Hierarchical Bayes Ensemble Kalman Filter

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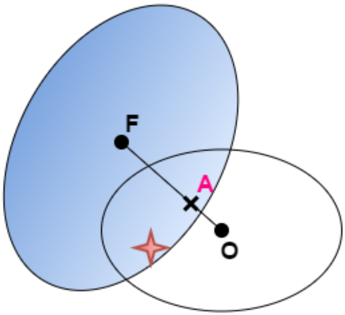
Specification of uncertainties in data assimilation is crucial to the analysis accuracy

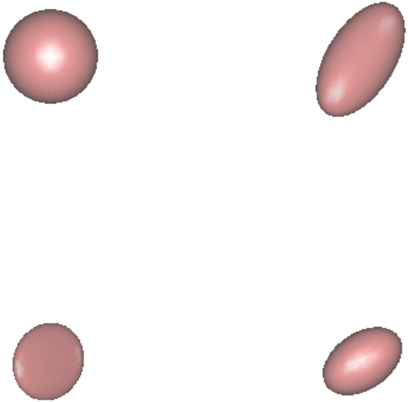
An example (see schematic below): the higher (assumed) background uncertainty (*left*) appears to be better than the lower assumed uncertainty (*right*).

(F stands for the forecast, O observations, A analysis, and the Star denotes the truth.)



Ensemble sample covariances are poor estimates

The true covariances: top left. Sample covariances (with the ensemble size 16) are the others.



Hierarchical Bayes Ensemble Kalman Filter (HBEF)

Starting point

1. $\mathbf{B}$ is random.
2. Ensemble members are observations on $\mathbf{B}$.

IDEA

- Build a secondary filter (a filter for the covariances)—just like the existing data assimilation techniques are built for the state.

Hypothesis

- The state is conditionally Gaussian given the covariances: $\mathbf{x}|\mathbf{B} \sim \mathcal{N}(\mathbf{m}, \mathbf{B})$

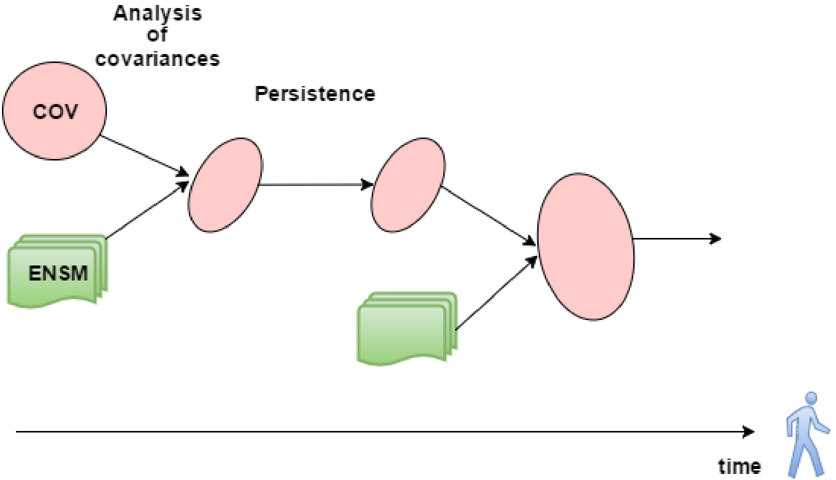
HBEF: Design

Simultaneously update both the state $\mathbf{x}$ *and the covariances* $\mathbf{B}$ *using:*

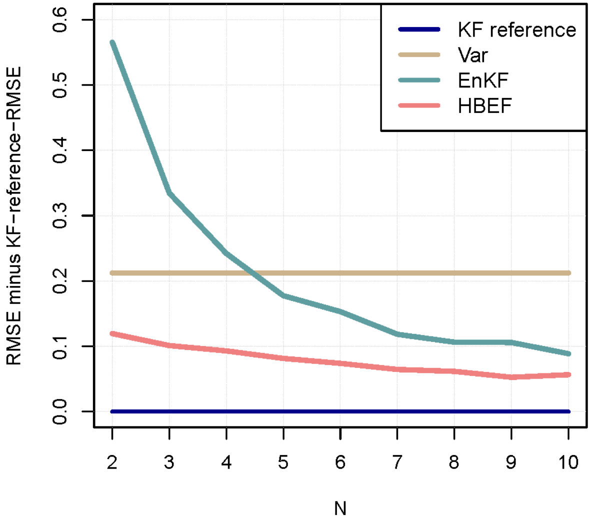
1. Background for
   - the state (traditional)
   - the covariances (new)
2. Observations for
   - the state (ordinary)
   - the covariances (ensemble members)

Secondary filter: start cycling the covariances

$$\mathbf{B}^a = w\mathbf{B}^f + (1-w)\mathbf{S}$$



Analysis RMSEs as functions of ensemble size



More info at the poster:

- How HBEF deals with non-Gaussian background-error distributions?
- How HBEF uses observations in the update of the covariances?
- Why is it worth splitting $\mathbf{B} = \mathbf{P} + \mathbf{Q}$?
- How HBEF can justify hybrid EnVar approaches?
- Which matrix variate probability distribution is more suitable to model the prior uncertainty in the covariances?
- How HBEF can be easily put into practice?
- Etc.

Conclusions

Key features of the HBEF

- Covariance matrices are treated as random matrices.
- Ensemble members are assimilated as generalized observations.
- The covariance matrices are subject to sequential Bayesian update.

Testing

- With a toy system, the HBEF (even in its simplest version) significantly outperformed Var and EnKF.

Implementation

- In its simplest version, the HBEF is computationally very affordable.

Download the manuscript from arXiv *or* ResearchGate.

Thank you and welcome to the poster!