Estimation of model errors due to unresolved scales using coarse-graining: a study with COSMO

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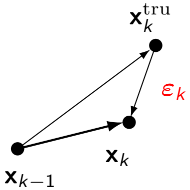
Motivation

- Definition: model error = model tendency error.
- Model error is an important source of uncertainty and is to be accounted for in both EDA and EPS.
- Still very little is known with certainty about model errors. Ad-hoc approaches like SPP or SPPT are in use.
- SPPT is not perfect:
  1. All variables are multiplied (at a grid point) by the *same* random number – as if the error is only in the magnitude of the physical tendency whilst the relationships between different variables is *correct*.
  2. Zero physical tendency implies *zero error*, which is also unphysical.
- Our previous approach to objectively estimate model errors by comparing model tendencies with observed tendencies failed (Tsyrulnikov and Gorin, COSMO Newsletter N 13, 2013): existing observation networks are far too scarce and observations too imprecise to detect the (very small nowadays) model error.
  - Therefore, we decided to estimate model errors by comparing model tendencies with those from a "true" model.

Model error definition

- Model equation: $\mathbf{x}_k = F(\mathbf{x}_{k-1})$
- "True model": $\mathbf{x}_k^{\rm tru} = F^{\rm tru}(\mathbf{x}_{k-1}^{\rm tru})$
- One-step error:

start the true model from $\boxed{\mathbf{x}_{k-1}^{\rm tru} = \mathbf{x}_{k-1}}$ (the "same start condition").



The difference $\boxed{\varepsilon_k = \mathbf{x}_k - \mathbf{x}_k^{\rm tru} = \mathsf{T}_k - \mathsf{T}_k^{\rm tru}}$ is the model error.

But what is the "same start condition" $\mathbf{x}_{k-1}^{\rm tru} = \mathbf{x}_{k-1}$ if the two models live in different spaces (having different resolutions and, possibly, different sets of model fields)?

Approach

For a low-resolution (coarse-grid) model in question, compute the model error with respect to a significantly higher-resolution (fine-grid) model.

That is, start the two models from "the same initial data", compute the two short-term tendencies and claim that their difference is the model error.

The fine-grid fields are upscaled (coarse-grained, smoothed) before being compared with the coarse-grid fields. *(Averaging over all fine-grid cells within each coarse-grid cell is performed.)*

A lesson from a previous stage of the study

- Model errors due to convection appear to be too complicated (and organized) to be treated with a purely stochastic field model. A physical model is needed for this purpose.
- Recommendation: A stochastic convection parameterization is to be used to represent the impact of unresolved convection in an ensemble prediction system.

Setup

1. The lowRes model was the local-area model COSMO-2.2 km L65.
   The hiRes ("true") model was COSMO-0.22 km L65 (with or without differences in physics).
2. Convection: we selected (presumably, non-convective) winter cases and switched off the convective parameterization.
3. Domain: 187*187 km in the North Sea.
4. The tendencies were computed after a 3–5h *pre-forecast*.
5. The length of the tendencies varied from 1 min to 30 min.

Generation of perfect model-error perturbations in an EPS (when a true model is available)

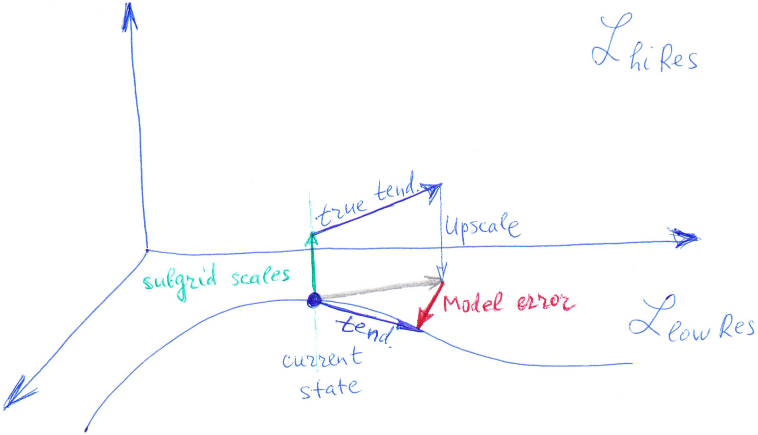
1. Take a lowRes-forecast (an ensemble member) at some lead time, $\mathbf{x}$. The lowRes tendency at this point is $F(\mathbf{x})$.
   We wish to compute the true (hiRes) model tendency at this point.
2. To do so, we have to (somehow) add sub-grid scales ξ to $\mathbf{x}$.
3. Start hiRes model from $\mathbf{x}+\xi$ and compute the hiRes tendency $F^{\mathrm{tru}}(\mathbf{x}+\xi)$.
4. Project $F^{\mathrm{tru}}(\mathbf{x}+\xi)$ onto lowRes-space $\mathcal{L}_{\mathrm{lowRes}}$ (i.e. smooth or *upscale* it), getting $\mathrm{Up}[F^{\mathrm{tru}}(\mathbf{x}+\xi)]$ – the ideal perturbed tendency.

Hence the model-error definition:

$$\varepsilon = F(\mathbf{x}) - \mathrm{Up}[F^{\mathrm{tru}}(\mathbf{x}+\xi)]$$

Note that, due to the presence of random ξ, model error is, largely, model uncertainty.
A lowRes model state is consistent with infinitely many hiRes states, whose difference from the lowRes state is essentially random.

Generation of perfect model-error perturbations



Having this definition of model error, how can we estimate $\varepsilon = F(\mathbf{x}) - \mathrm{Up}[F^{\mathrm{tru}}(\mathbf{x} + \boldsymbol{\xi})]$?

Estimation of perfect model-error perturbations

The major problem N1 is that a realistic stochastic model for sub-grid scales (multivariate, non-stationary, etc.) is not available. So, we cannot generate the sub-grid scales $\boldsymbol{\xi}$.

All we can do is to take the sub-grid scales from a hiRes field.

That is, take an hiRes field $\mathbf{x}^{\mathrm{tru}}$ and remove the sub-grid scales: $\mathbf{x} = \mathrm{Up}[\mathbf{x}^{\mathrm{tru}}] = \mathbf{x}^{\mathrm{tru}} - \boldsymbol{\xi}$:

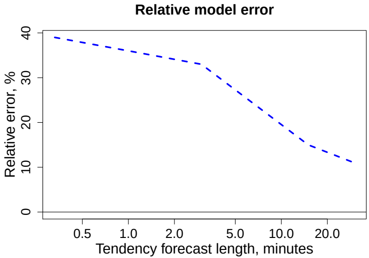
$$\widehat{\boldsymbol{\varepsilon}} = F(\mathbf{x}^{\mathrm{tru}} - \boldsymbol{\xi}) - \mathrm{Up}[F^{\mathrm{tru}}(\mathbf{x}^{\mathrm{tru}})]$$

instead of

$$\boldsymbol{\varepsilon} = F(\mathbf{x}) \quad - \quad \mathrm{Up}[F^{\mathrm{tru}}(\mathbf{x} + \boldsymbol{\xi})]$$

The major problem N2 is that $\mathbf{x}^{\mathrm{tru}} - \boldsymbol{\xi}$ appears to be not on the lowRes attractor (i.e. not balanced) so that the small model error is drowned in the initial lowRes shock, see the next slide.

Magnitude of the estimated model error (relative to the total tendency) in the default scheme: T, 3D averaging



The resulting estimate of the model error strongly depends on the length of the initial tendency and falls only to some 10% for the 20-min tendency (which is too poor for an operational-class COSMO model to be taken as its error). Why?

The culprit is **initial shock** in lowRes model started from **upscaled** hiRes fields.

The real model error is drowned in the initial model shock.

Fighting the imbalance

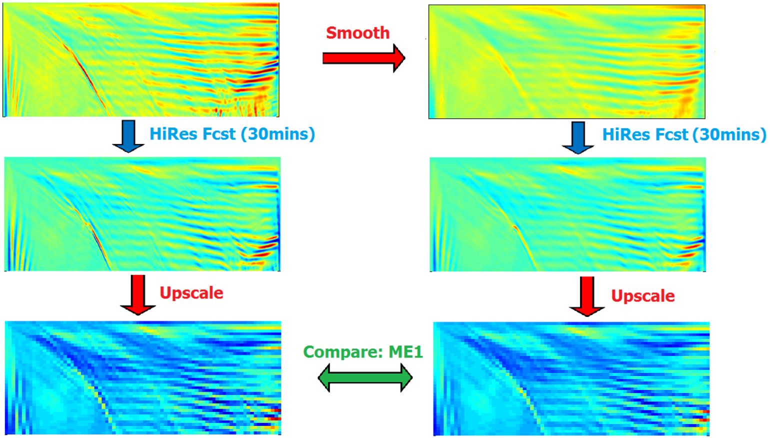
$$\widehat{\varepsilon} = F(\mathbf{x}^{\mathrm{tru}} - \boldsymbol{\xi}) - \mathrm{Up}[F^{\mathrm{tru}}(\mathbf{x}^{\mathrm{tru}})]$$

The proposed solution: split the model error into two components: ME1 and ME2:

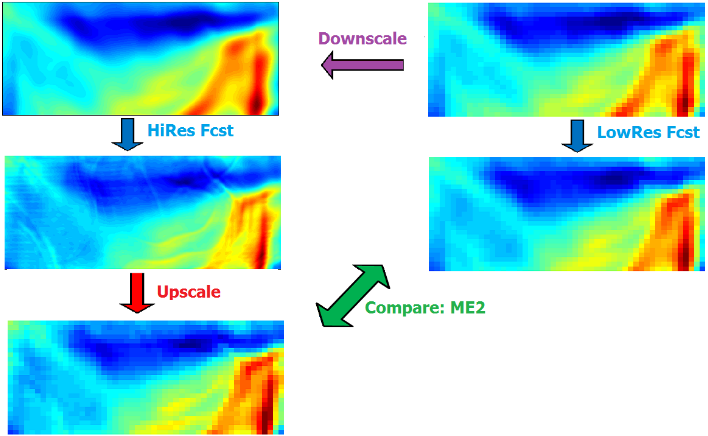
ME1 is the difference between forecasts of two hiRes models one of which starts from hiRes initial conditions and the other from smoothed initial conditions (same model, different initial conditions).
(ME1 is due to lack of subgrid scales in the lowRes-model state vector: true randomness, aleatory *uncertainty.)*

ME2 is the difference between forecasts of hiRes and lowRes models both started from lowRes initial conditions (same initial conditions, different models).
(ME2 is due to the difference between the two models: randomness due to lack of knowledge, epistemic *uncertainty.)*

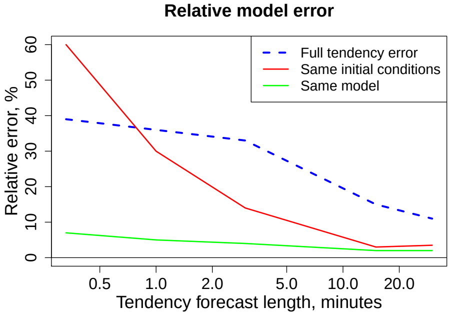
Computation of ME1 (start from a 3h hiRes pre-forecast, top-left)



Computation of ME2 (start from a 3h lowRes pre-forecast, top-right)

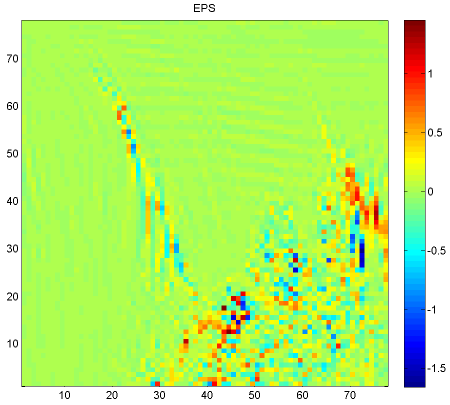


Magnitudes of ME1 and ME2 model-error components

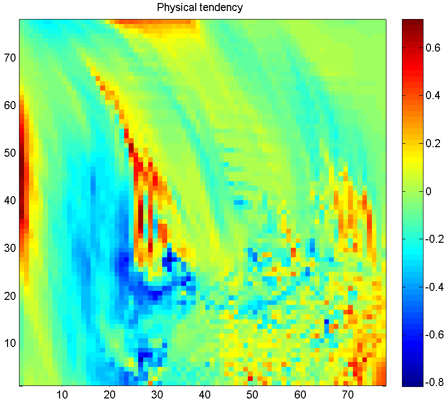


•Estimates of the two model-error components (ME1, green and ME2, red) fall to realistic 2% of the total tendency (in contrast to the default estimate of model error, the blue line).
•With ME1 and ME2, the spinup is seen to settle down more quickly than in the default scheme.
•So, 15-min or longer tendencies can be taken as initial-imbalance free, providing (hopefully) useful estimates of ME1 and ME2.

Example of ME1: V, level 40 (about 2 km above ground)



Physical tendency, V, level 40



ME2

Correlates with the physical tendency *only if* the physical parameterizations differ.

Tentative model

$$\varepsilon = \mathbf{P}(\mathbf{s}, t) \cdot \xi_{\rm mult}(\mathbf{s}, t) + \boldsymbol{\xi}_{\rm add}(\mathbf{s}, t)$$

where both $\xi_{\rm mult}$ and $\boldsymbol{\xi}_{\rm add}$ satisfy the SPG model (*Tsyrulnikov and Gayfulin. A limited-area spatio-temporal stochastic pattern generator for simulation of uncertainties in ensemble applications. Meteorol. Zeitschrift, 2017, v.26, 549–566*).

Variances, time scales, and length scales of the random fields are to be estimated from the training sample.

Conclusions

1. Non-convective model errors were estimated w.r.t. a higher-resolution model.
2. Splitting the model error into two components appeared to be necessary to mitigate imbalance in the lowRes model if it starts from *upscaled* hiRes fields.
3. The first model-error component is due to lack of subgrid scales in a lowRes state vector (true randomness, *aleatory* uncertainty).
   The second model-error component is due to the difference between the two models (randomness due to lack of knowledge, *epistemic* uncertainty).
4. Parameters of an additive-multiplicative SPG-based 4D multivariate random field model remain to be estimated.
5. The goal is a practical justified model-error model for use in EPS and EDA.