Regional Cooperation for
Limited Area Modeling in Central Europe

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Dynamics in RC LACE

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thanks to Jozef Vivoda, Mario Hrastinski, Alexandra Craciun
and other colleagues

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Outline

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- NH dynamics as a departure from HPE [Jozef Vivoda]
- VFE new formulation for HPE [Jozef Vivoda]
- Grey zone of turbulence? [Mario Hrastinski]

NH dynamics as a departure from HPE
[Jozef Vivoda]

NH dynamics as a departure from HPE

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ALADIN/HIRLAM system dynamics

- uses a hybrid terrain following vertical coordinate η based on hydrostatic pressure π
- uses hydrostatic primitive equation system (HPE) or fully compressible nonhydrostatic Euler equations (EE); recently implemented quasi elastic equation system (QE)
- prognostic variables $\vec{v}, T, q_s = \ln(\pi_s)$, in EE with $w, \hat{q} = \ln(\frac{p}{\pi})$
- let us consider adiabatic system with no moisture

Total time derivative

$$\dot{\psi} = \frac{\partial \psi}{\partial t} + \vec{v} \cdot \vec{\nabla} \psi + \dot{\eta} \frac{\partial}{\partial \eta}$$

Basic principles

Perfect gas law

$$p = \varrho R T$$

Mass conservation

$$\frac{\dot{\varrho}}{\varrho} = -D_3$$

Energy conservation

$$\frac{\dot{T}}{T} = \frac{R}{c_p} \frac{\dot{p}}{p}$$

Continuity equation

$$\vec{\nabla}_\pi \cdot \vec{v} + \frac{\partial \dot{\pi}}{\partial \pi} = 0$$

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Prognostic equations may be written as

Horizontal momentum

$$\dot{\vec{v}} = -RT\frac{\vec{\nabla}p}{p} - \frac{\partial p}{\partial \pi}\vec{\nabla}\phi$$

Vertical momentum

$$\dot{w} = \left(\frac{\partial p}{\partial \pi} - 1\right) g$$

Temperature

$$\dot{T} = \frac{RT}{c_p}\frac{\dot{p}}{p}$$

Pressure departure

$$\dot{\hat{q}} = -\left(\frac{\dot{\pi}}{\pi} + \frac{c_p}{c_v}D_3\right)$$

Surface pressure

$$\dot{q}_s = -\frac{1}{\pi_s}\vec{\nabla}\cdot\int_0^1 \frac{\partial \pi}{\partial \eta}\vec{v}\, d\eta'$$

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We introduce the measure of nonhydrostaticity ε_{NH}

Horizontal momentum

$$\dot{\vec{v}} = -RT\frac{\vec{\nabla}\pi}{\pi} - \varepsilon_{NH} RT\vec{\nabla}\hat{q} - \vec{\nabla}\phi - \varepsilon_{NH}\left(\frac{\partial p}{\partial \pi} - 1\right)\vec{\nabla}\phi$$

Vertical momentum

$$\dot{w} = \left(\frac{\partial p}{\partial \pi} - 1\right) g$$

Temperature

$$\dot{T} = \frac{RT}{c_p}\frac{\dot{\pi}}{\pi} + \varepsilon_{NH}\frac{RT}{c_p}\dot{\hat{q}}$$

Pressure departure

$$\dot{\hat{q}} = -\left(\frac{\dot{\pi}}{\pi} + \frac{c_p}{c_v} D_3\right)$$

Surface pressure

$$\dot{q}_s = -\frac{1}{\pi_s}\vec{\nabla}\cdot\int_0^1 \frac{\partial \pi}{\partial \eta}\vec{v}\, d\eta'$$

HYDROSTATIC SYSTEM with $\varepsilon_{NH} = 0$
+ NH DEPARTURE with $\varepsilon_{NH} = 1$

The system is closed with diagnostic relations

Hydrostatic pressure time change

$$\overset{\bullet}{\pi} = \vec{v} \cdot \vec{\nabla} \pi - \int_0^{\eta} \vec{\nabla} \cdot \frac{\partial \pi}{\partial \eta} \; \vec{v} \; d\eta'$$

Geopotential

$$\phi = \phi_s - \int_{\eta}^{1} \frac{RT}{p} \frac{\partial \pi}{\partial \eta} \; d\eta'$$

Total divergence and modified vertical divergence

$$D_3 = D + d \quad \textbf{where} \quad d = -\frac{gp}{RT}\frac{\partial w}{\partial \pi} + \frac{p}{RT}\frac{\partial \vec{v}}{\partial \pi}\vec{\nabla}\phi$$

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[Thanks to Pierre Bénard]

For the SI time scheme we define the isothermal, resting, horizontally homogeneous reference state X^* in the hydrostatic equilibrium given by 3 values T^*, T_a^*, π_s^*.

We use horizontal divergence D and modified vertical divergence d instead of $\vec{v}$ and w for stability reasons.

We define linear vertical operators

Linear vertical operators

$$G^* X = \int_\eta^1 \frac{m^*}{\pi^*} X d\eta' \qquad S^* X = \frac{1}{\pi^*} \int_0^\eta m^* X d\eta'$$

$$N^* X = \frac{1}{\pi_s^*} \int_0^1 m^* X d\eta \qquad L^* X = \frac{\pi^*}{m^*} \frac{\partial}{\partial \eta} \left(\frac{\pi^*}{m^*} \frac{\partial}{\partial \eta} + 1 \right) X$$

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When semi-Lagrangian advection is applied

$$\frac{dX}{dt} = \mathcal{M}(X)$$

We linearize around the reference state and get the linear terms $\mathcal{L} \cdot X$ and the nonlinear terms $\mathcal{N}(X)$. Then we treat linear terms implicitly and nonlinear terms in an iterative centered implicit manner.

$$\frac{X^{+i} - X^0}{\delta t} = \frac{\mathcal{L} \cdot X^{+i} + \mathcal{L} \cdot X^0}{2} + \frac{\mathcal{N}(X^{+(i-1)}) + \mathcal{N}(X^0)}{2}$$

We get the Helmholtz equation

$$\left[1 - \frac{\delta t}{2}\mathcal{L}\right] X^{+i} = X^{\bullet}_{a}$$

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Linearized Euler equations

Horizontal momentum

$$\frac{\partial D}{\partial t} = -RG^* \triangle T - RT^* \triangle q_s - \triangle \phi_s$$
$$+\varepsilon_{NH} RT^* \left(G^* - 1\right) \triangle \hat{q}$$

Vertical momentum

$$\frac{\partial d}{\partial t} = -\frac{g^2}{RT_a^*} L^* \hat{q}$$

Temperature

$$\frac{\partial T}{\partial t} = -\frac{RT^*}{c_p} S^* D +$$
$$+\varepsilon_{NH} \left[\frac{RT^*}{c_p} S^* D - \frac{RT^*}{c_v}\left(D + d\right)\right]$$

Pressure departure

$$\frac{\partial \hat{q}}{\partial t} = S^* D - \frac{c_p}{c_v}\left(D + d\right)$$

Surface pressure

$$\frac{\partial q_s}{\partial t} = - \quad N^* D$$

$\varepsilon_{NH} = 0$ **HPE**

$\varepsilon_{NH} = 1$ **Euler equations**

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[Thanks to Fabrice Voitus]

After discretization, the elimination of all variables except D yields (using $\varkappa = \frac{c_v}{c_p}$)

the reduced system

$$\left[1 - \delta t^2 \frac{RT^*}{\varkappa} B^* \triangle\right] D^+ = 0$$

with

vertical structure matrix

$$\begin{aligned}
B^* &= B^*_{HY} + \varepsilon_{NH} B^*_{NH} \\
B^*_{HY} &= \varkappa (1 - \varkappa) G^* S^* + \varkappa N^* \\
B^*_{NH} &= (I - \varkappa G^*) \left(I - \delta t^2 \frac{g^2}{\varkappa R T^*_a} L^* \right)^{-1} (I - \varkappa S^*)
\end{aligned}$$

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Conclusions:

- the whole procedure for the semi-implicit time scheme may be solved similarly for HPE and EE, only with different elimination matrix (Fabrice Voitus).
- Moreover, the elimination matrix may be formulated as the sum of a hydrostatic part and a nonhydrostatic departure.
- Values $0 < \varepsilon_{NH} < 1$ do not have a physical meaning in the full model. But we may use them in linear model for the semi-implicit time scheme and investigate the stability of the proposed solution.
- We may envisage values $0 < \varepsilon_{NH} < 1$ for example in dependence on the vertical coordinate η, allowing for smooth transition from fully elastic nonhydrostatic Euler equations to hydrostatic primitive equations near the model top where we care about stability more then accuracy (at least in LAM).

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Plans:

- analysis of the eigenvalues of the vertical structure matrix in case of $\varepsilon_{NH} \neq 0, 1$
- stability analysis for a system with ε_{NH} varying with the vertical coordinate η in the linear model for the SI scheme is (in progress)
- possible code redesign to include these ideas
- ...

VFE new formulation for HPE
[Jozef Vivoda]

VFE new formulation for HPE

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Hybrid mass based vertical coordinate definition with hydrostatic pressure π in both HPE and EE systems (Laprise, 1992).

Vertical discretization is based on finite difference method (VFD), or finite elements method (VFE).

Integral operator definition:

VFD

$$\int_0^1 \frac{\partial \pi}{\partial \eta} \psi d\eta \approx \sum_{l=1}^{L} \psi_l \delta \pi_l$$

VFE

$$\int_0^1 \frac{\partial \pi}{\partial \eta} \psi d\eta \approx (Km\psi)_L$$

Layer depth definitions differ:

VFD

$$\begin{aligned} \pi &= A(\eta) + \pi_s B(\eta) \\ \delta \pi_l &= \pi_{\tilde{l}} - \pi_{\widetilde{l}} \\ \pi_{\tilde{l}} &= A_{\tilde{l}} + \pi_s B_{\tilde{l}} \end{aligned}$$

η **remains implicit**

VFE

$$\begin{aligned} \pi &= A(\eta) + \pi_s B(\eta) \\ m_l = \frac{\partial \pi}{\partial \eta} &= \frac{\delta A_l}{\delta \eta_l} + \pi_s \frac{\delta B_l}{\delta \eta_l} \\ \delta \eta_l &= \eta_{\tilde{l}} - \eta_{\widetilde{l-1}} \end{aligned}$$

η **requires explicit definition**

VFE new formulation for HPE

To conserve mass we apply

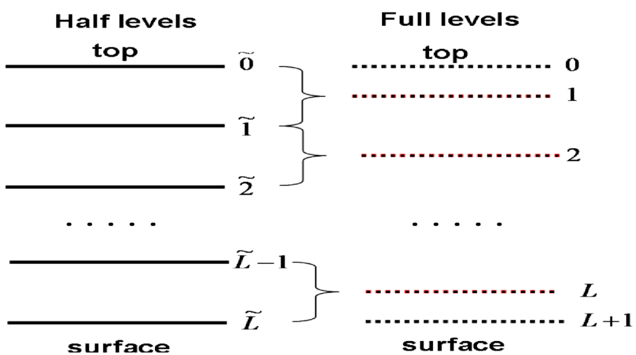
VFD

$$\sum_{l=1}^{L} \delta\pi_l = \sum_{l=1}^{L} \delta A_l + \pi_s \sum_{l=1}^{L} \delta B_l$$

$$\pi_{\tilde{L}} - \pi_{\tilde{1}} = (A_{\tilde{L}} - A_{\tilde{1}}) + \pi_s (B_{\tilde{L}} - B_{\tilde{1}})$$

satisfied through

$$\pi_{\tilde{L}} = \pi_s, \pi_{\tilde{1}} = 0$$
$$A_{\tilde{L}} = 0, A_{\tilde{1}} = 0, B_{\tilde{1}} = 0, B_{\tilde{L}} = 1$$



VFE

$$\pi_s = (Km)_L = \left(K\frac{\delta A}{\delta\eta}\right)_L + \pi_s \left(K\frac{\delta B_l}{\delta\eta}\right)_L$$

$$\left(K\frac{\delta A}{\delta\eta}\right)_L = 0, \quad \left(K\frac{\delta B}{\delta\eta}\right)_L = 1$$

adjustment needed

$$\widetilde{\delta B_l} = B_{\tilde{l}} - B_{\widetilde{l-1}}$$

$$\alpha = \left[K\frac{\widetilde{\delta B}}{\delta\eta}\right]_L$$

$$\delta B_l = \frac{1}{\alpha}\widetilde{\delta B_l}$$

$$\widetilde{\delta A_l} = A_{\tilde{l}} - A_{\widetilde{l-1}}$$

$$\beta = \left[K\left(\frac{\widetilde{\delta A}}{\delta\eta} + \pi_s \frac{\delta B}{\delta\eta}\right)\right]_L$$

$$\delta A_l + \pi_s \delta B_l = \frac{1}{\beta}\left(\widetilde{\delta A_l} + \pi_s \delta B\right)$$

VFE new formulation for HPE

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Explicit definition of the η coordinate in VFE:

VFE

$$\eta_l = \frac{\sum_{i=1}^{l} \delta\pi_l^{\alpha}}{\sum_{i=1}^{L} \delta\pi_l^{\alpha}}$$

$\alpha = 0$ gives regular η levels

Found unstable for high order VFE schemes with spline order ≥ 5.
Stabilisation requires higher density of levels close to BCs.

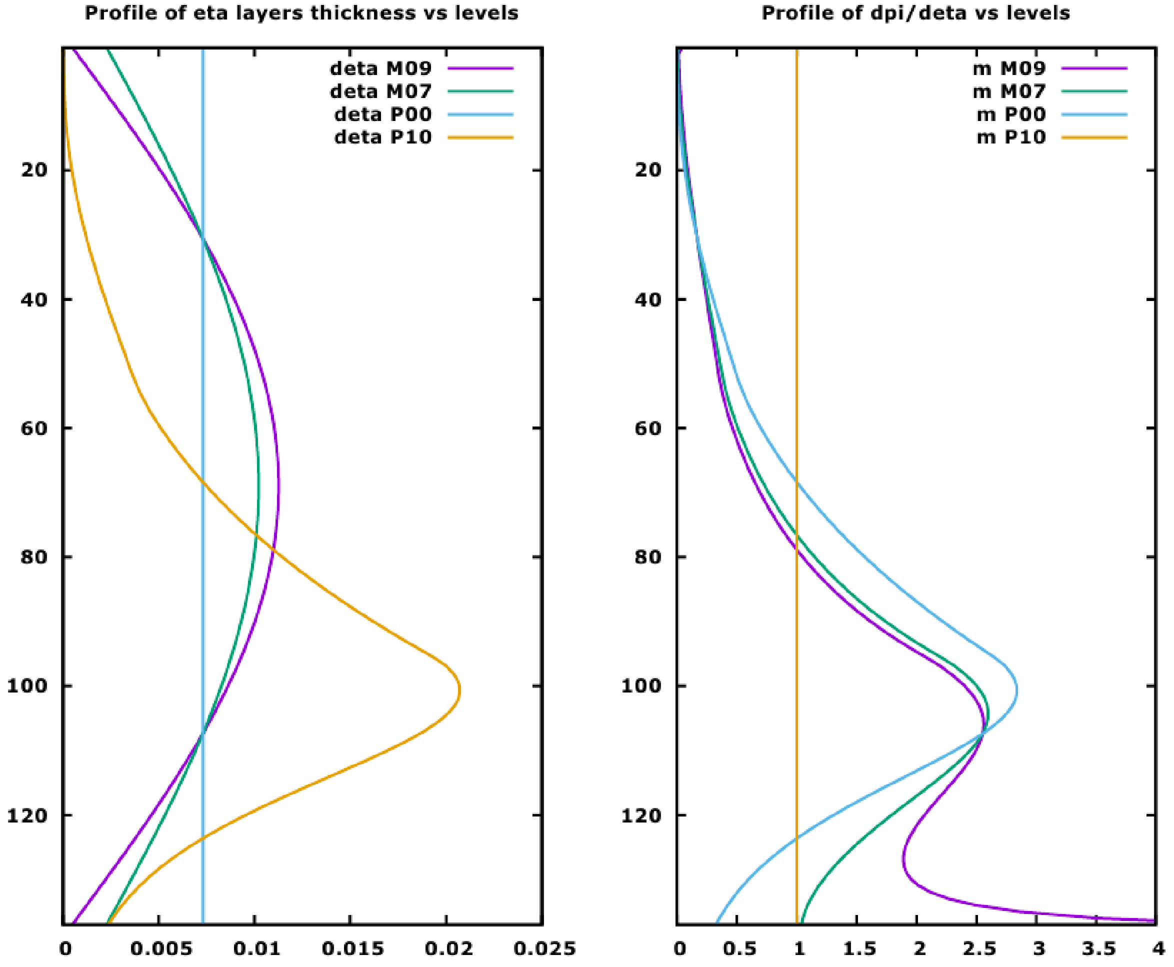
VFE

$$\eta_l = (1-\beta)\frac{l}{L} + \frac{\beta}{2}\left(1 - \cos(\frac{\pi l}{L})\right)$$

$\beta = 0$ gives regular η levels
$\beta \approx 1$ very dense close to BCs

Experimentally found that $\beta \approx 0.5$ is stable for high order operators and the eigenvalues of the linear model are purely imaginary as needed.

VFE new formulation in HPE

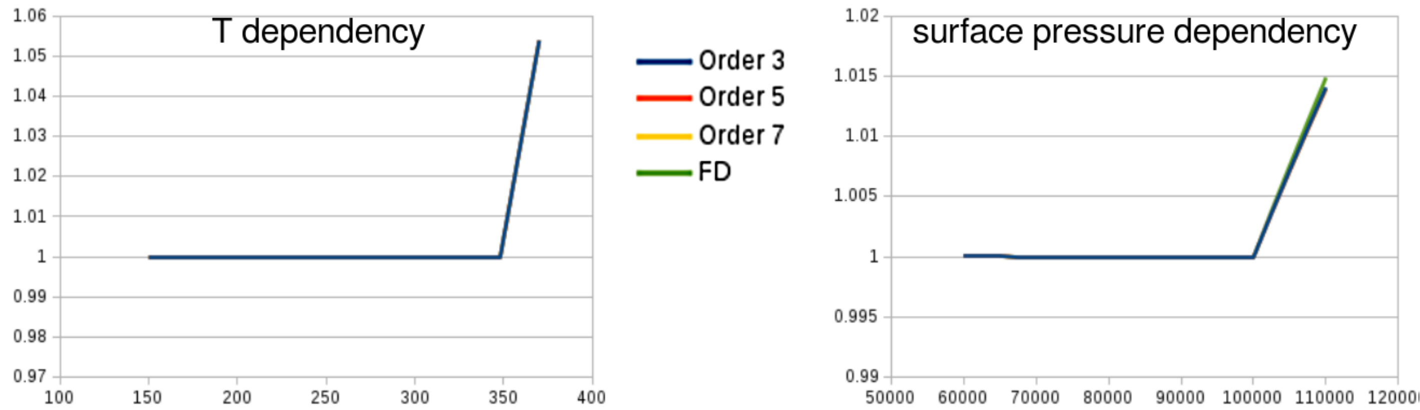


regular η levels, $\alpha = 1$, $\beta = 0.7$, $\beta = 0.9$

VFE new formulation for HPE

General order B-splines are available as the basis for the finite element method in the vertical.

Analysis of stability was implemented according to (Simmons, Hoskins, Burridge, 1978). Atmosphere is described by isothermal, resting, horizontally homogeneous and hydrostatically balanced state given by 2 values $\overline{T}, \overline{\pi_s}$. Nonlinear residual is treated with iterative centered implicit method with one predictor and one corrector step.



Performed for single wave. Reference state with $T^* = 350K$, $T_a^* = 50K$, $\pi_s^* = 1000hPa$ and vertical coordinate definition uses $\beta = 0.5$.

VFE new formulation for HPE

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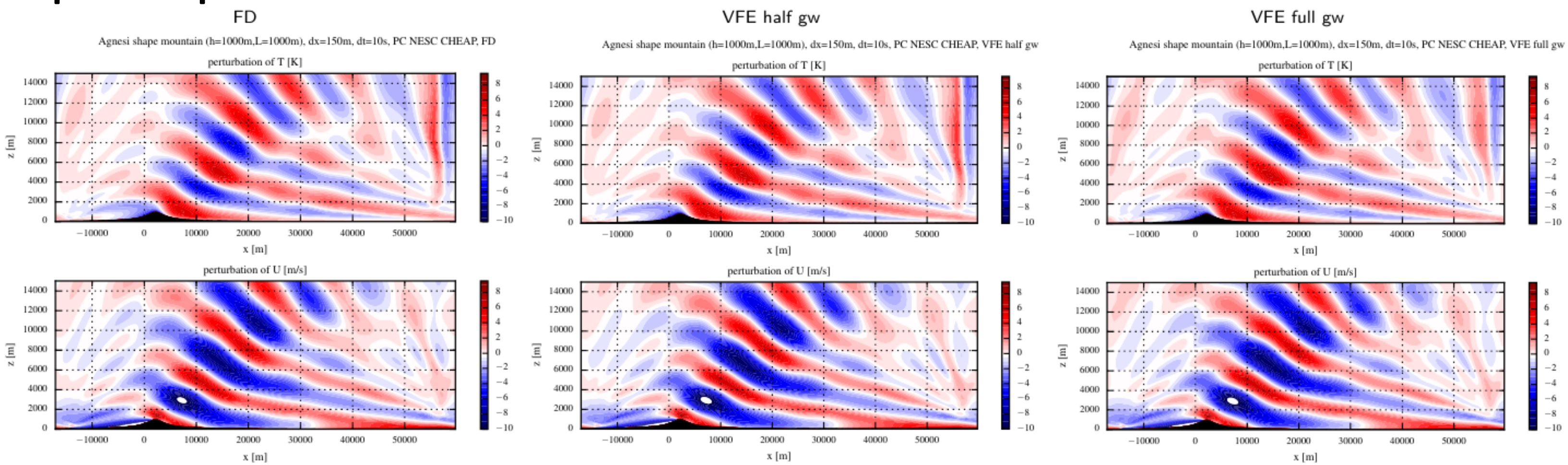
Vertical velocity may be defined either on half levels, or on full levels. In that case there is no vertical staggering at all, but it does not seem to pose a problem.



Experiment in IFS with elliptic Agnesi shaped mountain ($h = 1000m, L = 1000m$). Used resolution $\Delta x = 150m, \Delta t = 10s$ and $N = 0.02s^{-1}, U = 10ms^{-1}$. Vertical cross section through the mountain is shown.

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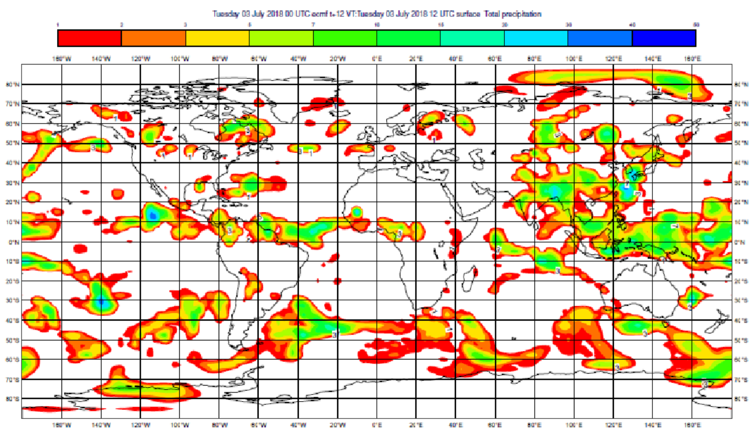
VFE new formulation for HPE

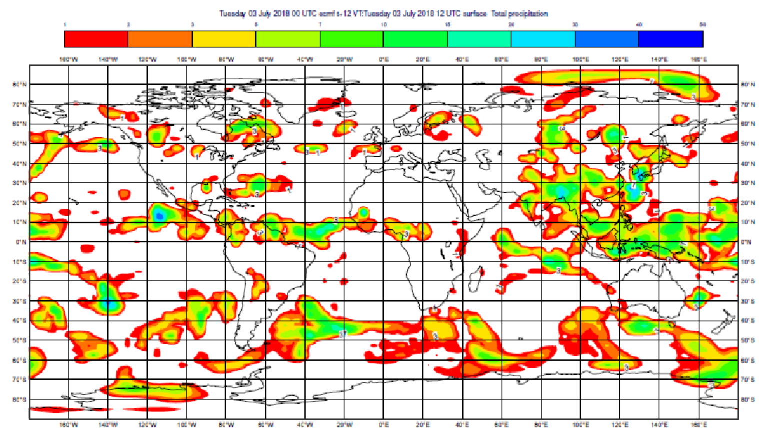
Real experiment with NH IFS

- total precipitation after 12 hours of integration

VFD



VFE with cubic B-splines and vertical η-levels with $\beta = 0.5$



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Grey zone of turbulence
[Mario Hrastinski]

- In ALARO configuration the turbulence scheme TOUCANS (Third Order Moments Unified Condensation Accounting and N-Dependent Solver) is applied, based on a unified treatment of stability functions, applicable in both stable and unstable regimes.
- It includes the prognostic equation for turbulent kinetic energy TKE and total turbulent energy TTE.

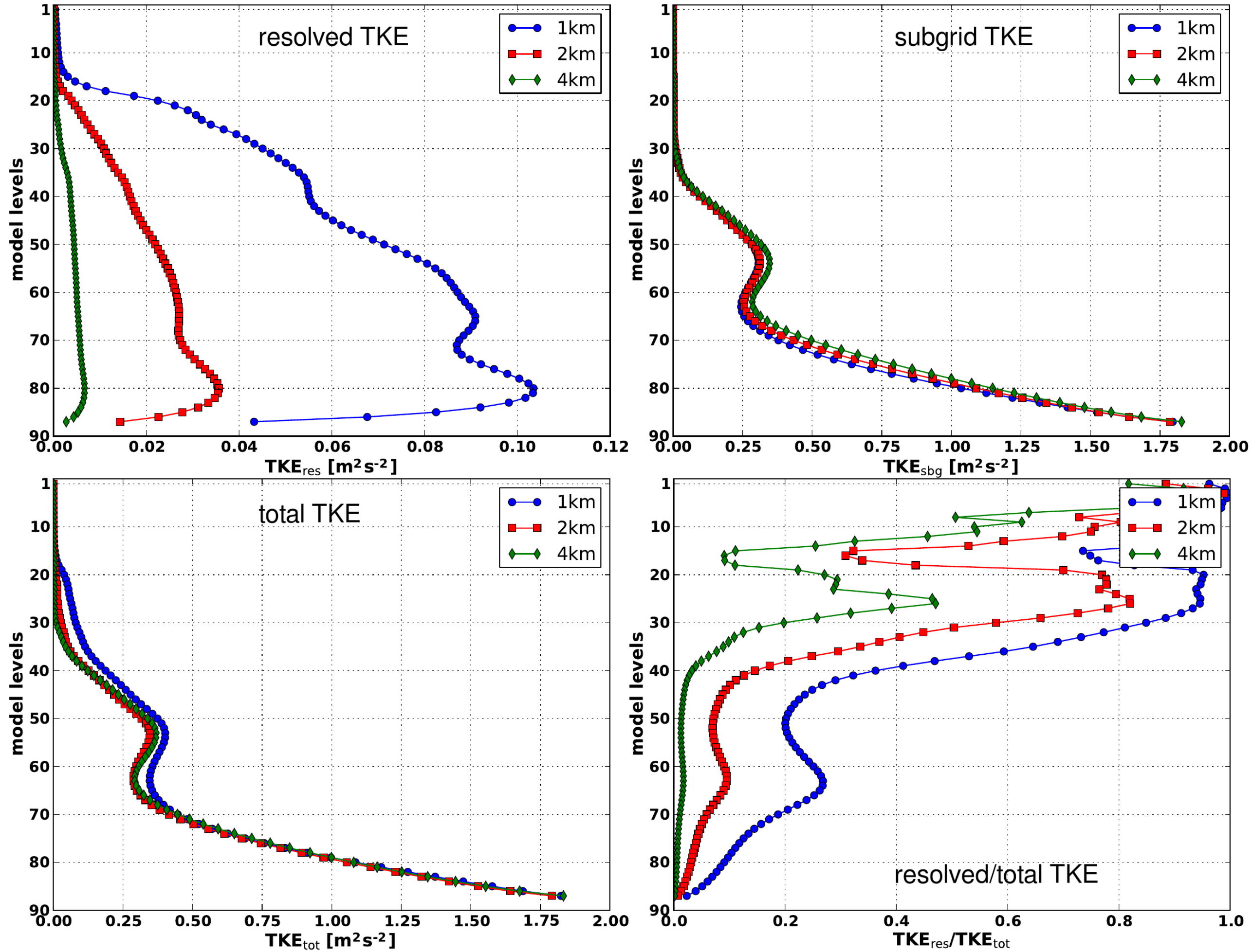
Are we entering the grey zone of turbulence?

R.Honnert: The grey zone exists from resolutions smaller than 2 times the boundary-layer height in convective boundary layers.

Do we have to adjust the turbulence scheme in the grey zone of turbulence?

We estimated the resolved and the subgrid part of TKE as the first step.

Grey zone of turbulence



Grey zone of turbulence

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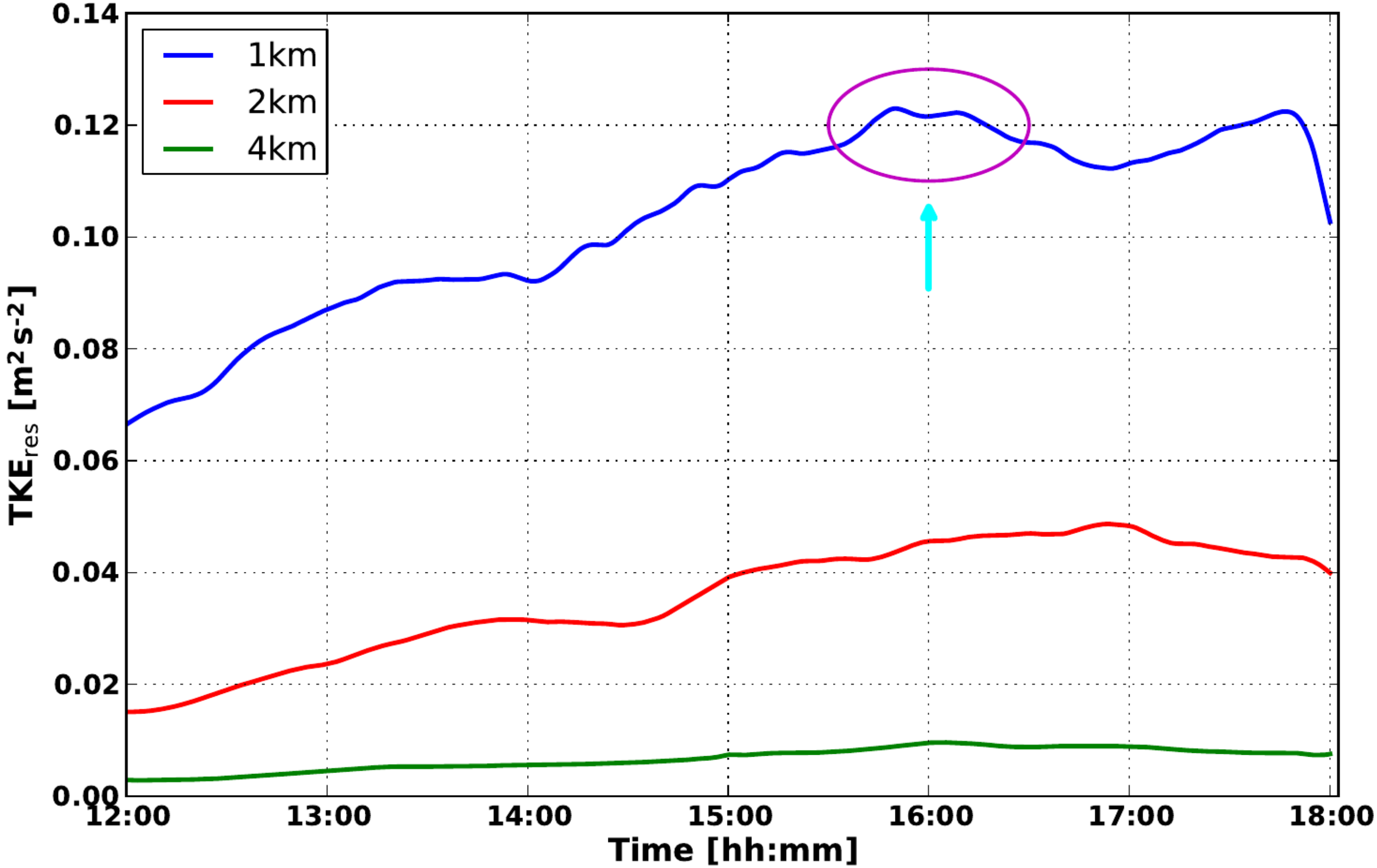

Time evolution of the spatially averaged resolved TKE for ALARO-CMC at model level 80 during a case of summer convection (21.06.2018.)

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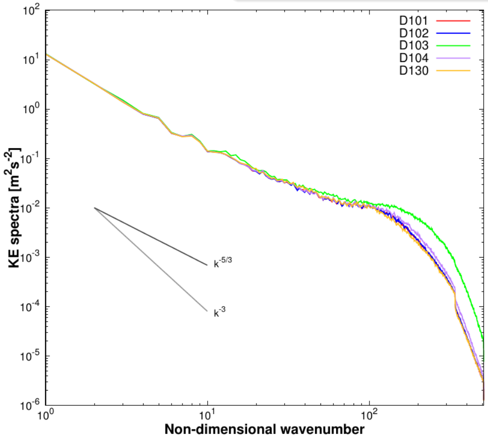
Grey zone of turbulence

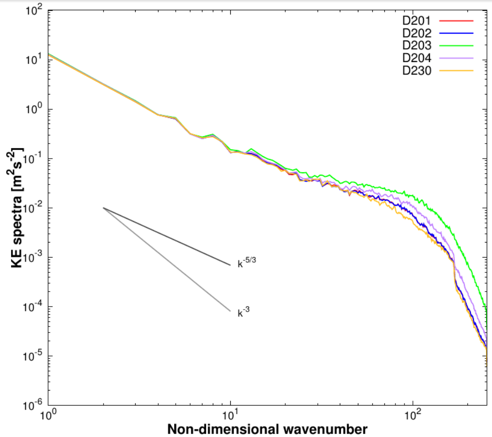
SLHD (Semi-Lagrangian Horizontal Diffusion, Váňa, 2008) is the nonlinear horizontal diffusion with three basic parts:

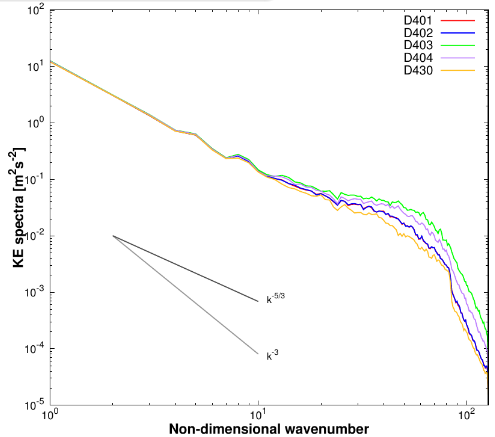
- grid-point diffusion dependent on the flow deformation
- reduced spectral diffusion acting on the upper domain
- supporting spectral diffusion

How the turbulence scheme interacts with SLHD?

Can we control SLHD to act as the horizontal part of the turbulence parametrization?







Благодаря ви за вниманието!