# Recent advances on Dynamics at Météo-France.

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# Context

**Main features of the current dynamical kernel of IFS/ARPEGE Global Models and ALADIN/HIRLAM/ALARO/HARMONIE/AROME (LAMs)**

- Access by a switch to Fully-compressible (EE) or Hydrostatic (HPE) governing equations cast in time-dependent hybrid mass-based terrain-following $\eta$-coordinate.
- Horizontal Spectral (SP) transform discretisation.
- Semi-implicit (SI) time-stepping techniques.
- Semi-Lagrangian (SL) transport scheme.

**Need for changes**

- In the prospect of future HPC architectures, current status of the dynamical core need to be revisited for more flexible, efficient and scalable numerical algorithms.

# Overview

1. SWITCH TO (MODERN) SOUND-PROOF NH EQUATIONS
   - A Quasi-elastic system cast in mass-based coordinate
2. ABANDONING SPECTRAL TECHNIQUE ?
   - Accurate grid-point derivatives and diffusion
   - Grid-point SI mass-based Solvers
3. AN ALTERNATIVE TO SEMI-IMPLICIT TIME-SCHEMES ?
   - RK-IMEX HEVI schemes

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## Switch to modern Sound-proof NH equations

### Motivations

- Suppress high-frequency vertically-propagating acoustic wave at their source
  $\Rightarrow$ Potential benefit in term of stability, and for initialization procedure.
- Design an approximate set of sound-proof NH equations in mass-based coordinate, accurate at both small- and large- scales, by exploiting Arakawa and Konor (2009) idea together with Laprise (1992) formalism.
- Formulate "blended" numerical model, where different model configurations (soundproof-to-fully compressible) are seamlessly accessed by straightforward switching within a single numerical framework.

## Switch to modern Sound-proof NH equations

### Definition of pseudo-hydrostatic reference-state

Defining $(\pi, \widetilde{\rho}, \widetilde{T})$, the pseudo-hydrostatic thermodynamic variables by

$$\begin{aligned} \frac{\partial \pi}{\partial z} &= -\widetilde{\rho}\, g \\ \widetilde{\rho} &= p_{00}^{\kappa} \frac{\pi^{1-\kappa}}{R\theta} \exp[\xi(1-\kappa)q] \\ \widetilde{T} &= \frac{\pi}{R\widetilde{\rho}} \exp[\xi\, q] \end{aligned}$$

where $\kappa = R/C_p$, and $q$ denotes the pressure departure from the pseudo-hydrostatic pressure $\pi$, so that the true thermodynamic variables $(p, \rho, T)$ are entirely determined by

$$\begin{aligned} p &= \pi \exp[q] \\ \rho &= \widetilde{\rho} \exp[(1-\xi)(1-\kappa)q] \\ T &= \widetilde{T} \exp[(1-\xi)\kappa\, q] \end{aligned}$$

$\xi$ : binary switch for Quasi-elastic soundproof approximation.

# Switch to modern Sound-proof NH equations

## Basic underlying idea

True density $\rho$ is replaced by the pseudo-density $\widetilde{\rho}(\theta, \pi, q)$ is the mass-continuity equation:

$$\frac{D\widetilde{\rho}}{Dt} = \left(\frac{\partial\widetilde{\rho}}{\partial\theta}\right)_{\pi,q}\frac{D\theta}{Dt} + \underbrace{\left(\frac{\partial\widetilde{\rho}}{\partial\pi}\right)_{\theta,q}}_{\text{H}-\text{compressibility}}\frac{D\pi}{Dt} + \underbrace{\left(\frac{\partial\widetilde{\rho}}{\partial q}\right)_{\theta,\pi}}_{\text{NH}-\text{compressibility}}\frac{Dq}{Dt} = -\widetilde{\rho}\,\mathbb{D}_3$$

where

$$\left(\frac{\partial\widetilde{\rho}}{\partial q}\right)_{\theta,\pi} = \xi(1-\kappa)\widetilde{\rho}$$

$\mathbb{D}_3$ : three-dimensional divergence of wind.

## QE approximation ($\xi = 0$)

- Minimal condition for filtering vertically propagating acoustic waves,
- NH compressibility of the fluid is suppressed,
- Hydrostatic compressibility is maintained for good accuracy at large-scales.

# Switch to modern Sound-proof NH equations

**Prognostic equations cast in mass-based $\eta$-coordinate with $\pi = A(\eta) + B(\eta)\pi_s$**

$$\frac{DV}{Dt} = -\frac{\widetilde{\rho}}{\rho}\left[\frac{1}{m}\frac{\partial p}{\partial \eta}\nabla\phi + \frac{1}{\widetilde{\rho}}\nabla p\right] + \mathcal{V}$$

$$\gamma\frac{Dw}{Dt} = g\frac{\widetilde{\rho}}{\rho}\left[\frac{1}{m}\frac{\partial(p-\pi)}{\partial \eta} - \frac{\rho - \widetilde{\rho}}{\widetilde{\rho}}\right] + \gamma\mathcal{W}$$

$$\frac{D\widetilde{T}}{Dt} = \frac{R\widetilde{T}}{C_p}\left(\frac{\dot{\pi}}{\pi} + \xi\frac{Dq}{Dt}\right) + \frac{\mathcal{Q}\widetilde{T}}{C_p T}$$

$$\xi\frac{Dq}{Dt} = -\frac{\dot{\pi}}{\pi} - \frac{C_p}{C_v}\mathbb{D}_3 + \frac{\mathcal{Q}}{C_v T}$$

$$\frac{\partial \pi_s}{\partial t} = -\int_0^1 \nabla\cdot(mV)\,\mathrm{d}\eta'$$

$\gamma$ : binary switch for Hydrostatic approximation.

# Switch to modern Sound-proof NH equations

## Diagnostic equations :

$$
\begin{aligned}
m &= \frac{\partial \pi}{\partial \eta} \\
\nabla \phi &= \nabla \phi_s + \int_{\eta}^{1} \nabla \left( m \frac{R\widetilde{T}}{\pi}\, e^{-\xi q} \right) \mathrm{d}\eta' \\
\frac{\dot{\pi}}{\pi} &= V \cdot \frac{\nabla \pi}{\pi} - \frac{1}{\pi} \int_{0}^{\eta} \nabla \cdot (mV)\, \mathrm{d}\eta' \\
m\, \dot{\eta} &= B(\eta) \int_{0}^{1} \nabla \cdot (mV)\, \mathrm{d}\eta' - \int_{0}^{\eta} \nabla \cdot (mV)\, \mathrm{d}\eta' \\
\mathbb{D}_3 &= \nabla \cdot V + \frac{\widetilde{\rho}}{m} \nabla \phi \cdot \frac{\partial V}{\partial \eta} - g\, \frac{\widetilde{\rho}}{m} \frac{\partial w}{\partial \eta}
\end{aligned}
$$

# Switch to modern Sound-proof NH equations

## Different model configurations : "Switchable" equation set

1. **EE case** $(\xi, \gamma) = (1, 1)$ : $[V, w, \widetilde{T}, q, \pi_s]$ are prognostic variables.
2. **HPE case** $(\xi, \gamma) = (1, 0)$ : $[V, \widetilde{T}, \pi_s]$ are prognostic variables, $p = \pi \Rightarrow q = 0$ and pressure Eq. is used to diagnose $w$.
3. **QE case** $(\xi, \gamma) = (0, 1)$ : $[V, w, \widetilde{T}, \pi_s]$ are prognostic variables, and $q$ is diagnosed from the pressure Eq., which becomes a divergence constraint relationship.

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## Abandoning Spectral technique: Derivatives and Diffusion

### Two parallel approaches explored at ECMWF + M-F

- FVM (Smolarkiewicz P., Kuehnlein C.,...) effort focused on: local coordinate, unstructured grid, conservation.
- GlobSWAR (Bénard, Caluwaerts,...) effort focused on: global spherical coord., structured grid, high-order in space also trying to keep most of the existing architecture of IFS/ARPEGE models

### "Keep most existing architecture" means:

- Global Spherical coordinate
- Reduced Lat-Lon grid (semi-structured)
- Unstaggered grid (A-grid)

# Abandoning Spectral technique: Derivatives and Diffusion

## Spherical coord. + Reduced A-grid : not the "easiest" choice!

Two psychological obstacles must be overcome:

- Well-known "pole problem(s)" of spherical coordinates, especially with reduced lat-lon grids
- Well-known gravity wave distortion problem of A-grid

## But some advantages also:

- semi-structured grid is a potential advantage (efficiency, easier access to high-order)
- A-grid has the best balance of DoFs ever.
- A-grid is much better for transport/advection than C-grid

**⇒ We build a Proof-of-Concept SW model: GlobSWAR**

## Pole problem – special care required :

- very accurate zonal derivatives near poles
  (e.g. Fourier representation in a small cap)
- very careful definition of BCs at poles for meridional derivatives
- some continuous formulations more convenient than others
  (e.g. curl form mom. eqs, flux form cont. eq, term grouping...)
- Zonal averages $\langle u \rangle, \langle v \rangle$ must have specific BCs for meridional derivatives
  (for stability reasons)

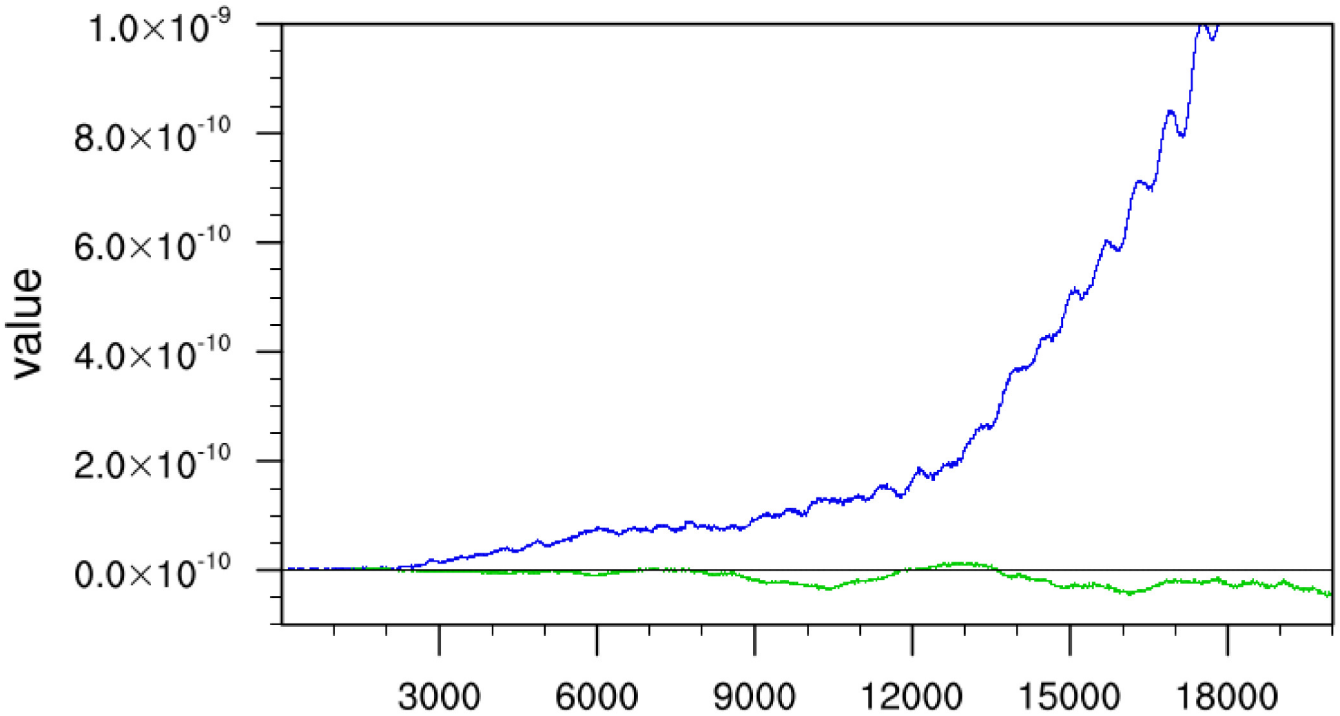
## A-Grid

- needs a good filter beyond $3\Delta x$ wave
  (required for anti-alias of $\psi^2$ in any case)
- no special problem encountered with A-grid

# Abandoning Spectral technique: Derivatives and Diffusion

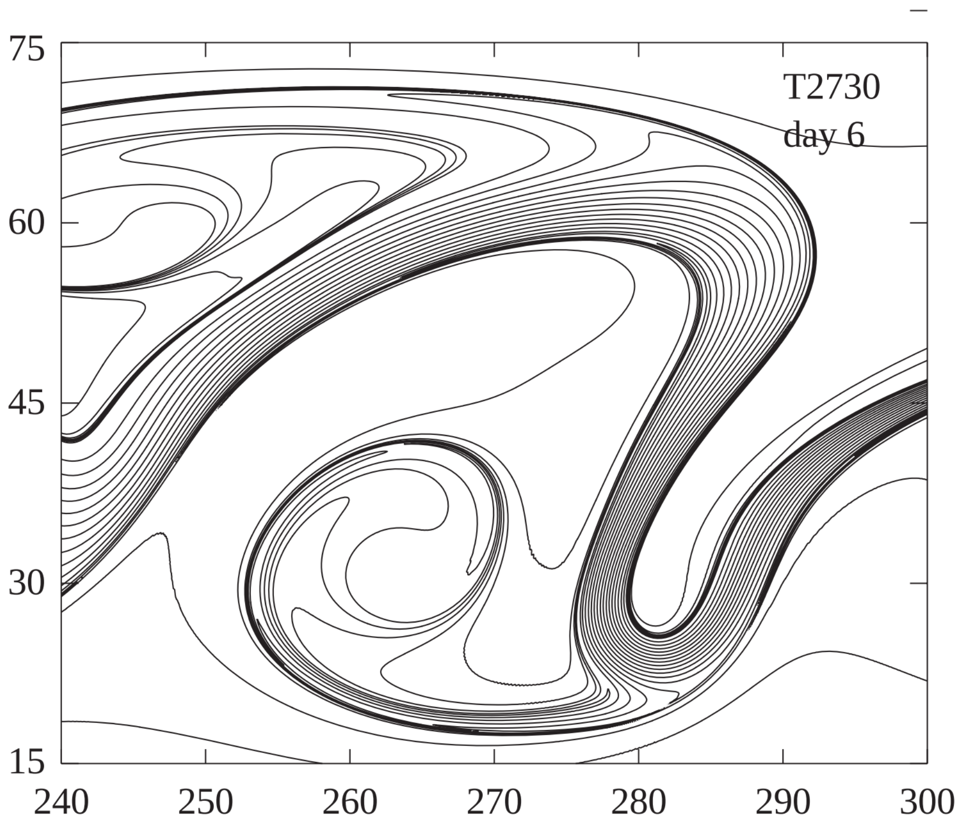
## GlobSWAR: PSBR test-case

- Demonstrate ability to keep simple stationary parallel flows, actually stationary without any damping/diffusion $\rightarrow$ tilted Solid-Body rotation test-case for $(\alpha = 0°, 45°, 90°)$.
  Blue : BC problem for $\langle u \rangle, \langle v \rangle$ not solved.
  Green: BC problem solved

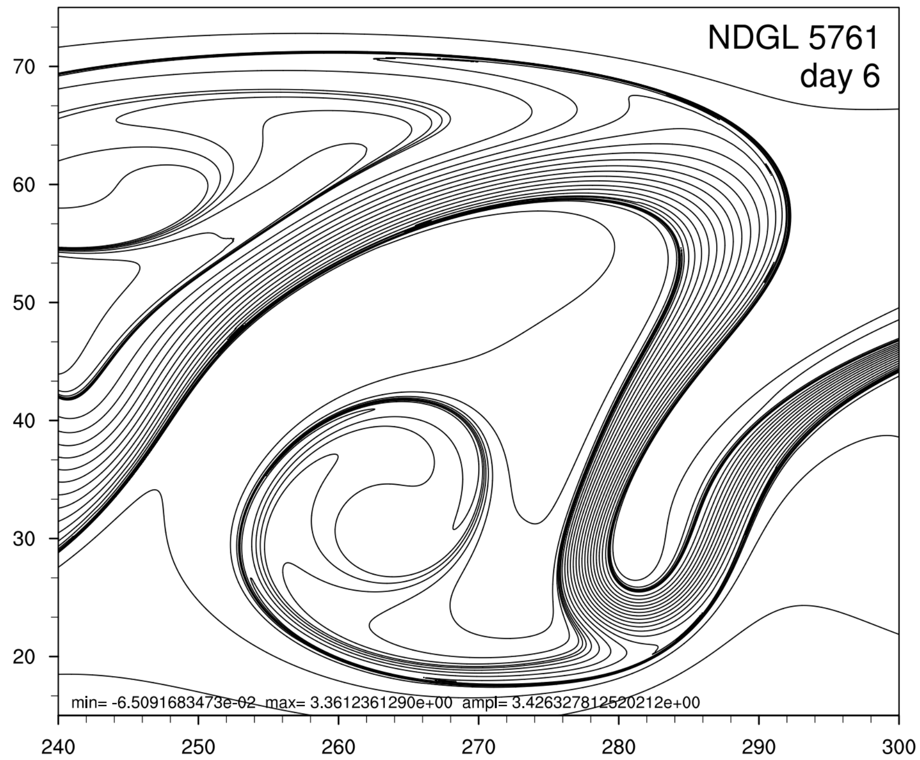


## GlobSWAR: Scott et al. (2016) baroclinic test-case

- Demonstrate ability to perform demanding cases at high resolution (at 3.5 km grid mesh). Barotropic instability pushed near end of determinism Reference solutions = models BOB (Columbia Univ.), and FV3 (GFDL)




BOB model




globSWAR model

# Grid-point SI mass-based solvers

## State of art of current constant-coefficient Spectral SI mass-based NH EE solver

- Horizontal and vertical divergence resp. $(D, d)$ are used as auxiliary prognostic variables in implicit part of the SI scheme.
- At the price of the fulfilment of two vertical constraints, the full linear implicit system is eventually reduced to

$$\left[I - \delta t^2 \mathbf{B}_{\mathrm{d}}^{*} \Delta\right] d^{+} = d^{\bullet}$$
$$\left[I - \delta t^2 c_{*}^{2} \Delta\right] D^{+} = D^{\bullet} + \delta t^2 \mathbf{G}_{\mathrm{d}}^{*} d^{+}$$

- $\mathbf{B}_{\mathrm{d}}^{*}$ is a definite positive constant-coefficient vertical matrix, $c_{*}$ is the constant speed of sound.
- $d^{+}$ and $D^{+}$ are obtained by solving successively **two** 2D horizontal Helmholtz-like equations for each vertical eigenmodes of $\mathbf{B}_{\mathrm{d}}^{*}$, enjoying the fact that the horizontal laplacian operator $\Delta$ is **diagonal** in spectral space.

# Grid-point SI mass-based solvers

## A constant-coefficient Grid-point SI mass-based NH solver

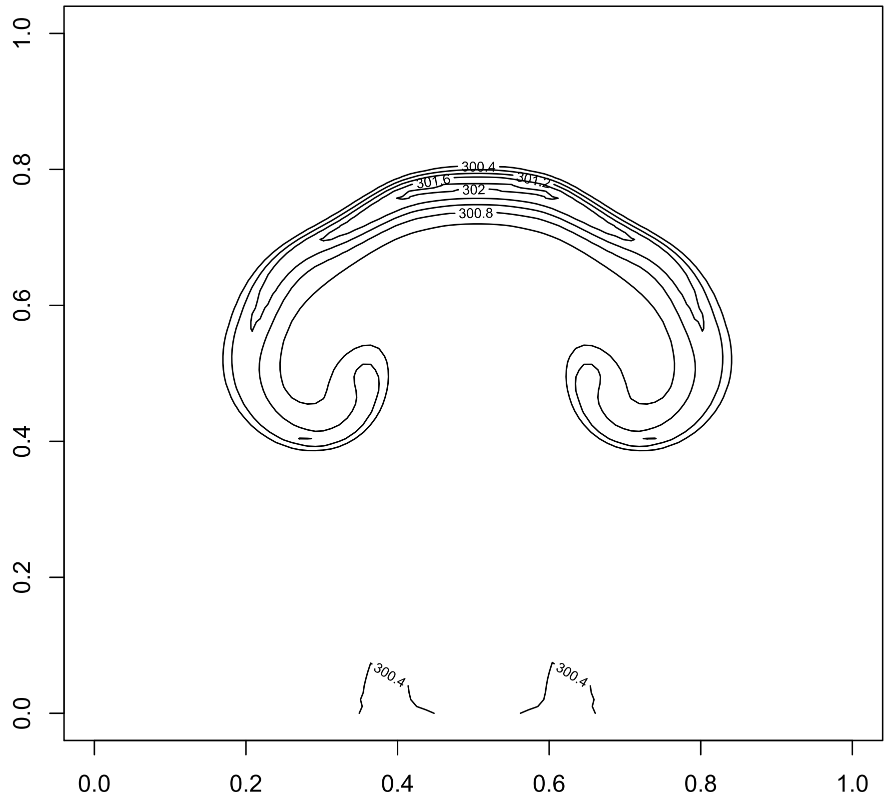
- Elimination must be performed in a different manner, relaxing the less restrictive of the two vertical constraints:

$$\left[I - \delta t^2 \mathbf{B}_\mathrm{D}^* \nabla^2\right] D^+ = D^{\bullet\bullet}$$
$$\mathbf{H}_\mathrm{v}^* q^+ = Q^\bullet + \delta t^2 \mathbf{S}_\mathrm{D}^* D^+$$

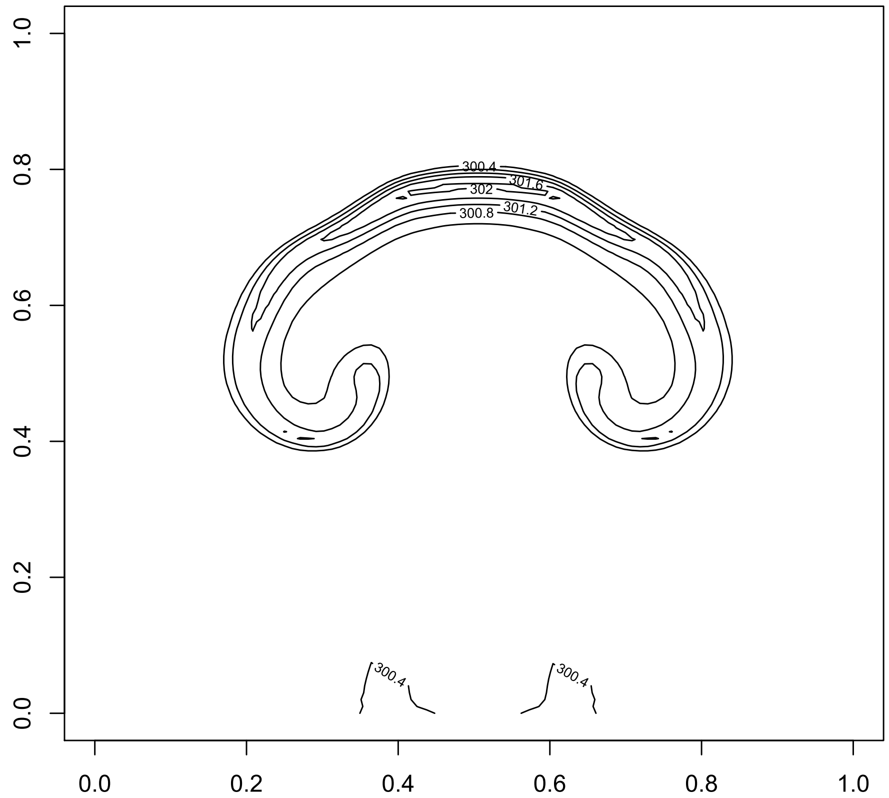
- $\mathbf{B}_\mathrm{D}^*$ is a newly defined constant definite positive vertical matrix ensuring that $\lambda(\mathbf{B}_\mathrm{D}^*) = \lambda(\mathbf{B}_\mathrm{d}^*)$, and $\mathbf{H}_\mathrm{v}^*$ is invertible tridiagonal constant vertical matrix.
- A **single** 2D horizontal Helmholtz-like equation for each vertical eigenmode of of $\mathbf{B}_\mathrm{D}^*$ is solved for projection of $D^+$ using iterative of Krylov solvers : GMRES or GCR(k).
- A simple tridiagonal **vertical inversion** to get $q^+$.

## Test in a vertical Slice ($x$-$\eta$) version of AROME model: Rising bubble test-case

- GMRES grid-point Krylov solver using 6th-order finite differences derivatives is compared to current Spectral solver.




Spectral solver




Grid-point solver

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## Alternative to SI time-schemes

### Basic principle of HEVI time-stepping approaches for solving EE equations

- Implicit time treatment of the vertical adjustment terms responsible for vertically propagating high-frequency acoustic modes ⇒ 1D vertical Helmholtz-like problems to solve.
- Explicit time treatment of the remaining processes ⇒ stability is mostly control by the horizontal wave Courant (CFL) number.

### "Our" Desirable properties for HEVI schemes

- No time filtering applied ⇒ Multi-steps time-schemes are discarded ⇒ Multi-stages time-stepping is favoured.
- No artificial divergence Damping either ⇒ Classical time-splitting approach is not retained ⇒ "monolithic" temporal treatment of all processes, i.e using a single time-step.

⇒ **Recent RK-IMEX HEVI schemes appear to be good candidates**

## Alternative to SI time-schemes

### Classical RK-IMEX HEVI schemes with double Butcher's tableaux

$$\partial_t X = \mathscr{M}(X) = \underbrace{\mathscr{E}(X)}_{\text{"explicit"}} + \overbrace{\mathscr{I}(X)}^{\text{"implicit"}}.$$

Runge-Kutta scheme for each part:

$$\overbrace{X^{(j)} - a_{jj}\delta t \mathscr{I}(X^{(j)})}^{1D\ Helmholtz\ equation} = \overbrace{X^t + \delta t \sum_{k=1}^{j-1} \tilde{a}_{jk}\mathscr{E}(X^{(k)}) + \delta t \sum_{k=1}^{j-1} a_{jk}\mathscr{I}(X^{(k)})}^{explicit\ part}$$

$$X^{t+\delta t} = X^t + \delta t \sum_{j=1}^{\nu} \tilde{b}_j \mathscr{E}(X^{(j)}) + \delta t \sum_{j=1}^{\nu} b_j \mathscr{I}(X^{(j)})$$

Coefficients are resumed in a double Butcher's tableau : $\{\tilde{c}, \tilde{\mathcal{A}}, \tilde{b}\}$ and $\{c, \mathcal{A}, b\}$ (where $\tilde{\mathcal{A}} = (\tilde{a}_{ji})_{1\leq j,i\leq\nu}$ et $\mathcal{A} = (a_{ji})_{1\leq j,i\leq\nu}$).

# Alternative to SI time-schemes

## HEVI schemes constraints

- *Vertical adjustment terms* must to be deat with implicitly and all *advection terms* have to be treated explicitly.
- The horizontal momentum equations and pressure (mass continuity) equation may be subjected to Foreward/backward time treament in the same spirit as Mesinger [1977] : Wind-Forward and Pressure-Backward (UFPreB) (Lock *et al.* [2014]).
- Pushing this idea further, we exploit the fact that Pressure-Forward and Wind-Backward (UBPreF) is also a possible option to elaborate more stable RK-IMEX HEVI scheme in presence of advection, Colavolpe *et al.* [2017].

# Alternative to SI time-schemes

## A proposed "Mixed" four Butcher's tableaux RK-IMEX HEVI scheme

$$\partial_t X = \underbrace{\overset{advection\ (explicit)}{\mathscr{A}(X)}} + \underbrace{\overset{vert.adj.\ (implicit)}{\mathscr{I}(X)}} + \underbrace{\mathscr{U}(X)}_{hor.grad(p)\ (foreward\,/\,backward)} + \underbrace{\mathscr{P}(X)}_{hort.div(\mathbf{v})\ (backward\,/\,foreward)}$$

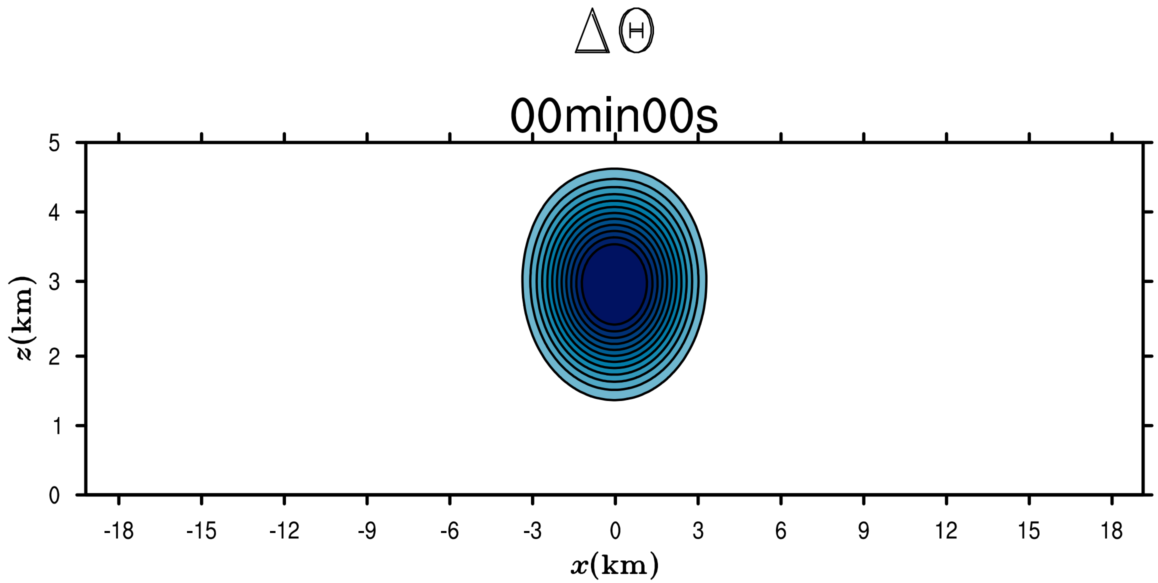
Hence

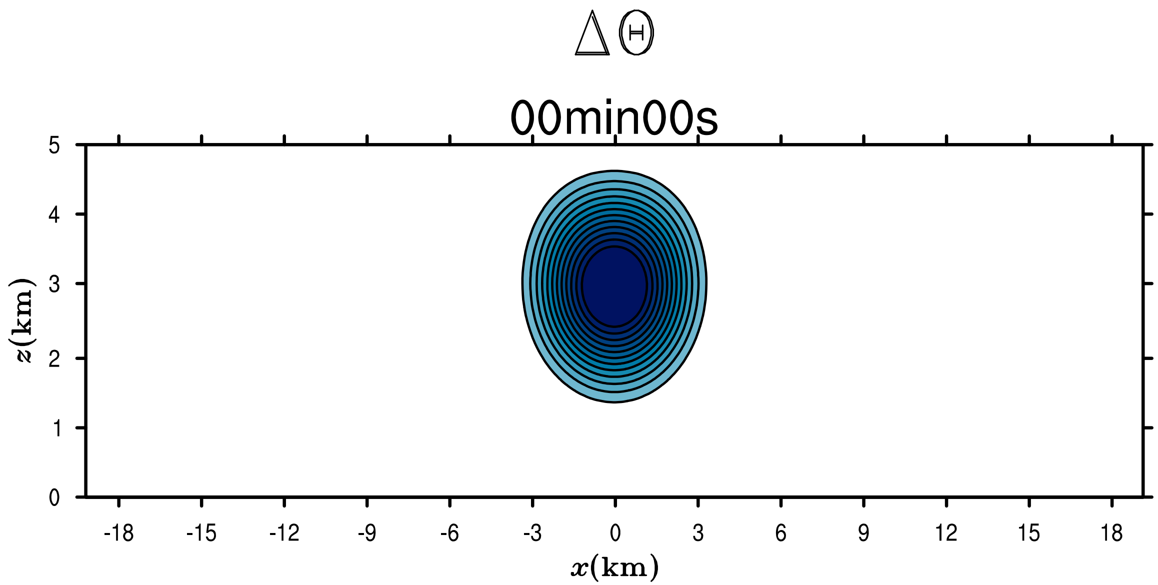
$$\frac{X^{(j)} - X^t}{\delta t} = \sum_{k=1}^{j-1} \tilde{a}_{jk} \mathscr{A}(X^{(k)}) + \sum_{k=1}^{j} a_{jk} \mathscr{I}(X^{(k)}) + \sum_{k=1}^{j} a^u_{jk} \mathscr{U}(X^{(k)}) + \sum_{k=1}^{j} a^p_{jk} \mathscr{P}(X^{(k)})$$

$$\frac{X^{t+\delta t} - X^t}{\delta t} = \sum_{j=1}^{\nu} \tilde{b}_j \mathscr{A}(X^{(j)}) + \sum_{j=1}^{\nu} b_j \mathscr{I}(X^{(j)}) + \sum_{j=1}^{\nu} b^u_j \mathscr{U}(X^{(j)}) + \sum_{j=1}^{\nu} b^p_j \mathscr{P}(X^{(j)})$$

- Coefficients of the four Butcher's tableau $\{\tilde{c}, \tilde{\mathcal{A}}, \tilde{b}\}$, $\{c, \mathcal{A}, b\}$, $\{c^u, \mathcal{A}^u, b^u\}$ and $\{c^p, \mathcal{A}^p, b^p\}$ must satisfied Colavolpe *et al.* [2017] conditions.
- Alternative "Foreward/Backward" treatment of "horizontal adjustment terms" : *pressure gradient/wind divergence*.
- No computational overcost, second-order accuracy, storage and number of vertical inversions.

# Abandoning SI time-schemes




**Straka et *al.* (1993) test-case**

$\Delta x == 75\,\mathrm{m}$, $\Delta z \approx 75\,\mathrm{m}$,
$\Delta t = 0.125\,\mathrm{s} \Rightarrow C^* \approx 1.8$

8-*th* order horizontal FD,
2-*nd* order vertical FD.

- **Top** : Classical UJ13(1,3,2) double butcher's tableaux.

- **Bottom** : New UJ13(1,3,2) mixed four butcher's tableaux.

# On-going Works

- Implementation of the QE dynamical system as a switch option in the full 3D common code is in progress (almost done). Testing and validations will be the next moves.
- A semi-implicit version of GlobSWAR model is first envisaged, before extending to multi-level GlobHPAR model.
- We are now ready to introduce a more complex linear SI operator including the variable-coefficient in order to deal with steep orography. This study will be pursued in the Vertical slice version of AROME using Krylov Grid-point Solver approach.
- 3D prototype of mass-based EE model integrating over a Cartesian domain through our best RK-IMEX HEVI time-scheme should be available soon.