COSMO — CONSORTIUM FOR SMALL SCALE MODELING

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

DWD

Recent numerics developments in the COSMO model and an outlook for the ICON model

41st EWGLAM / 26th SRNWP-meeting, Sofia, Bulgaria
30 Sept. – 03 Oct. 2019

Michael Baldauf (DWD), Damian Wojcik (IMGW)

Consortium in transition

The current **COSMO model** (with the ‚**Runge-Kutta**' dyn.core) is slowly phased out during the next years in the COSMO consortium.

- DWD plans to replace COSMO-D2 by **ICON**-D2 in Q4/2020
- …
- MeteoCH plans the replacement ~2023 (← adaptations to GPU computers!)

This migration is prepared by all COSMO partners in the priority project ‚*Transition of COSMO to ICON-LAM* (*C2I*); project leader: Daniel Rieger (DWD)

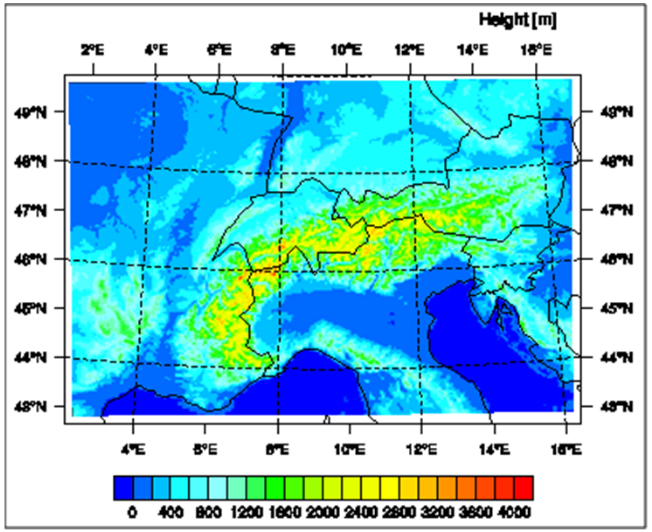
Therefore, no further development work at the dyn. core will be done from now on (exception: higher-order scheme by Univ. Cottbus, A. Will)

However, **COSMO-EULAG** will be further developed and probably will go into operations at IMGW (Poland) (currently pre-operational)

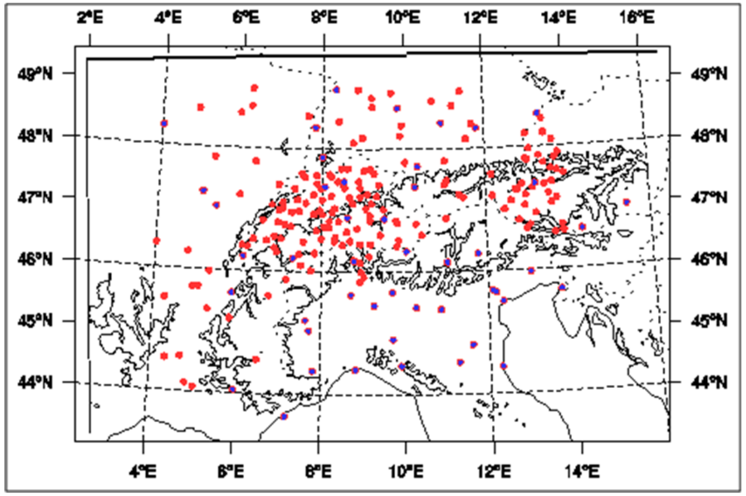
COSMO-EULAG

Investigations:

1. **TKE advection:** replacement of the Bott scheme by MPDATA-A
2. Selection of an optimal MPDATA advection version



Topographical map of the domain



Station network for surface verification

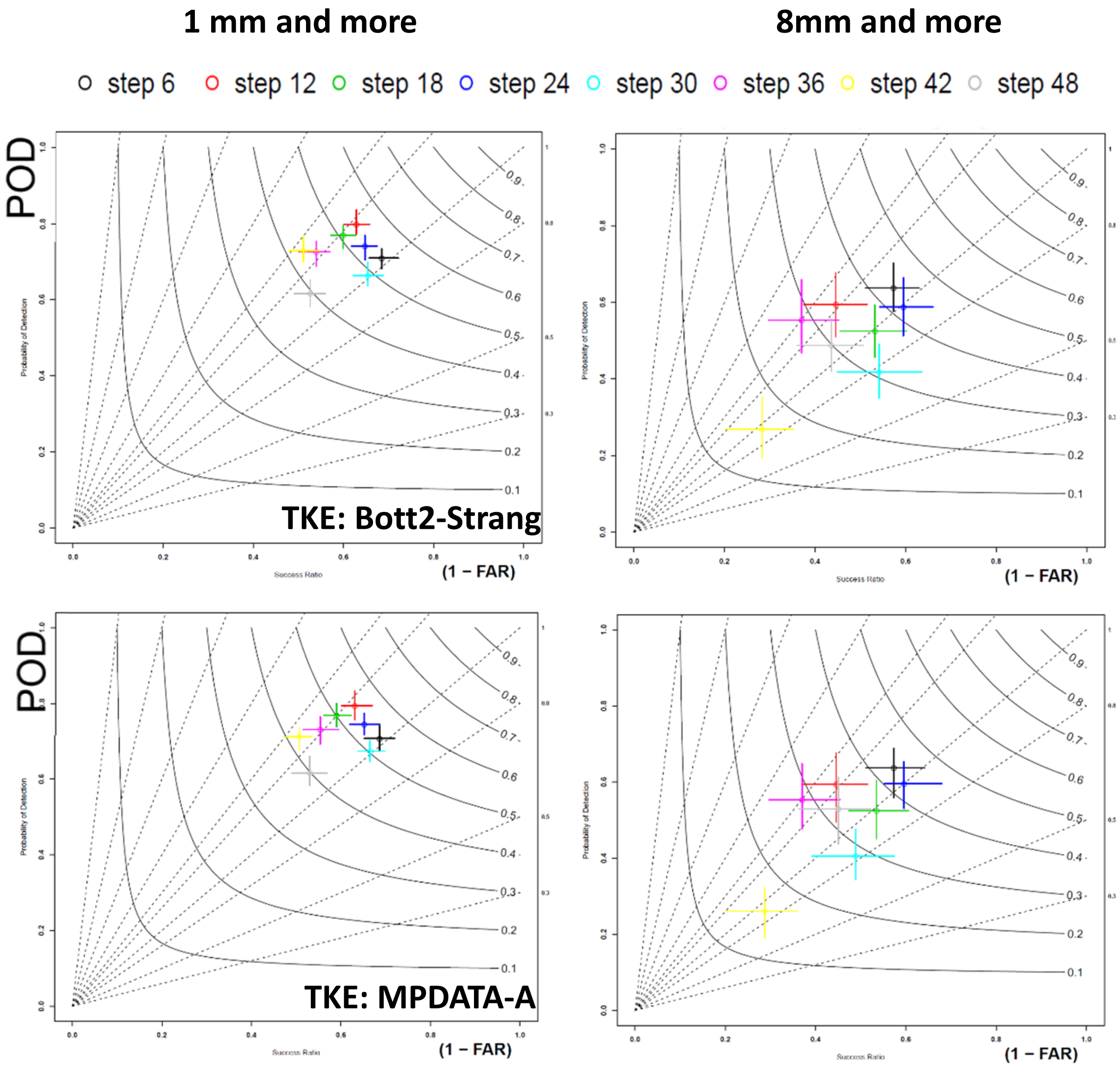
Setup of experiments:

- Operational COSMO-2 domain used by Meteo-Swiss, 60 vertical levels
- Entire June 2013, 48-hour forecasts, verification by VERSUS software
- Numerical and Smagorinsky diffusion are *turned off* for COSMO-EULAG

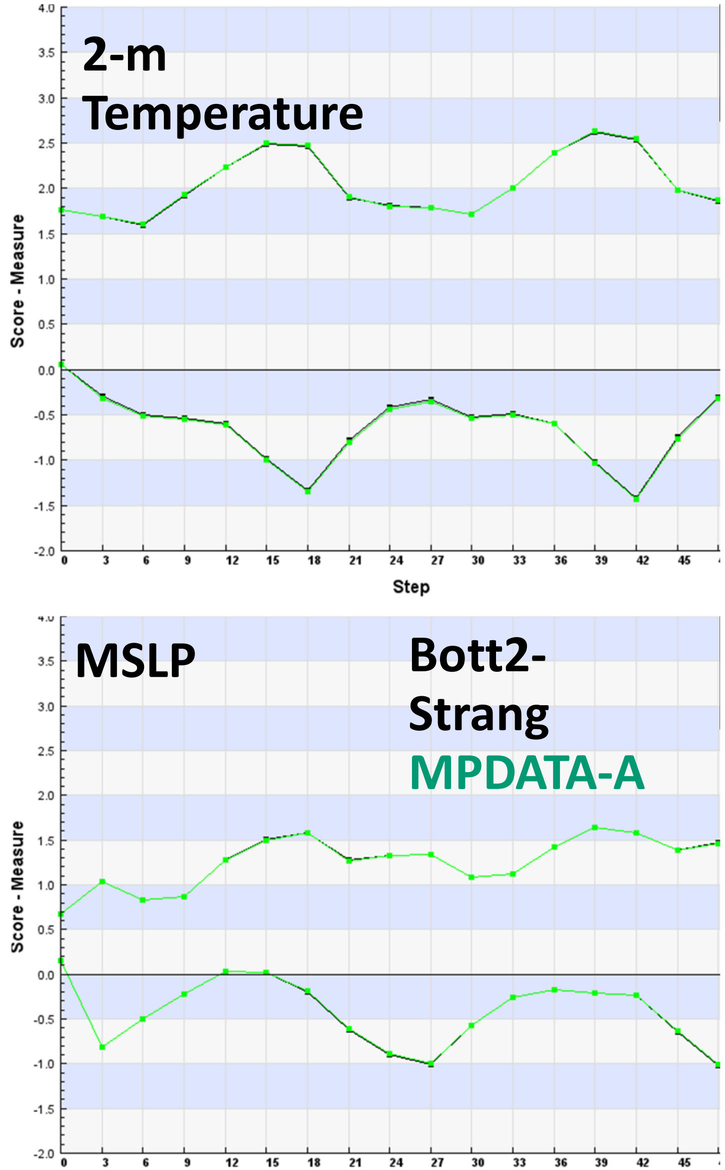
Damian Wojcik (IMGW)

Replacement of the Bott TKE adv. scheme by the MPDATA-A scheme

The verification scores of COSMO-EULAG do not alter significantly with that change.

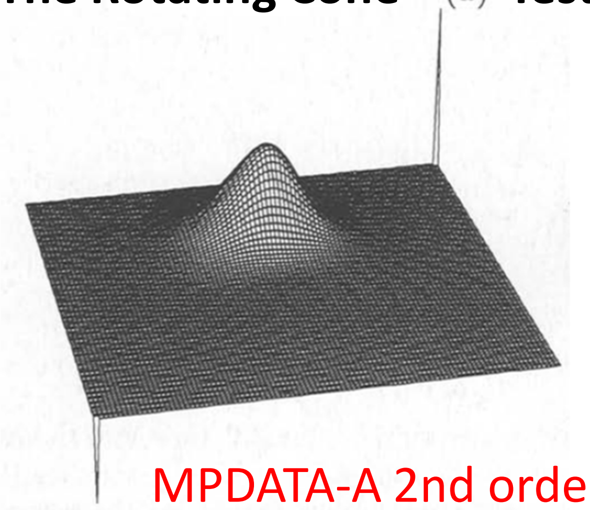


Reliability diagrams for precipitation

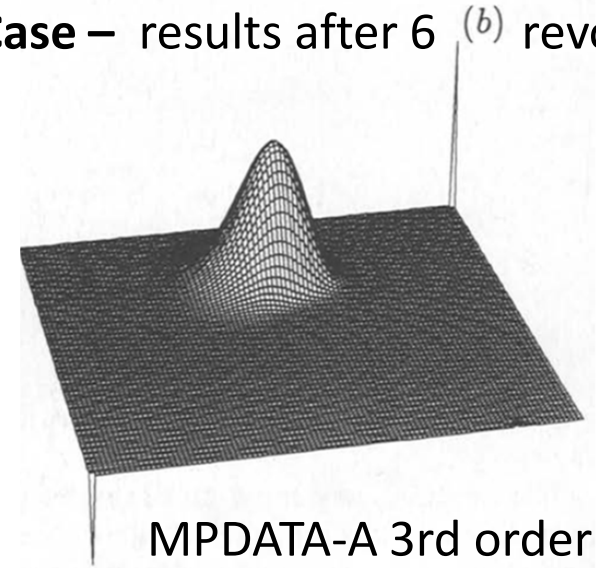


Selection of an optimal adv. sch. (MPDATA-A vs. MPDATA-M)

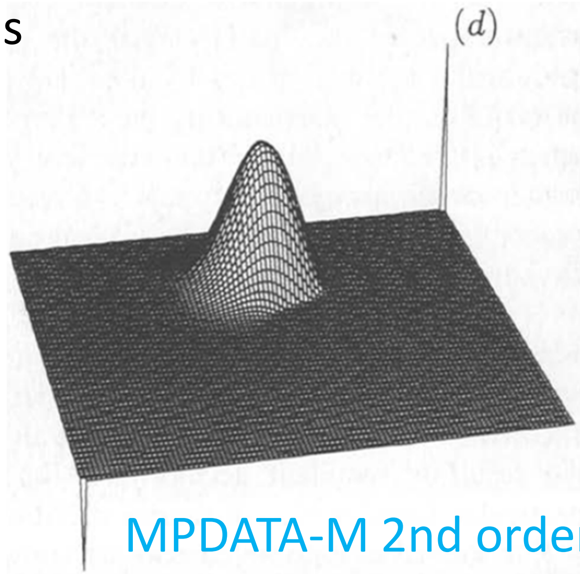
The Rotating Cone Test Case – results after 6 revolutions



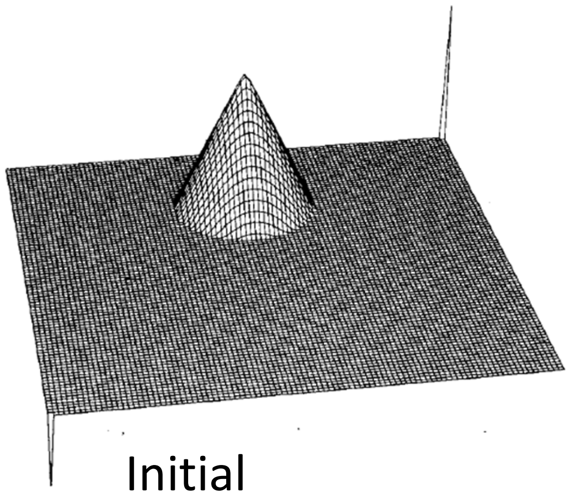
MPDATA-A 2nd order



MPDATA-A 3rd order



MPDATA-M 2nd order



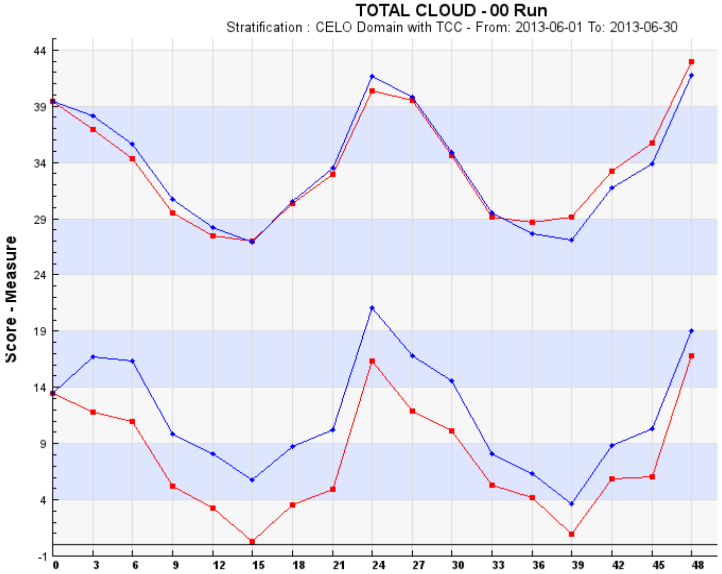
Initial condition

Smolarkiewicz and Clark (JCP, 1986), Smolarkiewicz and Grabowski (JCP, 1990).

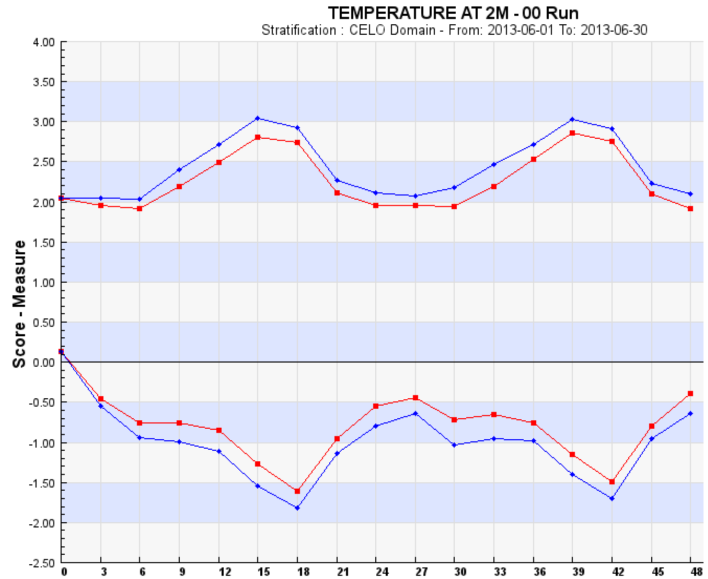
	MPDATA-A 2nd order	MPDATA-M 2nd order
Advected field	ψ	Ψ+c, c → ∞
Accuracy	Lower	Higher
Diffusion	More	Less

MPDATA = upwind advection with iterative improvement by 'anti-diffusive' fluxes (nonlinear)

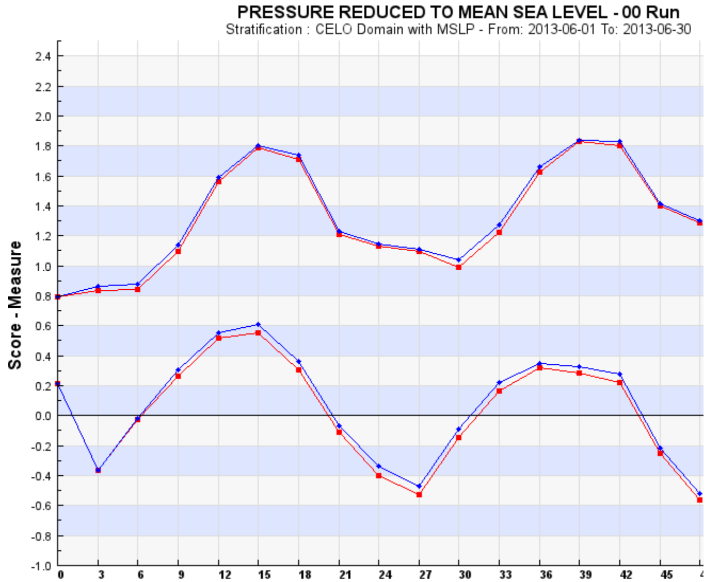
Selection of an optimal advection scheme (MPDATA-A vs. MPDATA-M)



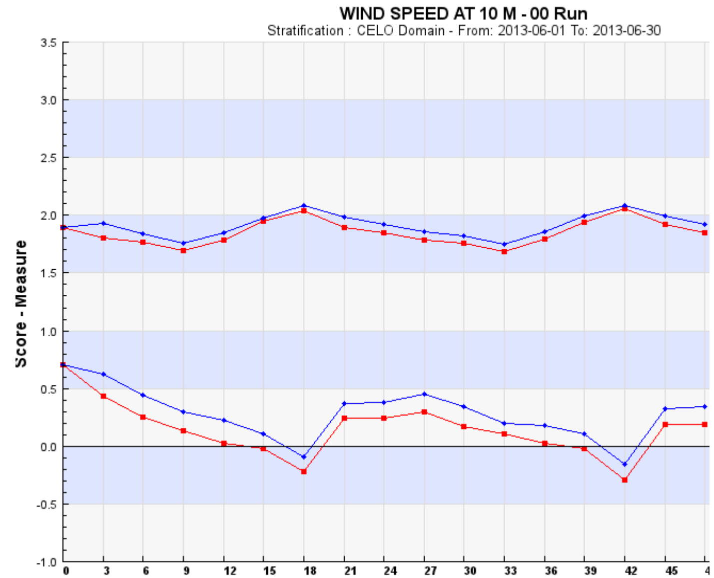
TCC



T_{2m}



MSLP

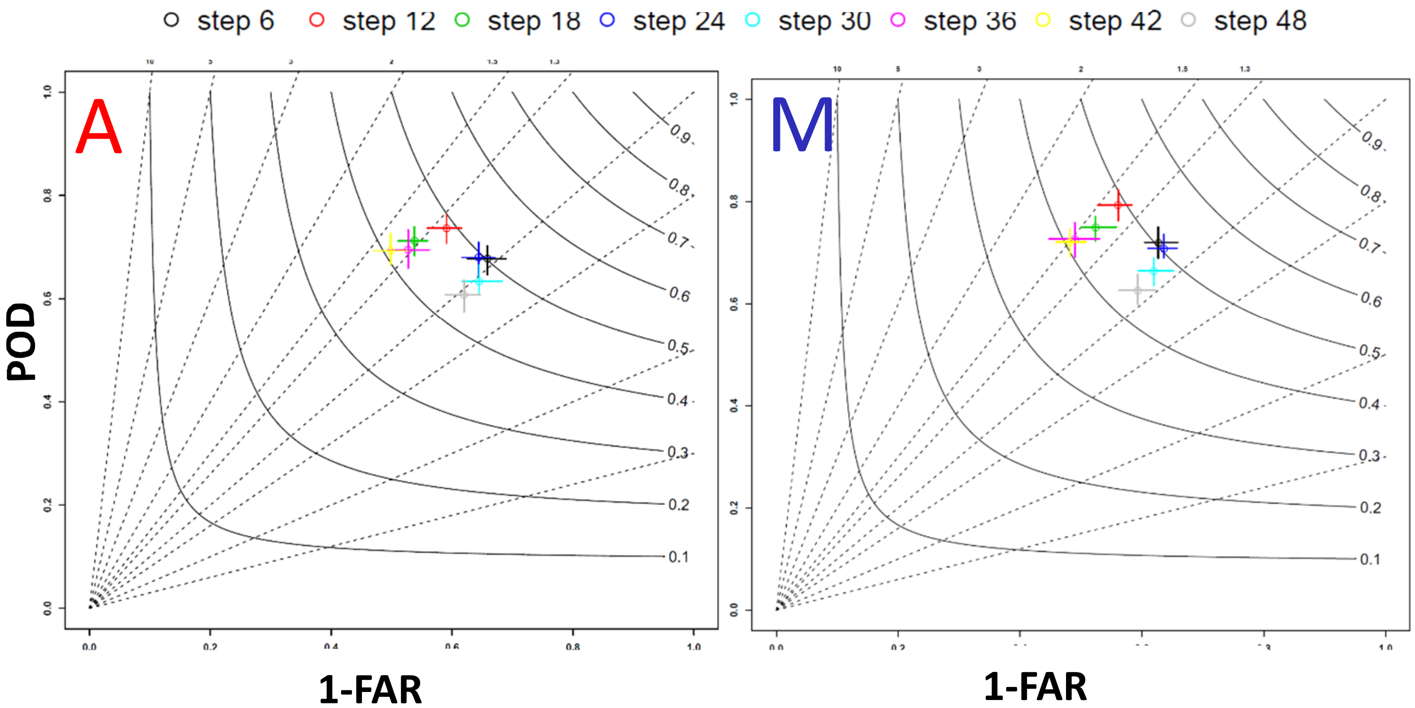


v_{10m}

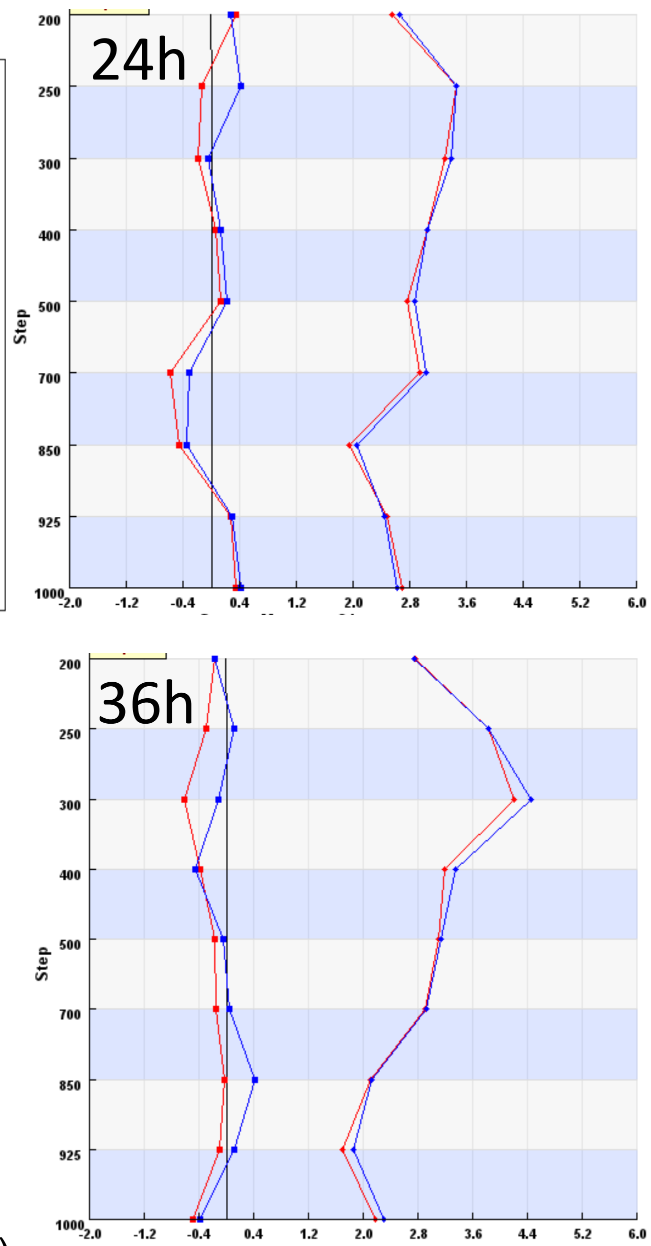
option A outperforms M for: T_{2m}, MSLP, 10-m wind speed,
and for total cloud cover (the latter except only 36-48 h for RMSE)

MPDATA-A vs. MPDATA-M

Precipitation: 1mm and more



Upper-air wind speed



- For upper-air wind speed RMSE is usually lower for A
- A provides precipitation forecasts with slightly improved frequency bias
- Additionally: in the A simulations lower vertical velocities within convective updrafts are observed (not shown)

COSMO-EULAG with the more diffusive scheme, MPDATA-A, provides forecasts having slightly better verification scores.

Damian Wojcik (IMGW)

Summary

1. The more accurate MPDATA-M advection delivers worse scores than MPDATA-A. Possible reasons
    1. might be a hint for too less (horizontal) diffusion ?
    2. verification issue: better scores for more diffusive fields ?
2. Consistent, optimized and extensively tested COSMO-EULAG v5.5
3. The computational performance was slightly improved
4. COSMO-EULAG works semi-operationally in IMGW-PIB since winter 2019 with nudging and with competitive verification scores
5. Future work may involve comparison of COSMO-EULAG and ICON-LAM for high spatial resolutions (over Poland) and with more advanced verification

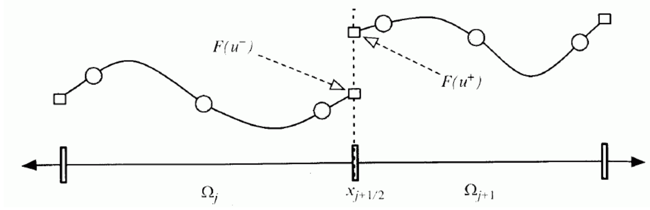
A possible alternative dynamical core for ICON based on Discontinuous Galerkin Discretisation

Michael Baldauf (Deutscher Wetterdienst)

MetStröm

Discontinuous Galerkin (DG) methods in a nutshell

$$\frac{\partial q^{(k)}}{\partial t} + \nabla \cdot \mathbf{f}^{(k)}(q) = S^{(k)}(q), \qquad k = 1, ..., K$$



From *Nair et al. (2011)* in *‚Numerical techniques for global atm. models'*

weak formulation $\int_{\Omega_j} dx \; v(\mathbf{x}) \cdot \ldots$

Finite-element ingredient

$$q^{(k)}(x,t) = \sum_{l=0}^{p} q_{j,l}^{(k)}(t) \; p_l(x - x_j)$$

e.g. Legendre-Polynomials

e.g.
Cockburn, Shu (1989) Math. Comput.
Cockburn et al. (1989) JCP
Hesthaven, Warburton (2008): Nodal DG Methods

Finite-volume ingredient

$$\mathbf{f}(q) \to \mathbf{f}^{num}(q^+, q^-) = \frac{1}{2}\left(\mathbf{f}(q^+) + \mathbf{f}(q^-) - \alpha(q^+ - q^-)\right)$$

Lax-Friedrichs flux

Gaussian quadrature for the integrals of the weak formulation

→ ODE-system for $q^{(k)}{}_{jl}$

DG – Pros and Cons

- **local conservation**
- any **order of convergence** possible
- flexible application on **unstructured grids** (also dynamic adaptation is possible, h-/p-adaptivity)
- very good **scalability**
- **explicit** schemes are easy to build and are quite well understood
- higher accuracy helps to **avoid several awkward approaches** of standard 2nd order schemes: staggered grids (on triangles/hexagons, vertically heavily stretched), numerical hydrostatic balancing, grid imprints by pentagon points or along cubed sphere lines, …

- high computational costs due to
  - (apparently) **small Courant numbers**
  - higher number of DOFs
- **well-balancing** (hydrostatic, perhaps also geostrophic?) in Euler equations is an issue (can be solved!)
- basically ‚only' an **A-grid-method**, however, the ‚spurious pressure mode' is very selectively damped!

Target system: **ICON model**
(Zängl et al. (2015) QJRMS)

- operational at DWD since Jan. 2015
  (global (13km) and nest over Europe (6.5km))
- convection-permitting (2.2km): Q4/2020
- horiz.: icosahedral triangle C-grid, vertic.: Lorenz-grid
- non-hydrostatic, compressible
- mixed finite-volume / finite-difference (mass, tracer mass conservation)
- predictor-corrector time-integration → overall 2nd order discretization

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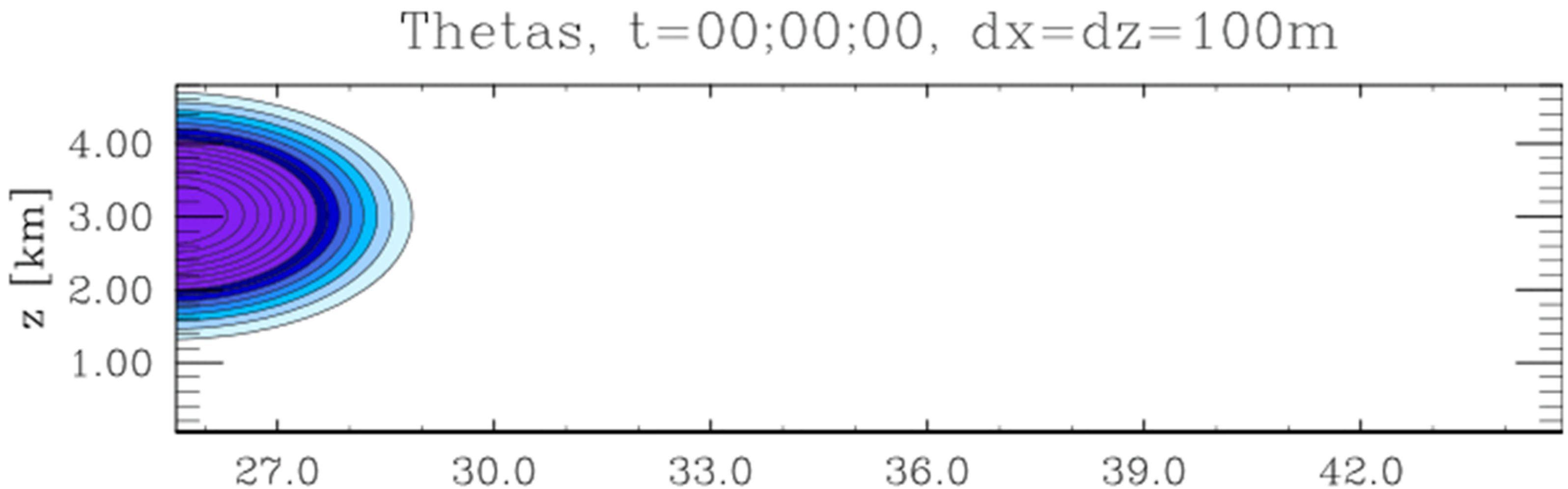
but currently far away from this, only a **toy model for 2D problems exists** with:

- explicit time integration DG-RK (with Runge-Kutta schemes) or
  horizontally explicit-vertically implicit (DG-HEVI) (with IMEX-Runge-Kutta)
- ‚local DG' (LDG) option for PDEs with higher spatial derivatives
- use of a triangle grid (also on the sphere) is optional

Test case: falling cold bubble

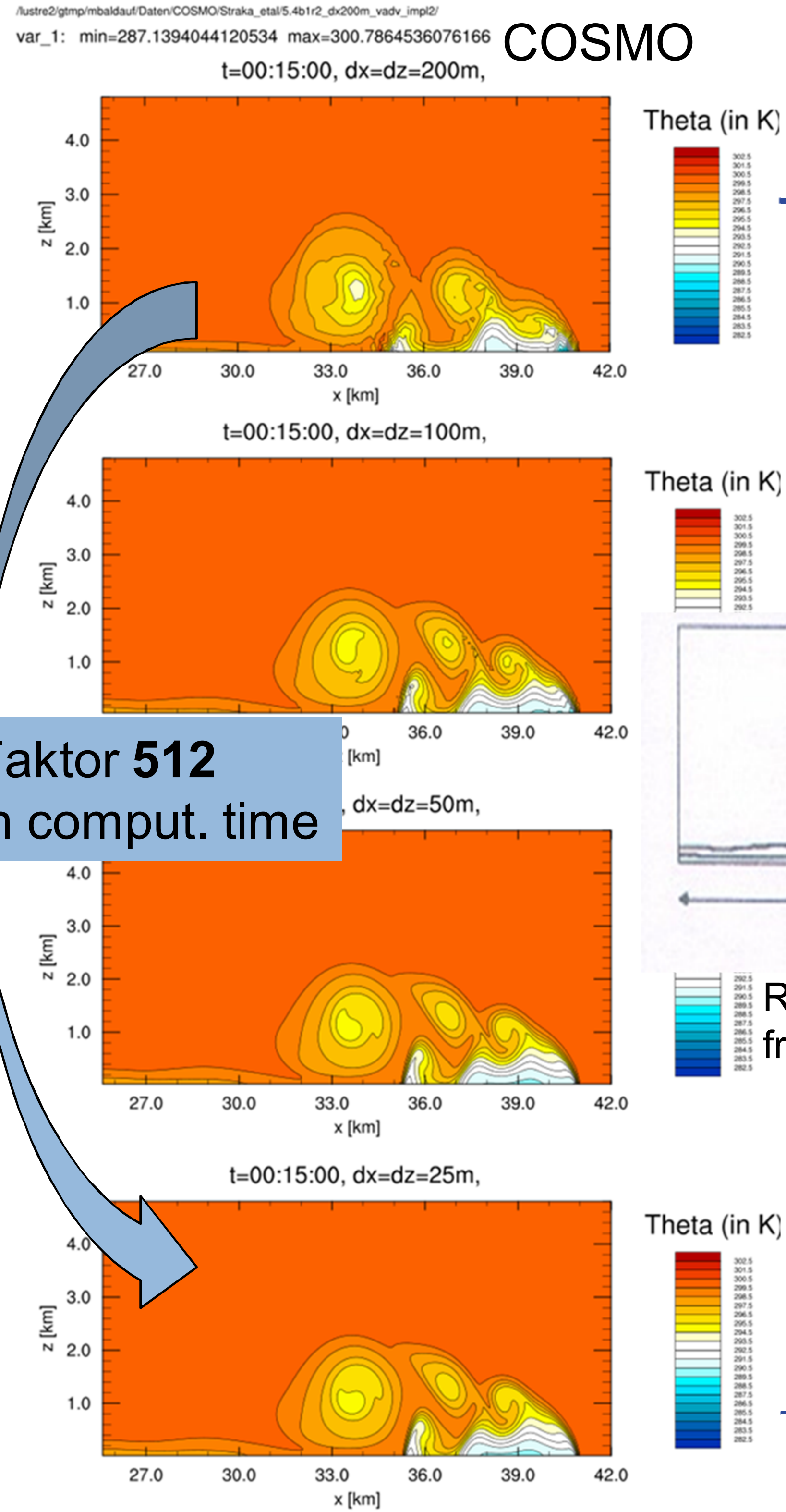
Testsetup by *Straka et al (1993)*



Test properties:

- test of dry Euler equations (without Coriolis force)
- unstationary
- strongly nonlinear
- comparison with reference solution from paper

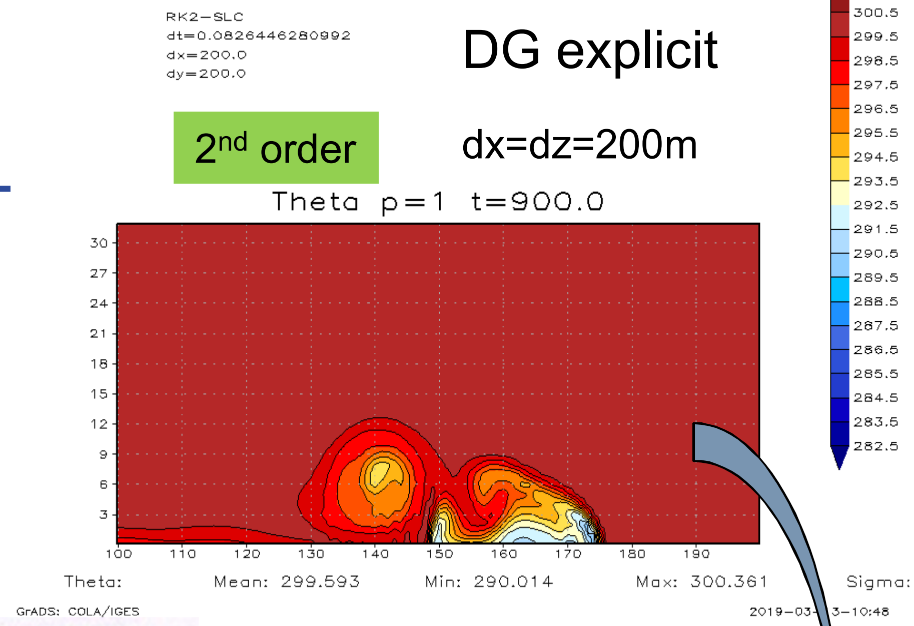
COSMO



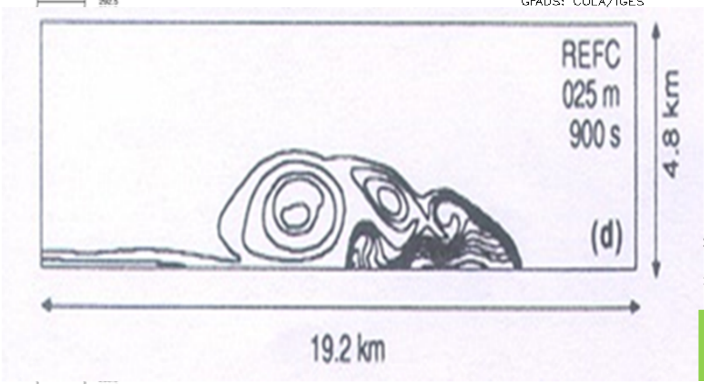
Faktor 512 in comput. time

DG explicit

2nd order dx=dz=200m

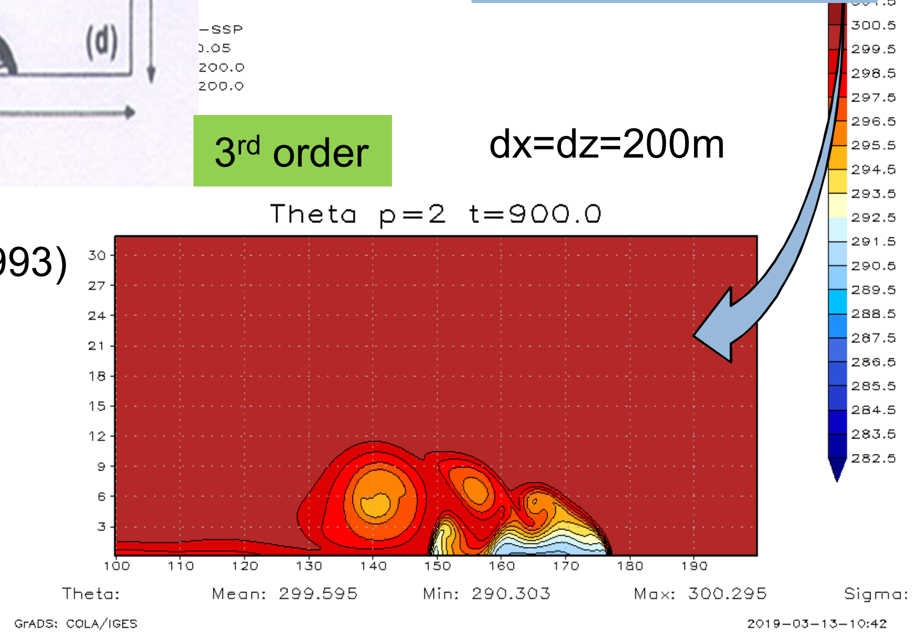


Faktor 4.3 in comput. time



Reference solution from Straka et al. (1993)

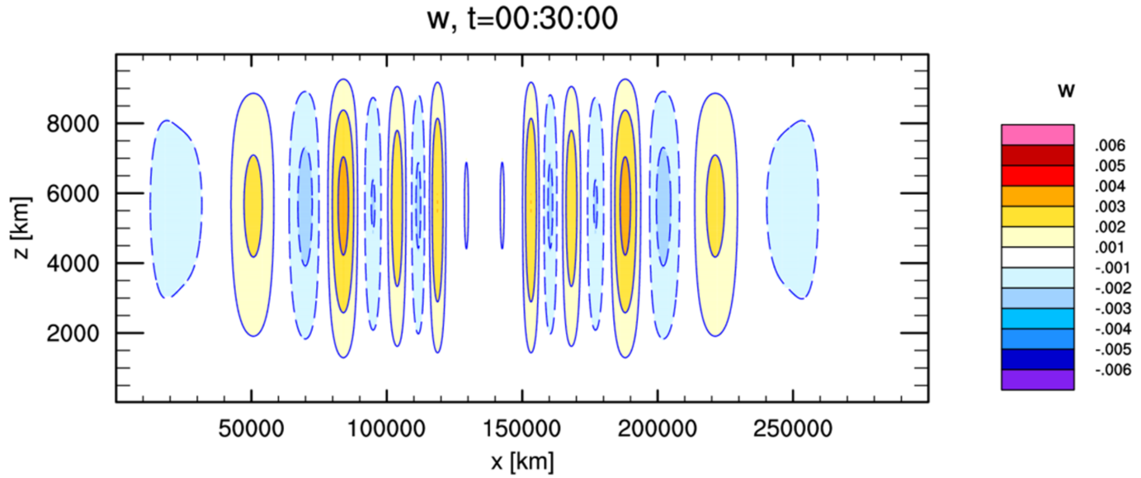
3rd order dx=dz=200m

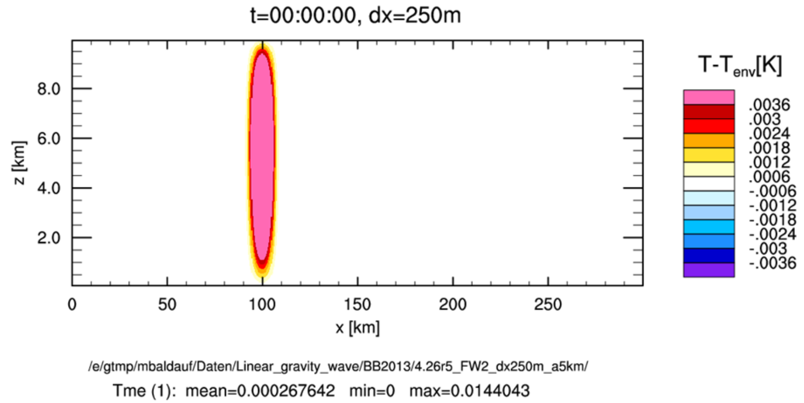


Linear gravity/sound wave expansion in a channel

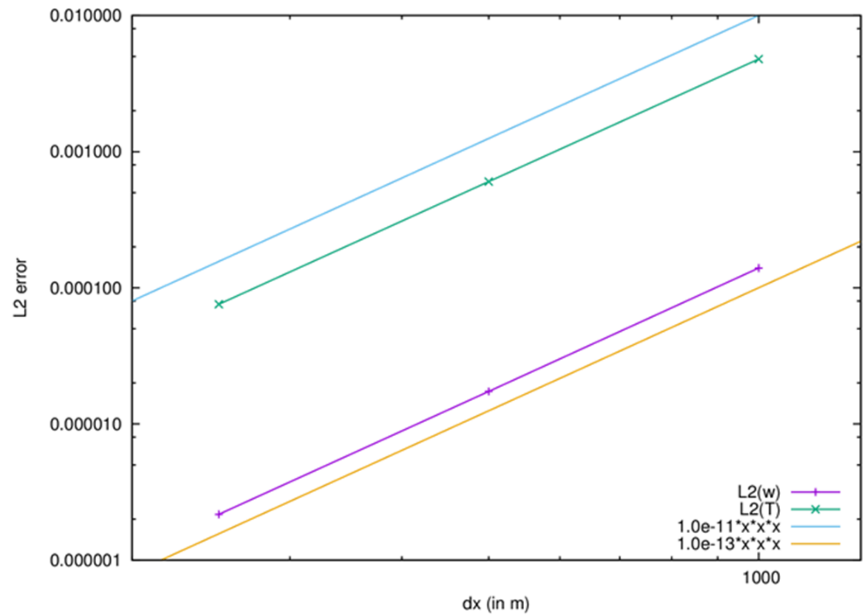
setup similar to *Skamarock, Klemp (1994) MWR*

Δx=500m, Δz=250m





Exact 3rd order convergence for w and T':



colors : simulation with p=2/RK3-SSP (i.e. 3rd order explicit DG)

blue lines: analytic solution for compressible, non-hydrostatic Euler eqns. (*Baldauf, Brdar (2013) QJRMS*)

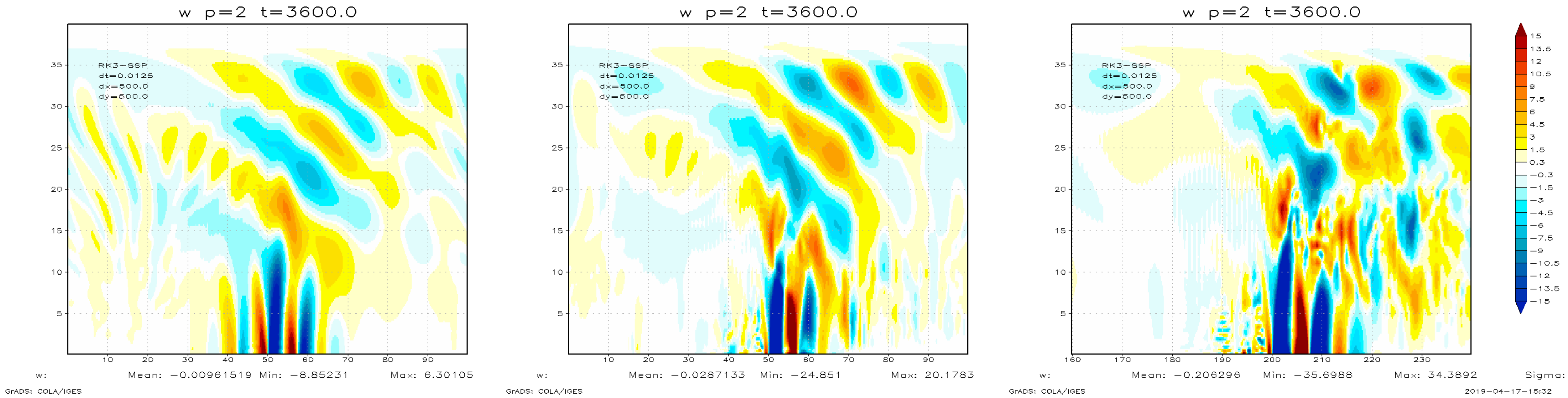
Test case: flow over steep mountains, vertically stretched grid

Schaer et al. (2002) MWR (case 5b: U_0=10m/s, N=0.01 1/s)

H_{oro} = 1000m, α_{max} = 38°

H_{oro}= 2000m, α_{max} = 57°

H_{oro} = 3000m, α_{max} = 61°



with vertical grid stretching ~1:20, Δz_{min}~50m

Explicit DG simulation (3rd order) remains stable even for steeper slopes!
(remark: diffusion switched off → strong gravity wave breaking occurs)

Horizontally explicit - vertically implicit (HEVI)-scheme with DG

Motivation: get rid of the strong time step restriction by vertical sound wave expansion in flat grid cells (in particular near the ground)

$$\frac{\partial q^{(s)}}{\partial t} + \underbrace{\nabla \cdot \mathbf{f}^{(s)}_{slow}}_{\text{explicit}} + \underbrace{\nabla \cdot \mathbf{f}^{(s)}_{fast}}_{\text{implicit}} = \underbrace{S^{(s)}_{slow}}_{\text{explicit}} + \underbrace{S^{(s)}_{fast}}_{\text{implicit}}$$

$$\mathbf{f}^{(s)}_{fast} = f^{(s)}_{z,fast}\mathbf{e}_z$$

$$f^{(s)}_{z,fast} = \sum_{s'} H^{ss'} q^{(s')}$$

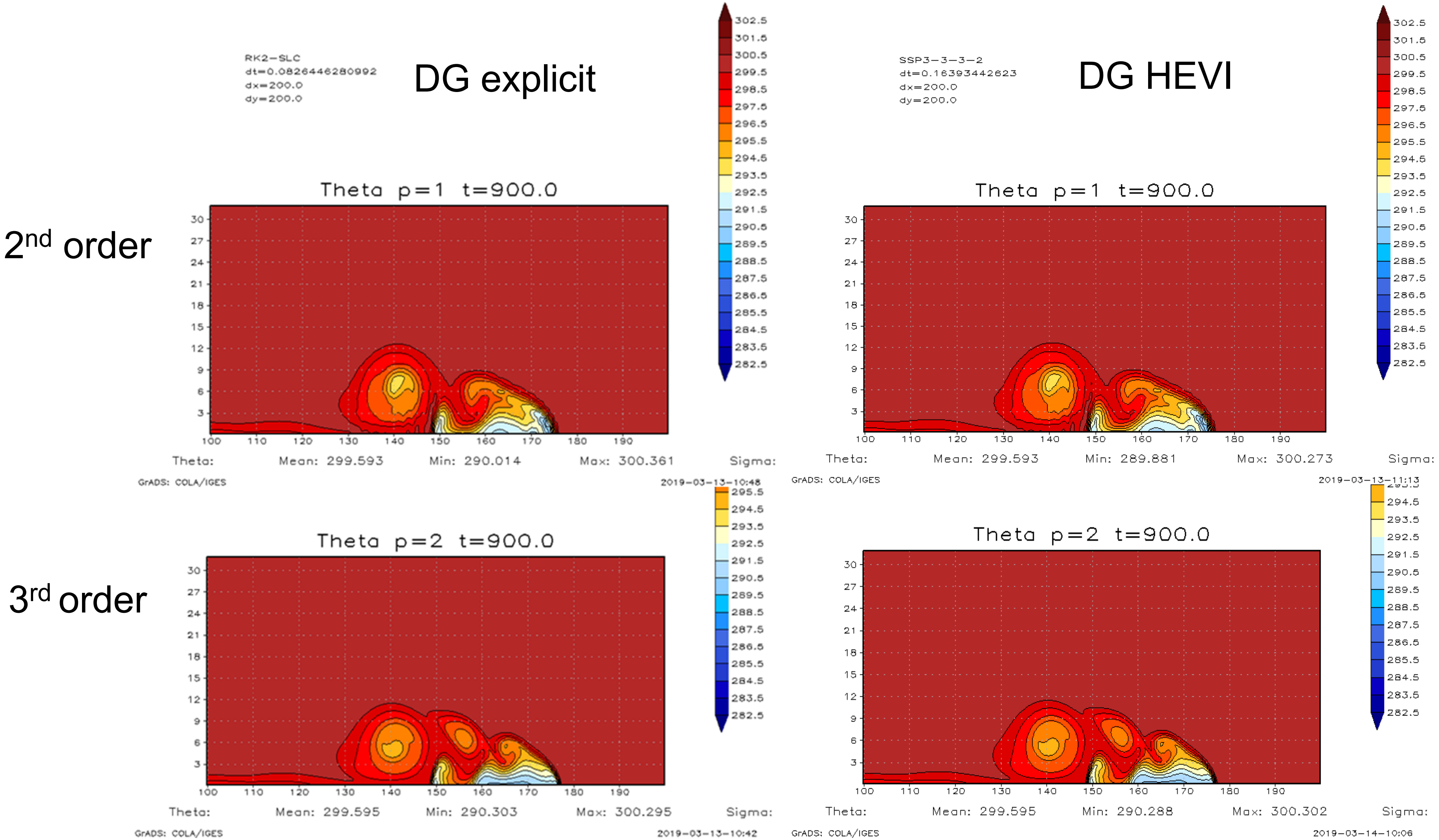
- Use of IMEX-RK (SDIRK) schemes: SSP3(3,3,2), SSP3(4,3,3) (*Pareschi, Russo (2005) JSC*)
- The implicit part leads to several band diagonal matrices
  → here a direct solver is used (expensive!)

References:
Giraldo et al. (2010) SIAM JSC: propose a HEVI semi-implicit scheme
Bao, Klöfkorn, Nair (2015) MWR: use of an iterative solver for HEVI-DG
Blaise et al. (2016) IJNMF: use of IMEX-RK schemes in HEVI-DG
Abdi et al. (2017) arXiv: use of multi-step or multi-stage IMEX for HEVI-DG

Test case: falling cold bubble (Straka et al. (1993)

Comparison explicit vs. HEVI scheme

Deutscher Wetterdienst DWD



How to bring DG on the sphere …

Idea to avoid pole problem and to keep high order discretization:
use **local (rotated) coordinates** for every (triangle) grid cell,
i.e. rotate every grid cell towards $\lambda \approx 0$, $\varphi \approx 0$.
→ geometry is treated exactly
→ transform fluxes between neighbouring cells

shallow water equations
covariant formulation (here: without bathymetry)

$$\begin{aligned}
\frac{\partial \sqrt{G} H}{\partial t} + \frac{\partial}{\partial x^i} \sqrt{G} m^i &= 0 \\
\frac{\partial \sqrt{G} m^i}{\partial t} + \frac{\partial}{\partial x^j} \sqrt{G} T^{ij} &= \sqrt{G} (F^i_{Cor} - \Gamma^i_{jk} T^{jk}) \\
T^{ij} &= \frac{m^i m^j}{H} + \frac{1}{2} g^{ij} g_{grav} H^2
\end{aligned}$$

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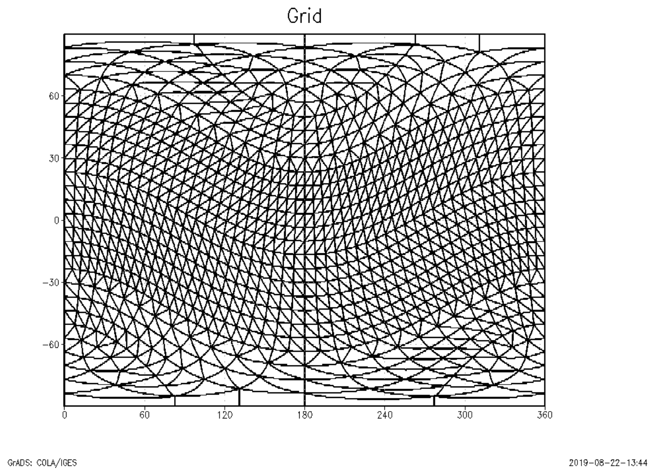
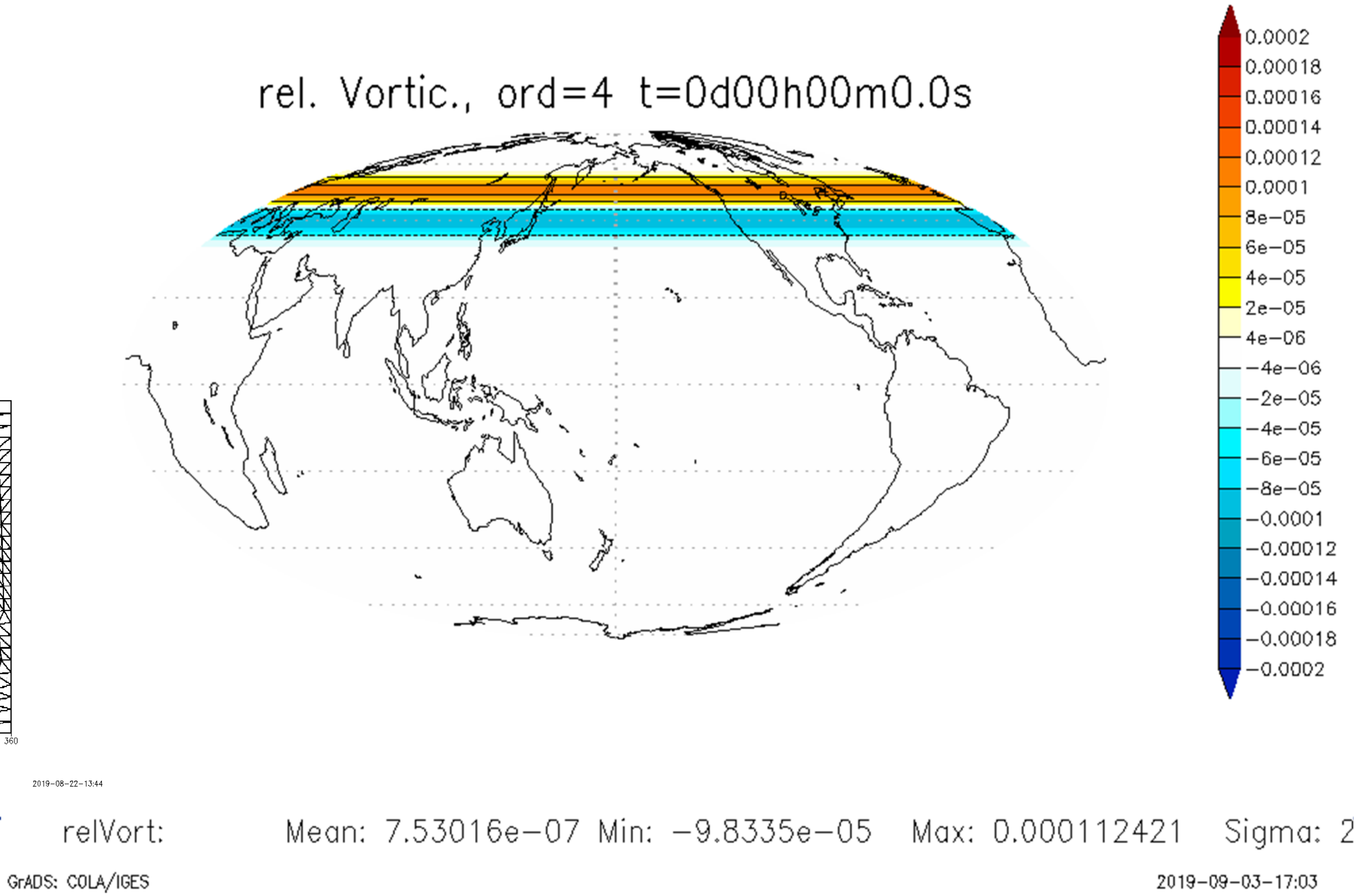
Barotropic instability test
Galewsky et al. (2004)

4th order DG scheme
without additional diffusion
dx~67 km, dt=15 sec.

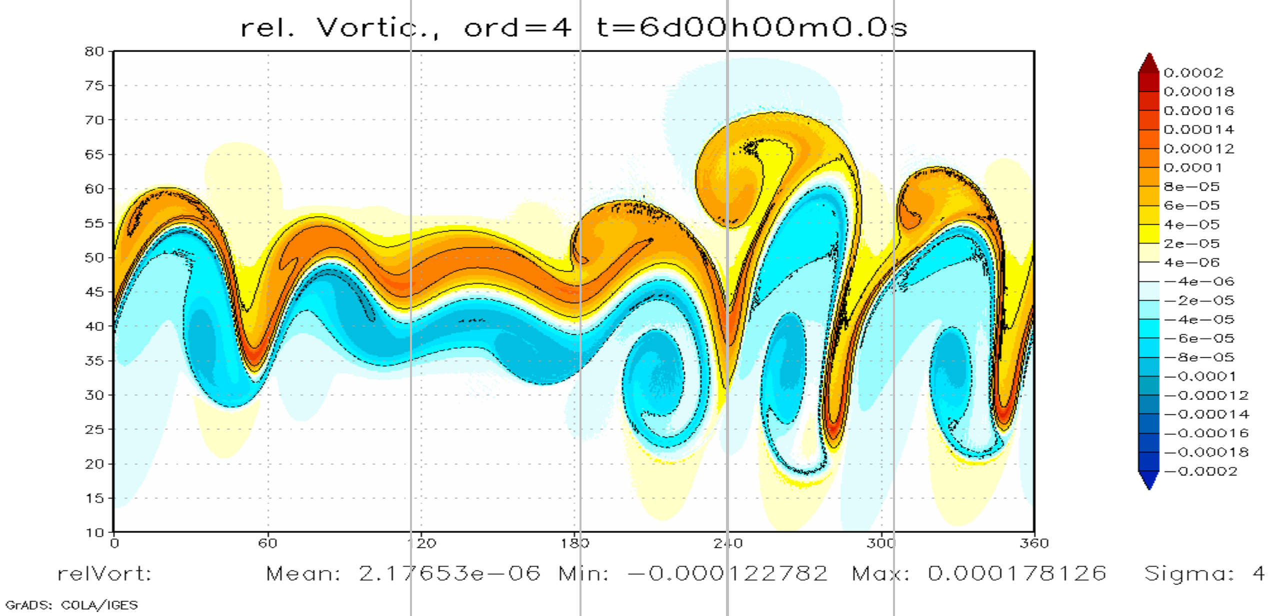
simple triangle grid
on the sphere
dx ~ 500km:



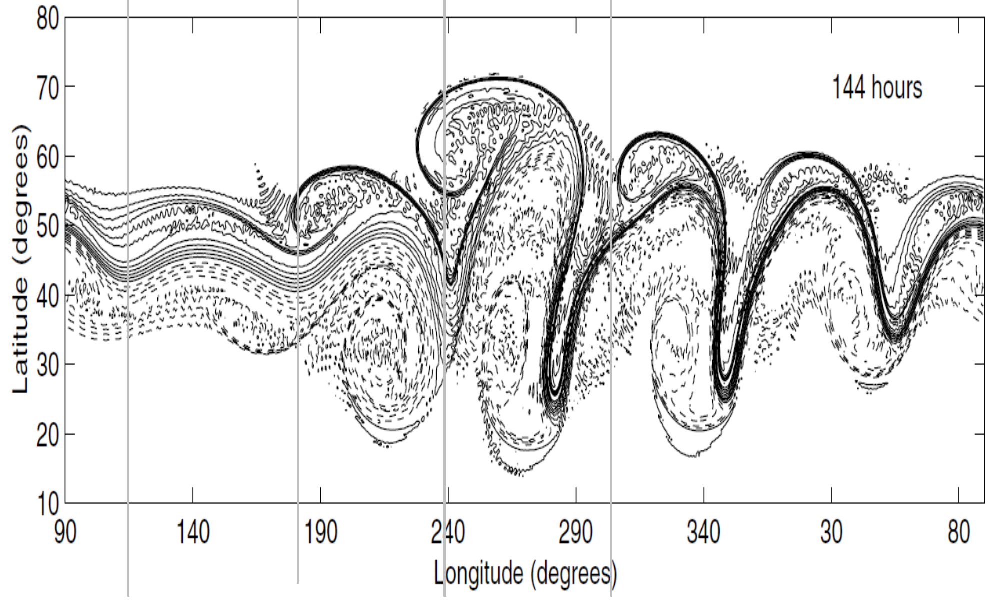


Barotropic instability test
Galewsky et al. (2004)

relative vorticity



4th order DG scheme
without additional diffusion
dx~67 km, dt=15 sec.



FMS-SWM (Geophys. Fl. Dyn. Lab.)
without additional diffusion
dx~60 km (T341), dt=30 sec.

Fig. 4 from *Galewsky et al. (2004)*

Summary

- **2D toy model** for
  - explicit DG-RK (on arbitrary unstructured grids with triangle or quadrilateral grid cells) and
  - **HEVI DG-IMEX-RK**

  works for several idealized tests (also Euler equations with terrain-following coordinates), correct convergence behaviour, …
- **DG on the sphere** by use of local (rotated gnomonial) coordinates

Outlook

- **further design decisions**: nodal vs. modal, local DG vs. interior penalty vs. …, ..
- coupling of **tracer advection** (mass-consistency)?
- improve **efficiency** in the HEVI direct solver
- further **milestones** (for the next years!)
  - development of a 3D prototype DG-HEVI solver
  - choose optimal convergence order *p* and grid spacing
    estimated: $p_{horiz} \sim 3 \ldots 6$, $p_{vert} \sim 3 \ldots 4$ ($p_{time} \sim 3 \ldots 4$)

PDEs
THE WORKSHOP ON PARTIAL DIFFERENTIAL EQUATIONS ON THE SPHERE

Announcement:

The next

„Partial differential equations on the sphere“ – workshop

will take place at
DWD, Offenbach, Germany
5-9 October 2020