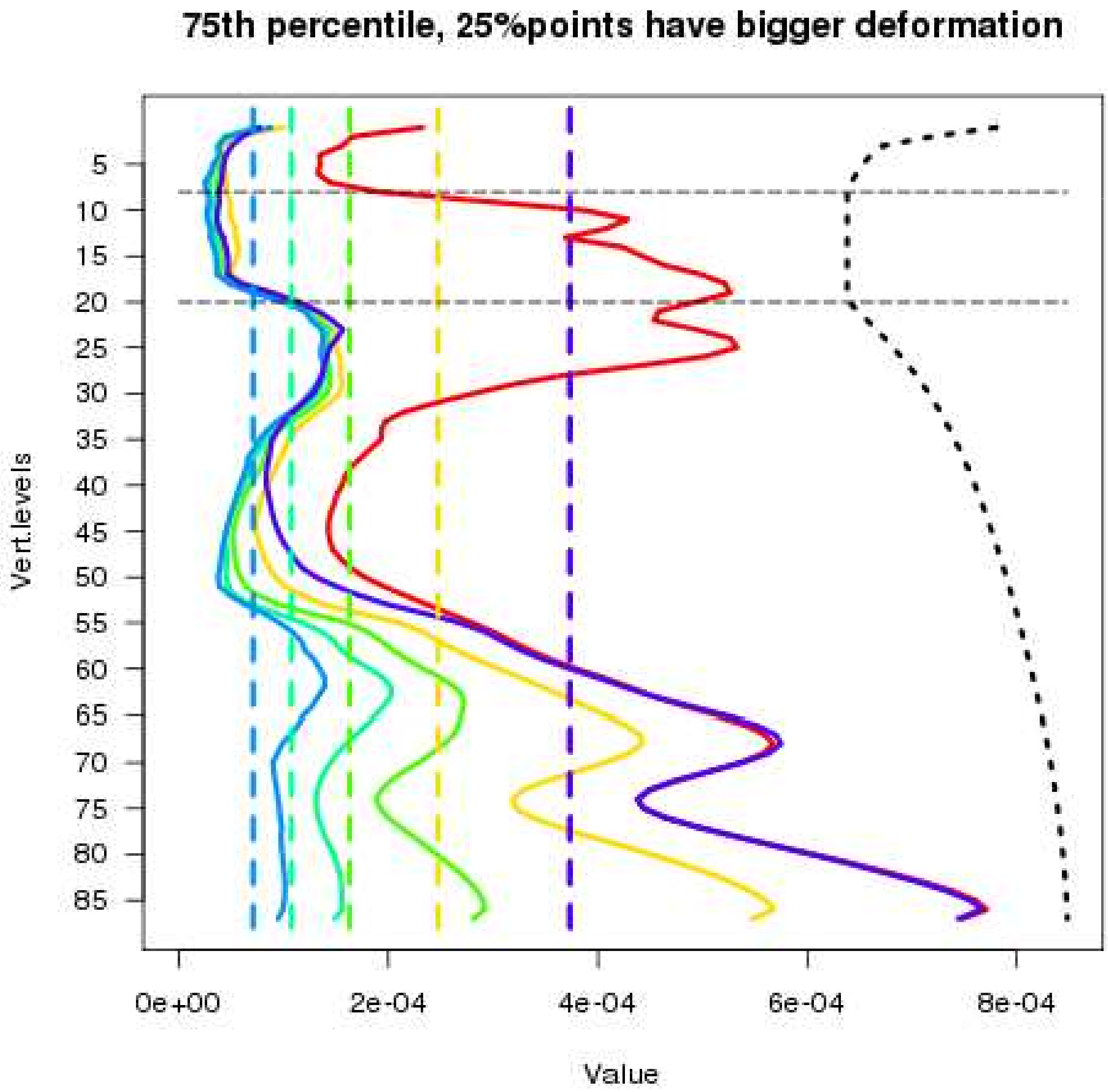


- - - temperature profile

Horizontal resolution:
1km
2km
4km
8km
16km

--- characteristic deformation based on resolution

SLHD tuning in ALARO 1km



- - - temperature profile

Horizontal resolution:
1km tuned
1km
2km
4km
8km
16km

--- characteristic deformation based on resolution

SLHD tuning in ALARO 1km

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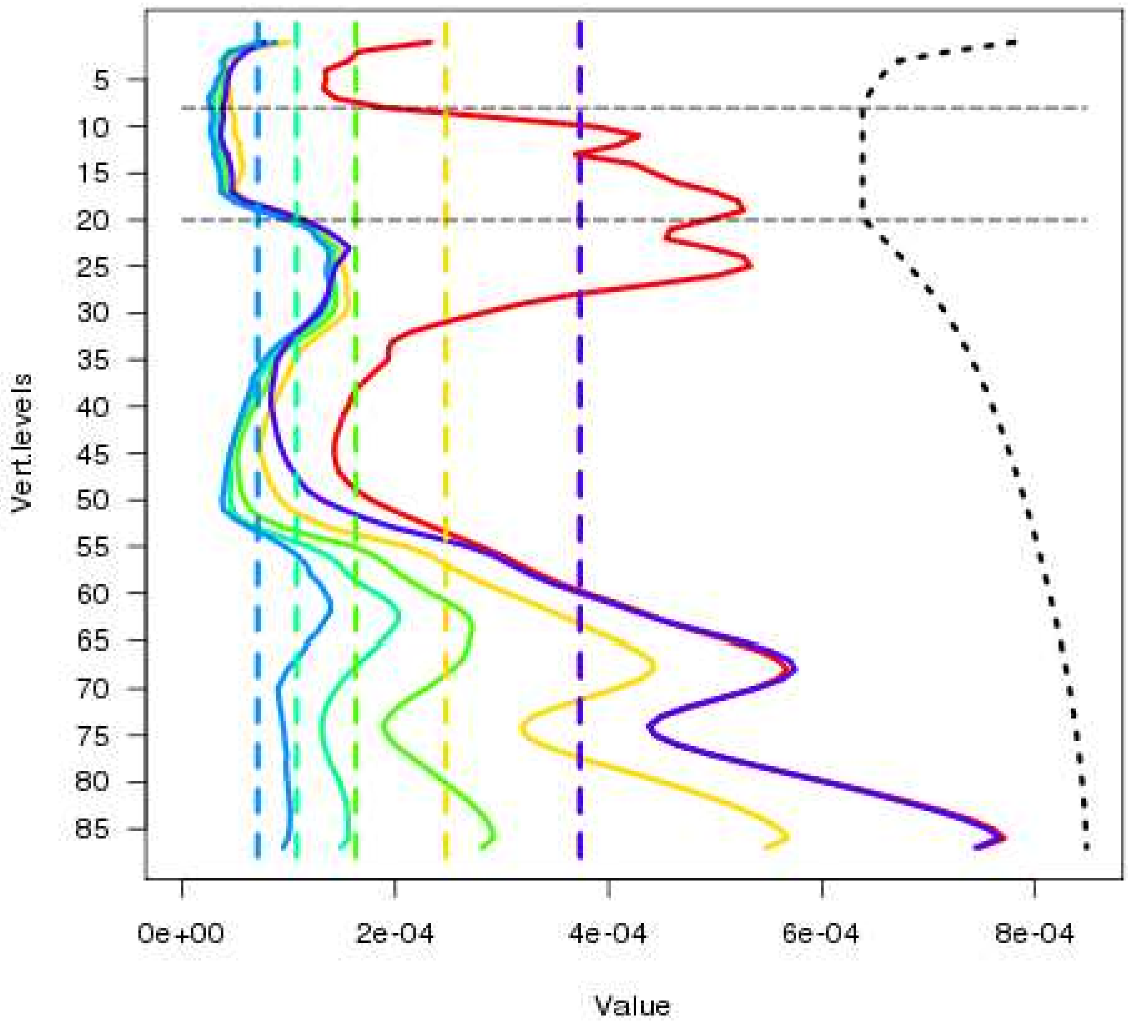
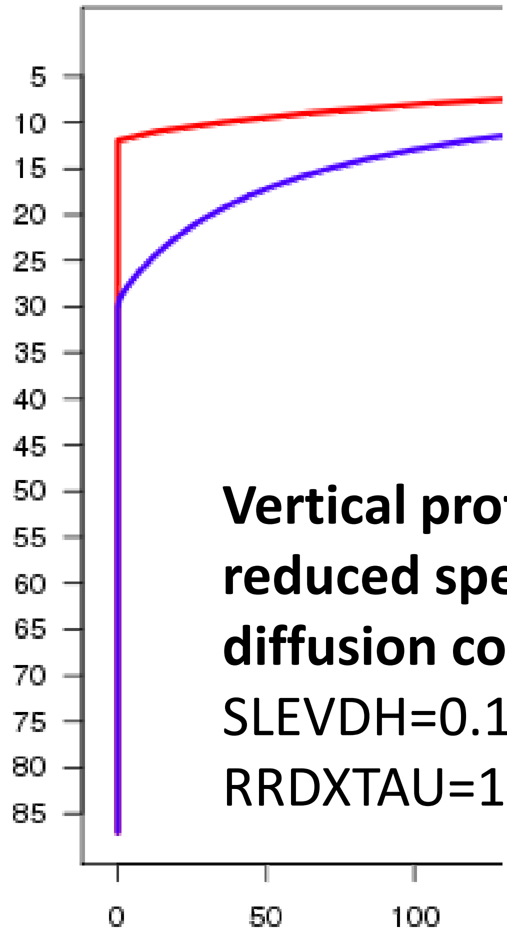
75th percentile, 25%points have bigger deformation



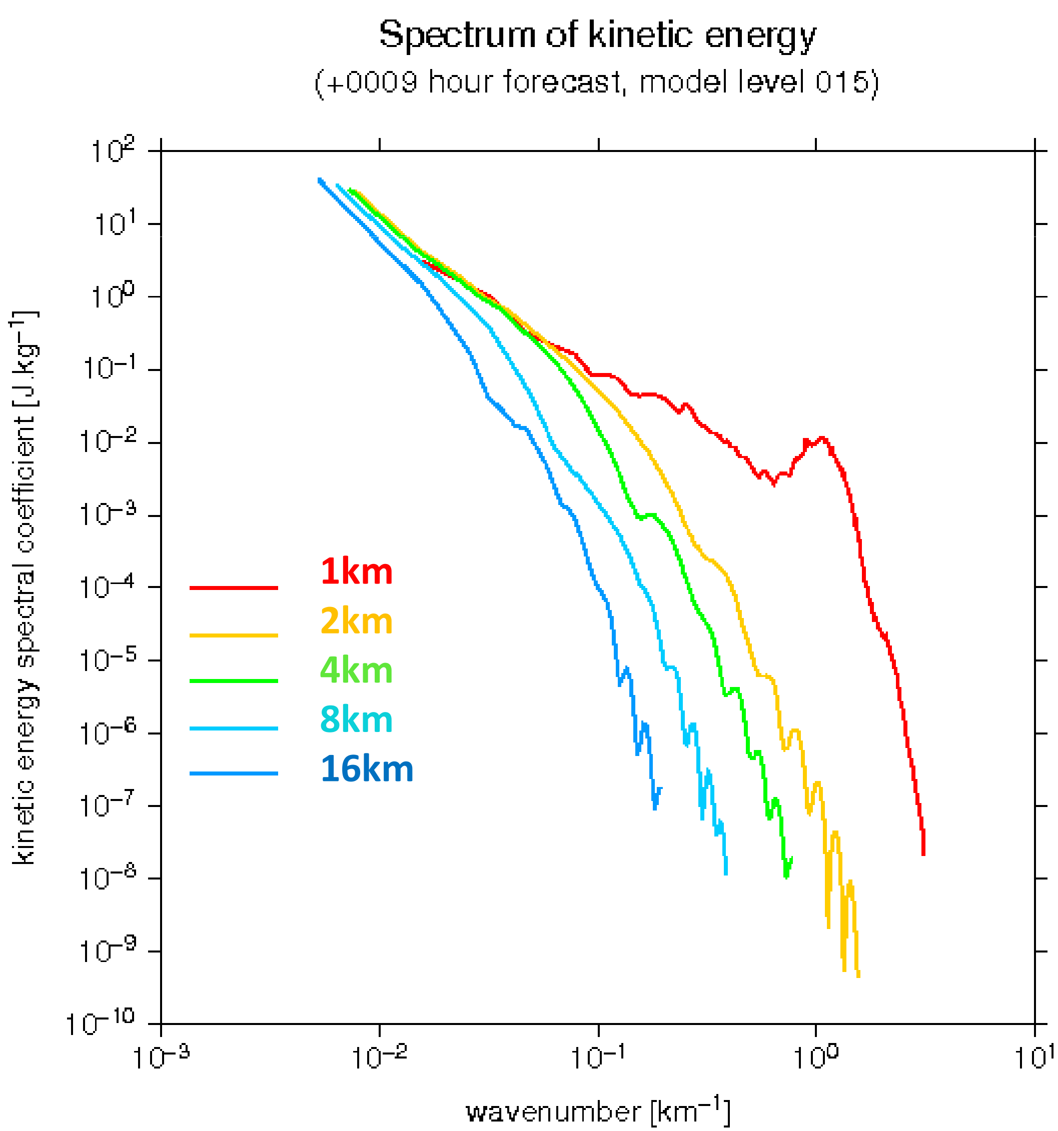


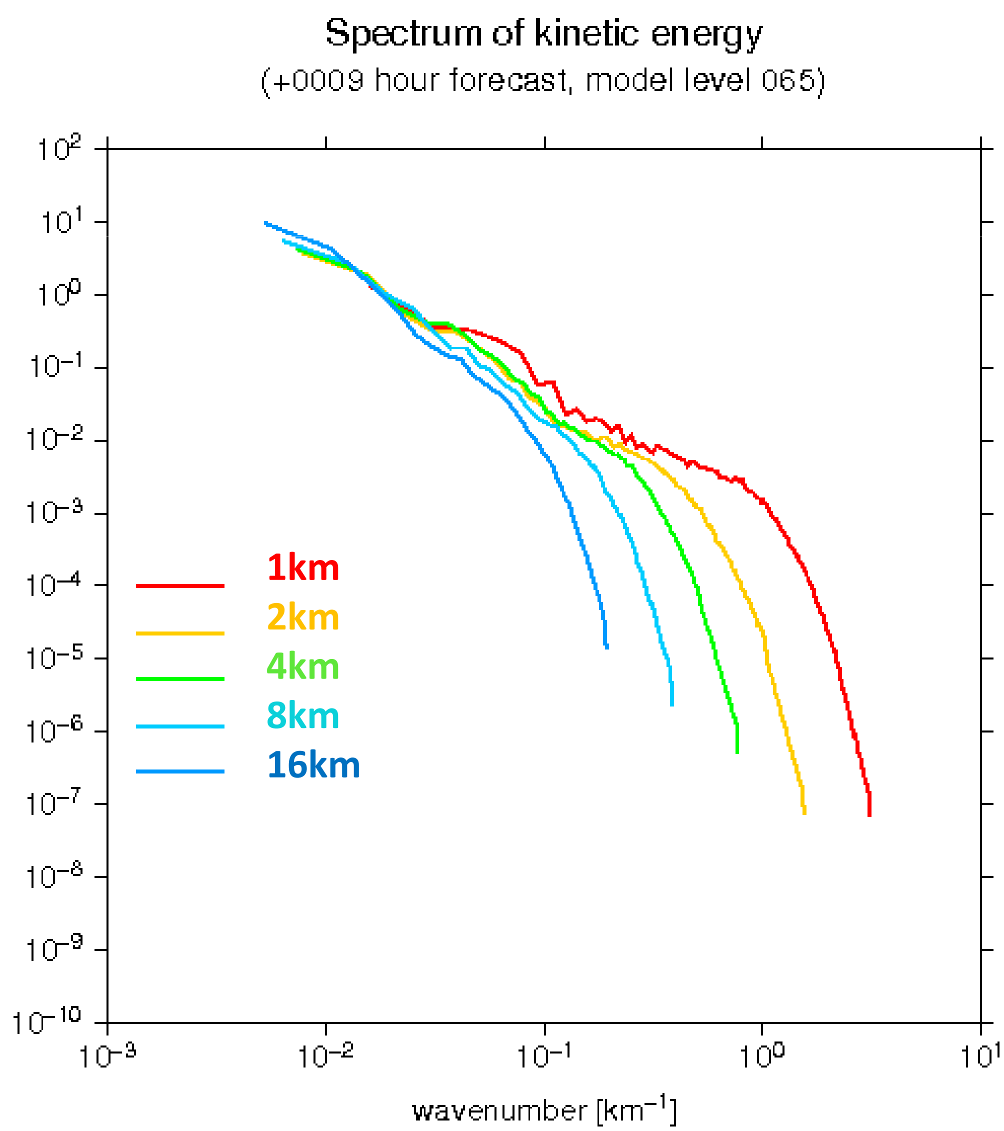
Vertical profile of reduced spectral diffusion coefficient:

SLEVDH=0.1 -> 0.4

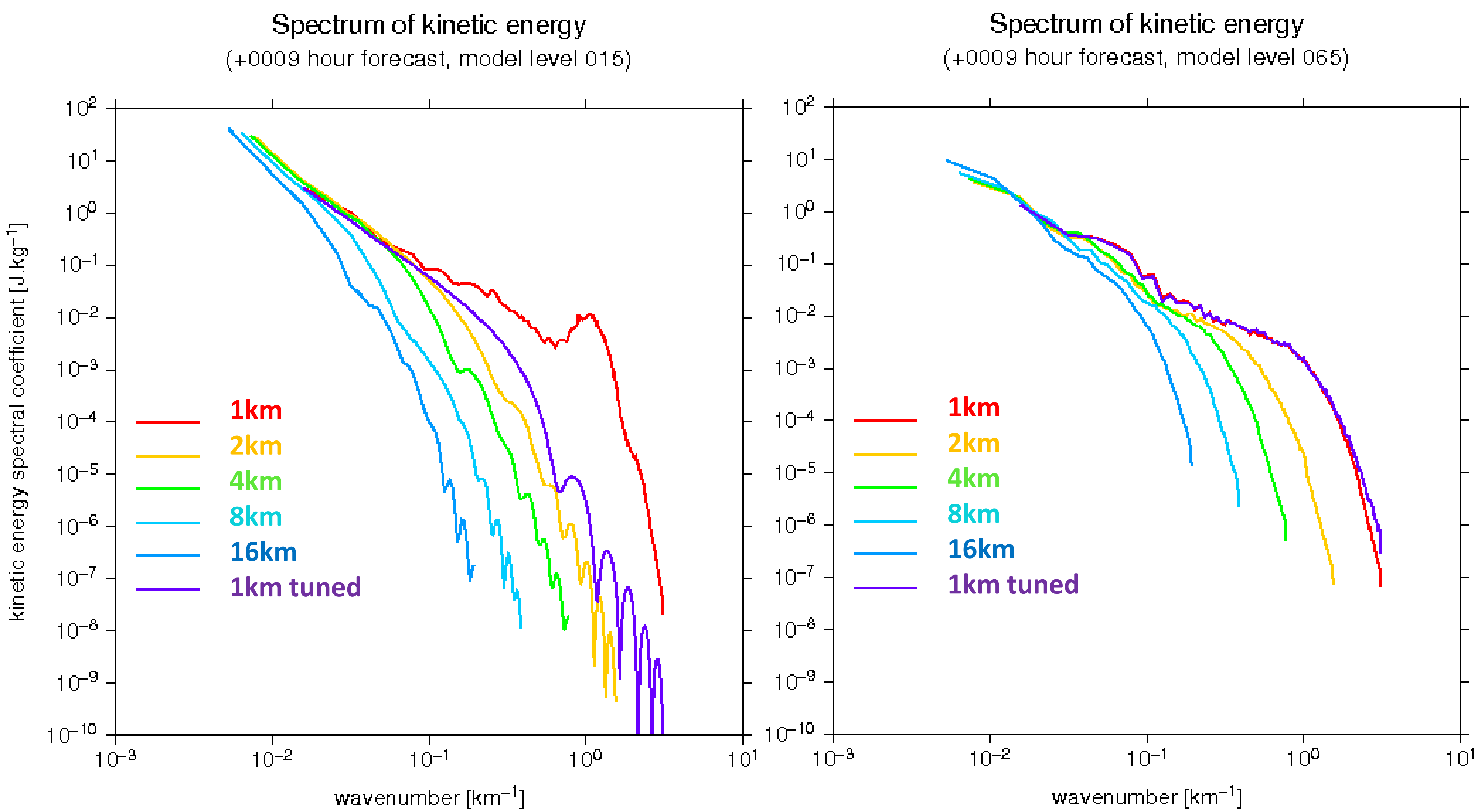
RRDXTAU=123. -> 31.

SLHD tuning in ALARO 1km

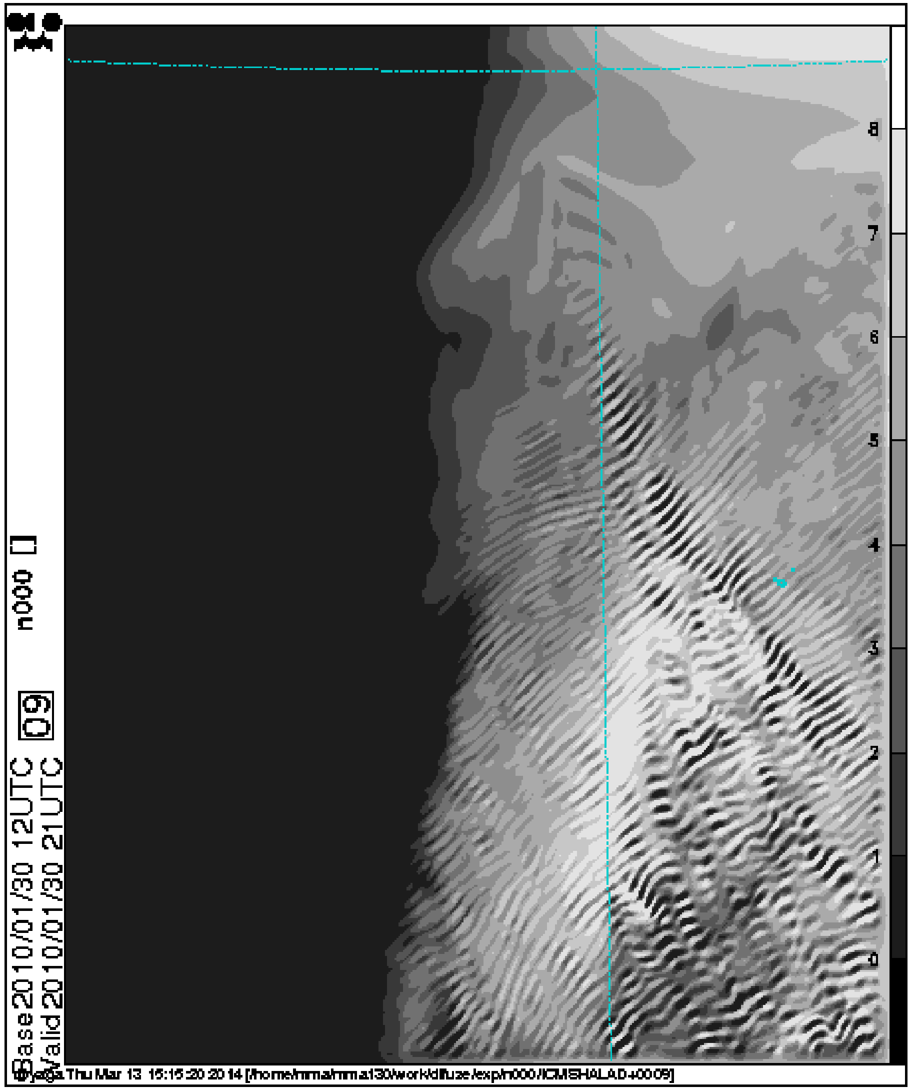




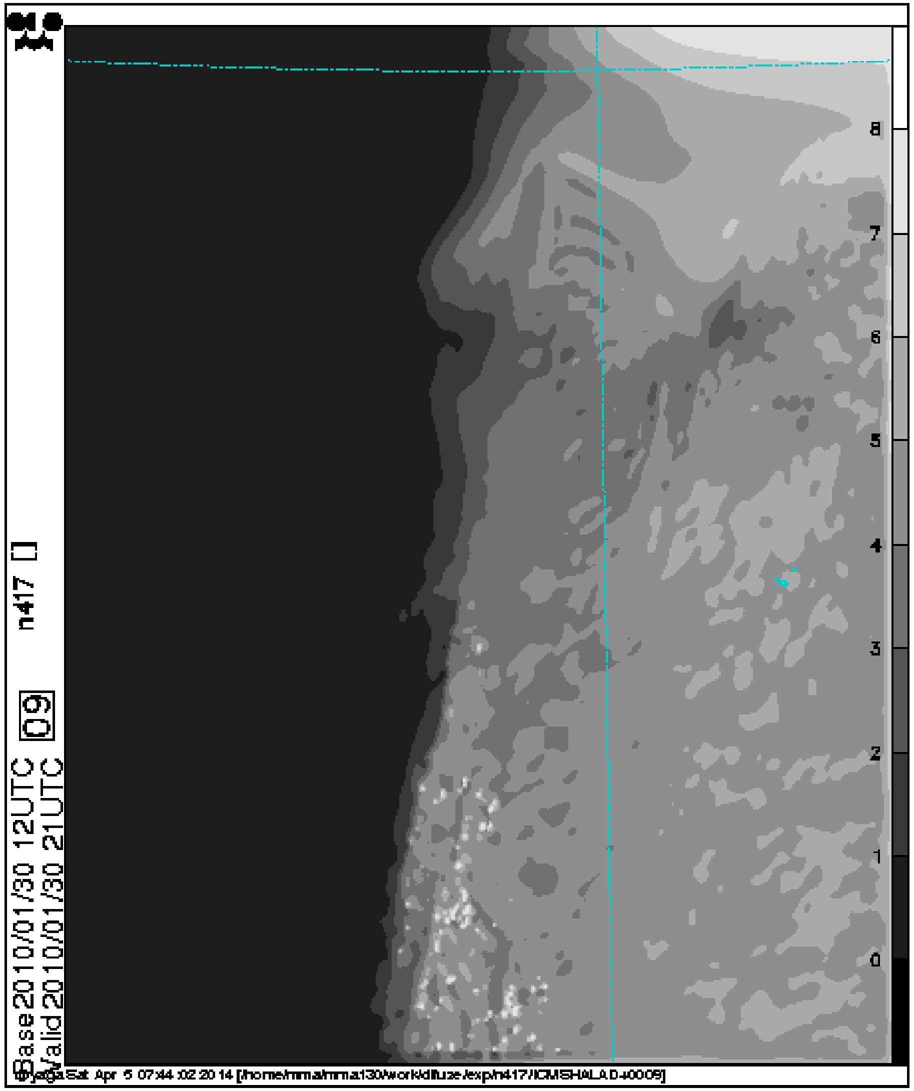
SLHD tuning in ALARO 1km



SLHD tuning in ALARO 1km



High level cloudiness: reference



tuned with SLEVDH=0.4,RRDXTAU=0.31


Thank you for your attention !
Ich danke Ihnen für Ihre Aufmerksamkeit!