A non-spectral Helmholtz solver for the ALADIN-HIRLAM dynamics

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Outline

- Introduction: the semi-implicit spectral ALADIN-HIRLAM dynamics
- Challenges for the spectral solver
- Evolution of the dynamical core
- An iterative solver
  - Multigrid preconditioner
  - Guaranteed convergence speed
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Introduction

- The ALADIN-HIRLAM System uses a semi-Lagrangian, semi-implicit, spectral dynamical core
- The vertical coordinate is a terrain-following mass-based coordinate
- The main reason for using a spectral dynamical core is to solve the Helmholtz problem arising in the semi-implicit timestepping
- The reference state for the semi-implicit scheme is constant in space and time

Challenges for the current dynamics: scalability

- Spectral transforms require data-rich global communications (MPI_ALLTOALL)

- (These communications are not a bottleneck on current HPC's yet)

- On future massively parallel machines, the scalability of spectral transforms may become problematic.

Challenges for the current dynamics: steep slopes

- Going to higher resolutions, the resolved slopes become steeper

- The horizontally constant reference state of the semi-implicit scheme may deviate substantially from the actual atmospheric state

- The (explicitly treated) nonlinear residual term becomes more important, thus negatively affecting stability

Evolution of the dynamics

- Keep as much as possible of the ALADIN-HIRLAM System intact, while addressing these two challenges:

  - keep semi-implicit timestepping
  - keep semi-Lagrangian advection
  - keep vertical coordinate system
  - keep physics
  - keep boundary condition formulation
  - …

- Only get rid of the spectral transforms

  $\Rightarrow$ develop a non-spectral solver for the Helmholtz problem

An iterative solver: problem formulation

- Thanks to the constant-coefficient semi-implicit formulation, the 3D Helmholtz problem can be split into a series of 2D Helmholtz problems:

$$(1 - c_\ell^2 \delta t^2 \nabla^2)\psi_\ell = RHS_\ell$$

  - Each 2D HH problem has its own wave speed
  - The domain is rectangular
  - Boundary conditions are biperiodic (like for the spectral solver; physical boundary conditions are applied with Davies relaxation)

- Solving 2D Helmholtz problems is pretty common in computational physics; typically done with iterative solvers. But:

How to comply with tight operational constraints when using an iterative solver?

An iterative solver: multigrid preconditioner

- A multigrid preconditioner can be used to speed up convergence
- This preconditioner has 4 parameters:
  - the multigrid depth d
  - the number of pre-relaxations ν_1
  - the number of post-relaxations ν_2
  - the number of relaxations at the coarsest resolution ν_0
- The choice of these parameters will be discussed further
- Relaxation is done with a red-black Gauss-Seidel scheme

An iterative solver: convergence speed

- Convergence of an iterative solver is determined by the spectrum, i.e. by the extreme eigenvalues of the system.
- Thanks to the specific formulation of the ALADIN-HIRLAM dynamics, i.e.
  - biperiodic rectangular domain;
  - constant-coefficient semi-implicit reference state;
  - vertical decoupling into 2D Helmholtz problems,

the eigenvalues of the (preconditioned) system can be determined analytically!

- The eigenvalues depend on the wave courant number

An iterative solver: convergence speed

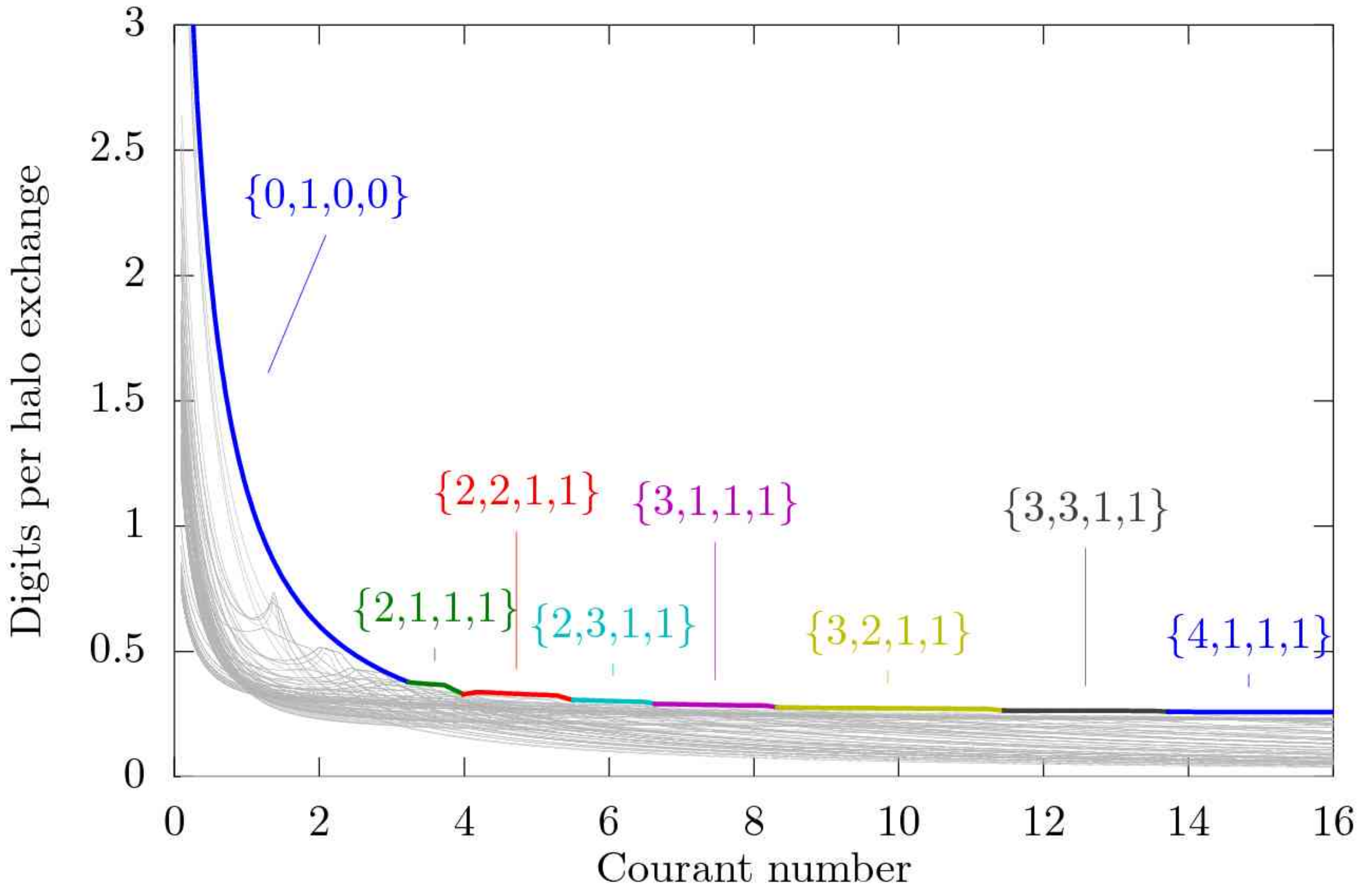
Knowledge of the extreme eigenvalues has important advantages:

1. The **convergence speed is guaranteed**: one knows – even before starting the forecast - how many iterations will be required.
   In an operational context, this is very valuable!
2. It becomes possible to compare different preconditioner settings, thus allowing to pick the optimal parameter values

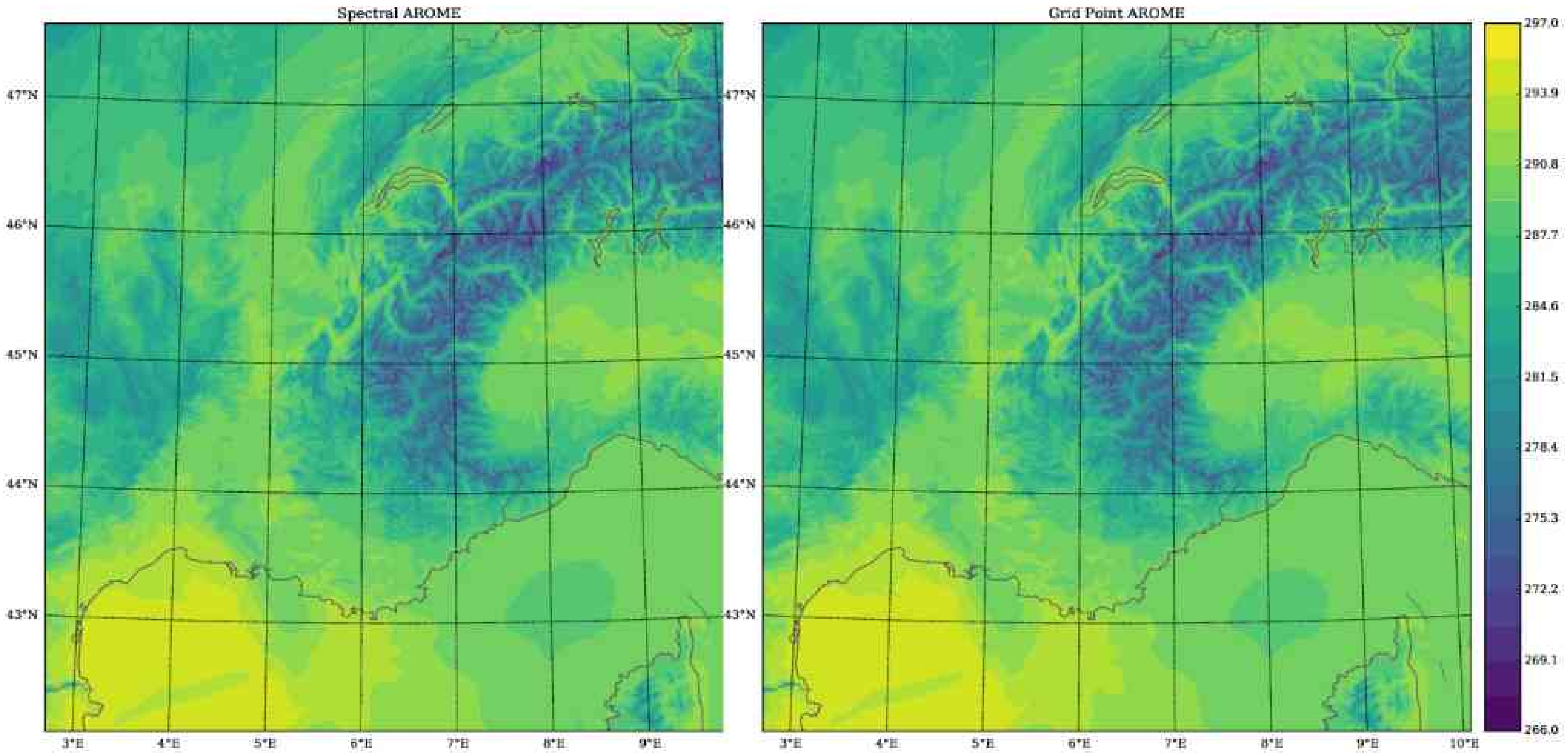
An iterative solver: convergence speed

- Optimal preconditioner parameters { d, ν_0, ν_1, ν_2 } for given wave Courant number of the 2D Helmholtz problems

- No multigridding is necessary (d=0) for slow modes;
- Multigridding pays off for fast modes



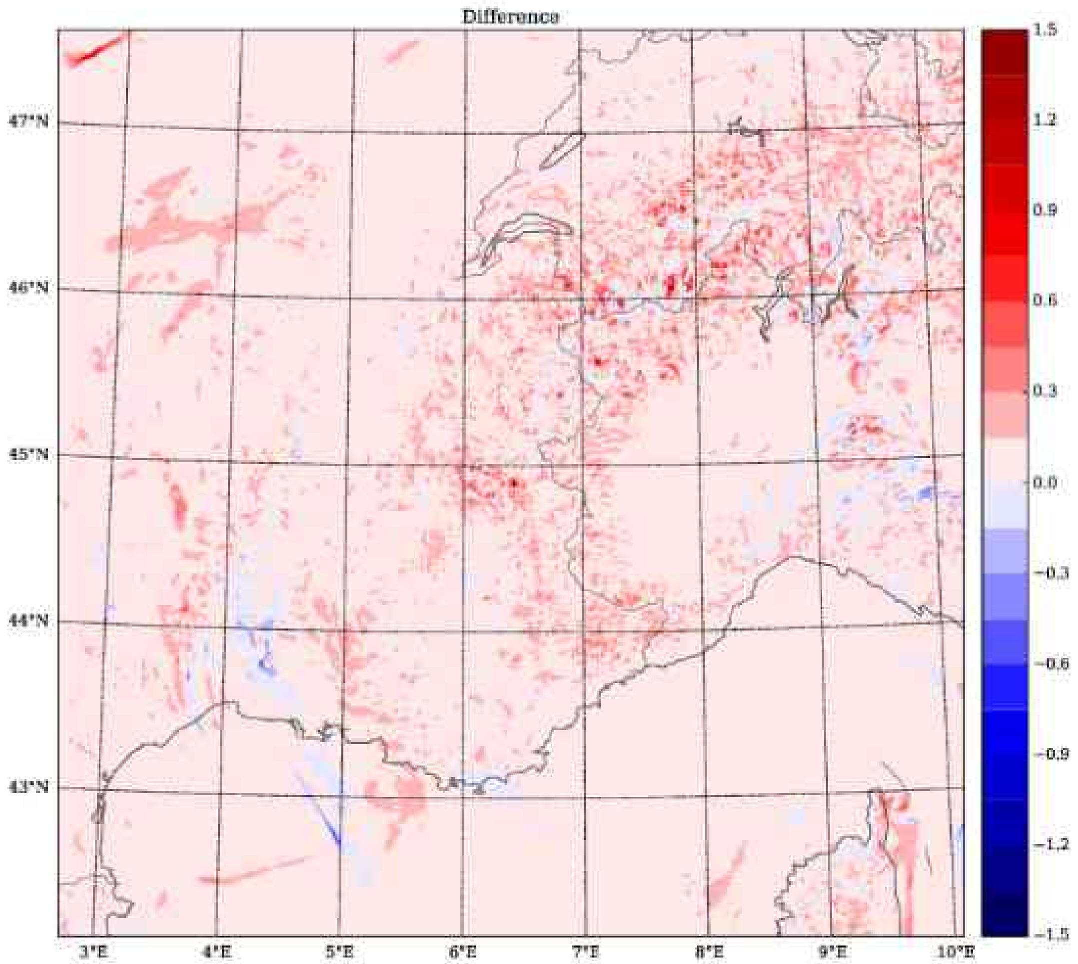
An iterative solver:



$$T80,\ \delta t = 50\ s,\ T = 2\ h,\ \Delta x = 1.3\ km,\ N_{iter} \approx 13,\ 8^{th}\ \text{order}$$

An iterative solver: meteorological results

- Difference between spectral and (high-order) non-spectral is small
- Further evaluation and verification needed
- Some calculations are still done spectrally (e.g. implicit diffusion)

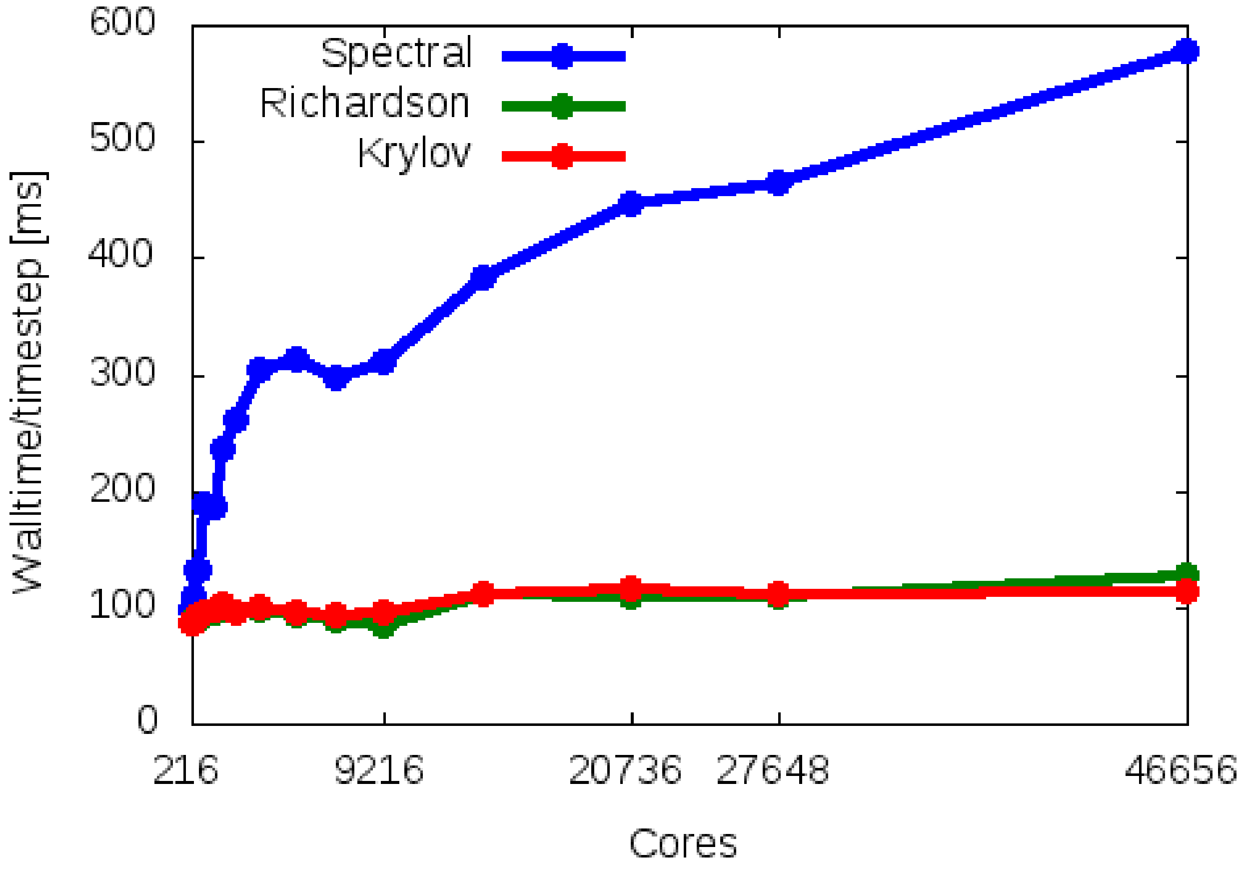


$$\delta(T80),\ \delta t = 50\ s,\ T = 2\ h,\ \Delta x = 1.3\ km,\ N_{iter} \approx 13,\ 8^{th}\ \text{order}$$

An iterative solver: scalability results

- Weak scalability test on ECMWF’s cca

- Note: only scalability of Helmholtz solver,
  not of entire atmospheric model



Conclusions

- The current spectral dynamics of the ALADIN-HIRLAM model faces important challenges regarding scalability and steep slopes;
- Scalability does not seem to be a pressing issue at current resolutions on current machines;
- An alternative, non-spectral, iterative solver is being developed:
  - Keep as much as possible intact of the ALADIN-HIRLAM model
  - Krylov solver with a multigrid preconditioner
- Specific choices in the formulation of the semi-implicit scheme lead to predictable convergence speed, making the solver appropriate for use in an operational context
- Scalability of iterative solver is superior to that of the spectral solver
- Meteorological results are not affected when using high-order finite differences