Recent numerics developments in the COSMO and ICON model

43rd EWGLAM / 28th SRNWP meeting, online
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Outline

- **COSMO (Runge-Kutta)**: no further developments in dyn. core
  - Transition to ICON in the consortium:
    DWD has switched off COSMO completely; ICON-D2 since Feb. 2021.
    Many COSMO partners run ICON pre-operationally, too.
- **COSMO-EULAG:** developed by and running operationally at IMGW, Poland
- **ICON**
  - Almost no further developments in the current dyn. core during last year
    COSMO Tech. Report No. 44: ‚Comparison of the dynamical cores of ICON and COSMO'
    (available soon on COSMO web page)
  - **Discontinuous Galerkin-discretisation** for ICON
    - Further work on the 2D toy model
    - BRIDGE: a 3D ICON-prototype

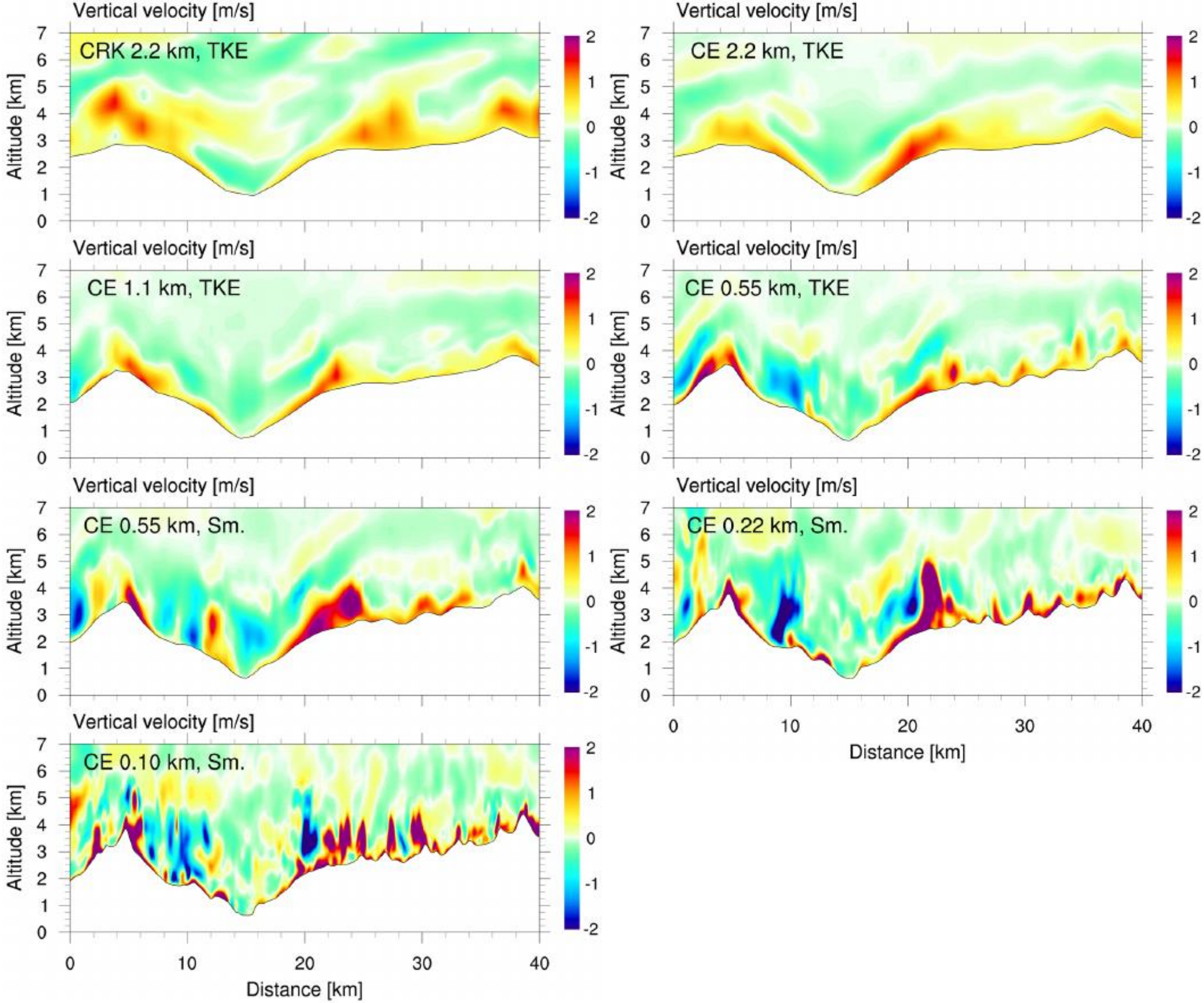
COSMO-EULAG publication

M. Ziemiański

An article „Compressible EULAG dynamical core in COSMO: convective-scale Alpine weather forecasts” by M. Ziemiański, D. Wójcik, B. Rosa, and Z. Piotrowski was accepted for publication in Monthly Weather Review (August 2021)

- it contains :
  - description of the semi-implicit compressible EULAG dynamical core
  - discussion of the coupling of EULAG dynamical core with the COSMO computational and physical framework (version 5.05 was used)
  - comparison of standard verification statistics for 2.2 km COSMO-Runge-Kutta (CRK) and COSMO-EULAG (CE) for warm and colder season over Alpine domain
  - verification case-study for representation of summer convective clouds with CRK at 2.2 km grid and CE at 2.2, 1.1, and 0.55km grid
  - demonstration of the CE forecast of Alpine convection at 0.22 and 0.1 km grid
- it demonstrates the competitive CE verification scores and realism and robustness of its Alpine forecasts at O(100 m) horizontal grid with slopes reaching 85 deg.

CRK and CE vertical velocity over the Alps



Vertical velocity (m/s) over the Rhone valley (Bietschhorn on the left, Weisshorn on the right) at 1230 UTC of 19 July 2013 for horizontal grids between 2.2 and 0.1 km and different turbulence schemes

(TKE or Smagorinsky)

Properties of the dynamical core of ICON

- uses non-hydrostatic, compressible Euler eqns.
- exactly mass- and tracer mass-conserving.
- It is a true 2nd order scheme (as long as parameterizations are switched off).
- stable in very steep mountainous regions.
- useable both for global and regional applications.
- computationally very efficient and scales well on current parallel comp.

(Zängl et al. (2015) QJRMS, Zängl (2012) MWR)

Some numerical details:

- staggering: horizontal: icosahedral, triangle C-grid, vertical: Lorenz-grid
- mixed finite-volume / finite-difference
- predictor-corrector time-integration
- several damping mechanisms are used (divergence damping (2D and quasi-3D), off-centering in the vertically implicit solver, artificial horizontal diffusion…)

Discontinuous Galerkin (DG) methods in a nutshell (I)

$$\frac{\partial q^{(k)}}{\partial t} + \nabla \cdot \mathbf{f}^{(k)}(q) = S^{(k)}(q), \qquad k = 1, ..., K$$

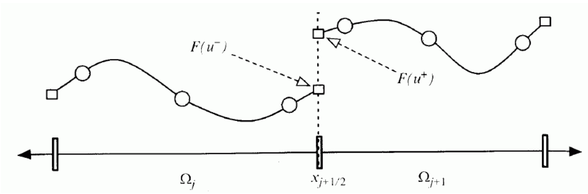
e.g.
Cockburn, Shu (1989) Math. Comput.
Cockburn et al. (1989) JCP
Hesthaven, Warburton (2008)

1.) weak formulation $\int_{\Omega_j} dx\, v(\mathbf{x}) \cdot \ldots$

$$\rightarrow \quad \frac{d}{dt}\int_{\Omega_j} q^{(k)}\, v\, dV + \int_{\partial\Omega_j} f^{(k)num,\perp}\, v\, da - \int_{\Omega_j} \mathbf{f}^{(k)} \cdot \nabla v\, dV = \int_{\Omega_j} S^{(k)}\, v\, dV$$

2.) Finite-element ingredient

$$q^{(k)}(x,t) = \sum_{l=0}^{p} q_{j,l}^{(k)}(t)\; p_l(x - x_j)$$



Galerkin-idea: identify $v \equiv p_l$

Modal base: orthogonal functions e.g. Legendre-Polynomials
Nodal base: interpolation (Lagrange) polynomials

From *Nair et al. (2011)* in *‚Numerical techniques for global atm. models'*

Discontinuous Galerkin (DG) methods in a nutshell (II)

Weak formulation

$$\frac{d}{dt}\int_{\Omega_j} q^{(k)}\, v\, dV + \int_{\partial\Omega_j} f^{(k)num,\perp}\, v\, da - \int_{\Omega_j} \mathbf{f}^{(k)} \cdot \nabla v\, dV = \int_{\Omega_j} S^{(k)}\, v\, dV$$

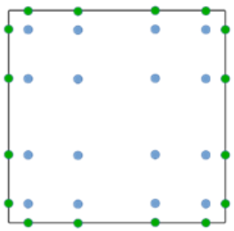
3.) Finite-volume ingredient:

Replace physical flux by a numerical flux in the surface integral
→ couple two neighbouring cells

Often used: simple Lax-Friedrichs flux

$$\mathbf{f}(q) \to f^{num,\perp}(q^+, q^-) = \frac{1}{2}\left(\mathbf{f}(q^+) + \mathbf{f}(q^-)\right) \cdot \mathbf{n} - \frac{\alpha}{2}(q^+ - q^-)$$

4.) Gaussian quadrature for the volume and surface integrals



→ ODE-system for $q^{(k)}{}_{jl}(t)$

5.) Use a time-integration scheme (Runge-Kutta, …)

Step 1:

bring DG on the sphere …

How to construct a higher order numerical scheme on the sphere

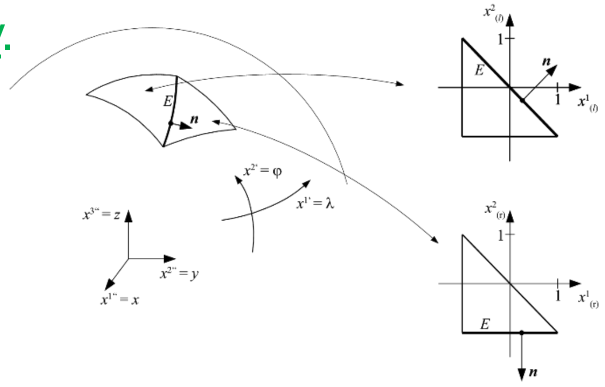
annoying: the sphere doesn‘t allow a single coordinate system without singularities ☹

Straightforward approach to avoid this (for any 2D manifold!)

1. generate a triangulation for an arbitrary set of *points* on the manifold and by connecting them
   a) by *geodetic lines* (=great circle arcs on the sphere) → curved triangles
   b) and by *straight lines* in the embedding Euclidean space → flat triangles → unit triangles
2. map every unit triangle (with *local coordinates* x^1, x^2) to the related curved triangle; this can be done *exactly* (and without any ‚holes‘ or overlappings) for the
   - sphere: by *gnomonial projection* (e.g. *Läuter, Giraldo, … (2008) JCP*)
   - ellipsoid: by *gnomonial + affine projection*

→ all geometric properties (g_{ij}, Γ^i_{jk}, ...) are treated exactly.

→ higher order discretizations are straightforward.



In a FV scheme, one only has to *transform the fluxes* between neighboring unit triangles by

$$f^i_{(r\to l)} = \frac{\partial x^i_{(l)}}{\partial x^j_{(r)}} f^j_{(r)}, \qquad f^i_{(l\to r)} = \frac{\partial x^i_{(r)}}{\partial x^j_{(l)}} f^j_{(l)}$$

This is simplified by using the *covariant form* of the equations ...

Shallow-water equations in covariant form, i.e. only tensors occur
→ equations are valid *on any 2D manifold* (at least from a mathematical viewpoint)

$$\frac{\partial H}{\partial t} + \nabla_j M^j = 0,$$

$$\frac{\partial M^i}{\partial t} + \nabla_j T^{ij} = S^i, \quad i, j = 1, 2,$$

momentum flux tensor:

$$T^{ij} = \frac{M^i M^j}{H} + \frac{1}{2} g_{grav} H^2 g^{ij},$$

source vector of momentum:

$$S^i = -g_{grav} H g^{ij} \nabla_j h_B + f_c g^{ij} E_{jl} M^l,$$

E_{jl} : 2[nd] rank Levi-Civita pseudo tensor,
f_c : Coriolis parameter (a pseudo scalar field)

express covariant derviative ∇_j
by partial derivative and Christoffel symbols
→ accessible to a numerical implementation:

$$\frac{\partial \sqrt{g} H}{\partial t} + \frac{\partial}{\partial x^j} \sqrt{g} M^j = 0,$$

$$\frac{\partial \sqrt{g} M^i}{\partial t} + \frac{\partial}{\partial x^j} \sqrt{g}\, T^{ij} + \sqrt{g}\, \Gamma^i_{jk} T^{kj} = \sqrt{g} S^i.$$

Baldauf, M. (2020): *Discontinuous Galerkin solver for the shallow-water equations in covariant form on the sphere and the ellipsoid,* J. Comp. Phys. 410

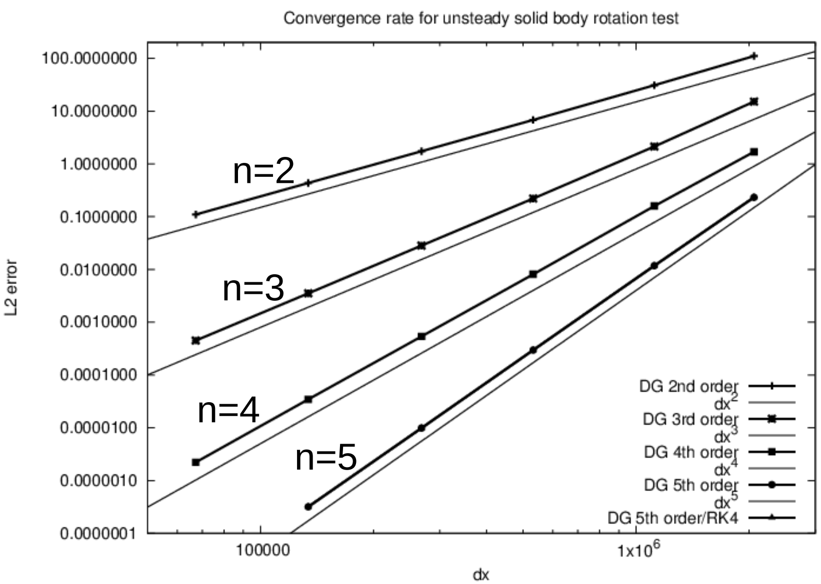
Unsteady solid body rotation

Läuter et al. (2005) JCP

Exact analytic solution available! → calculate error measures L_2 (=RMSE)

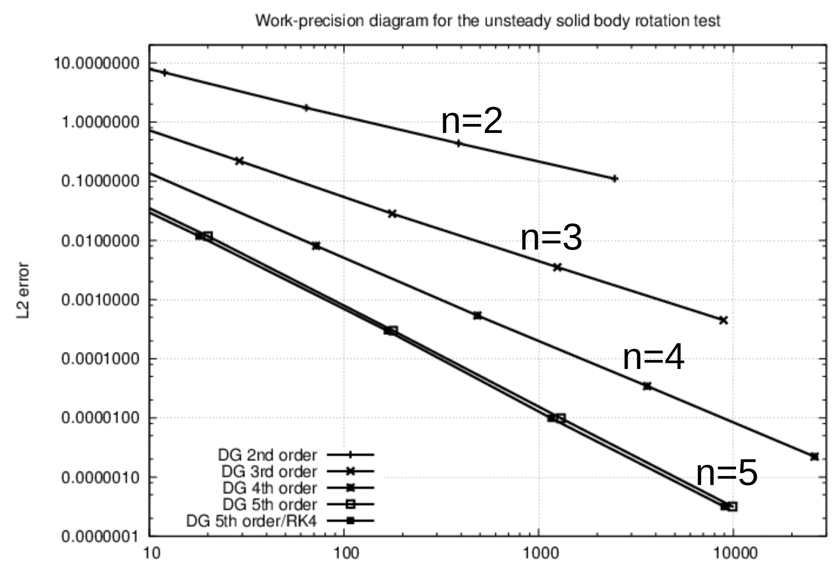
DG: combine polynomials of degree n-1 with n-th order Runge-Kutta scheme

Convergence plot:



dx (in m)

Work-precision diagram:



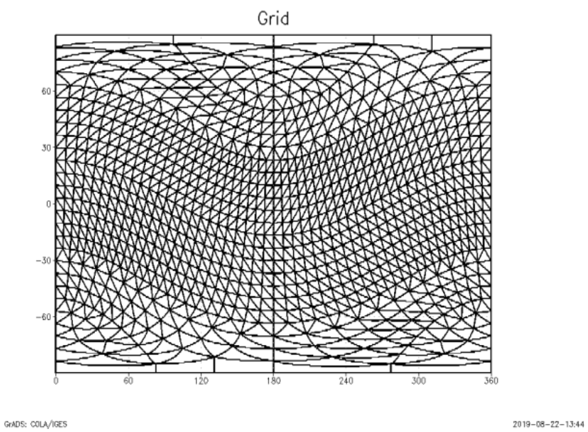
CPU wall clock time (in sec.)

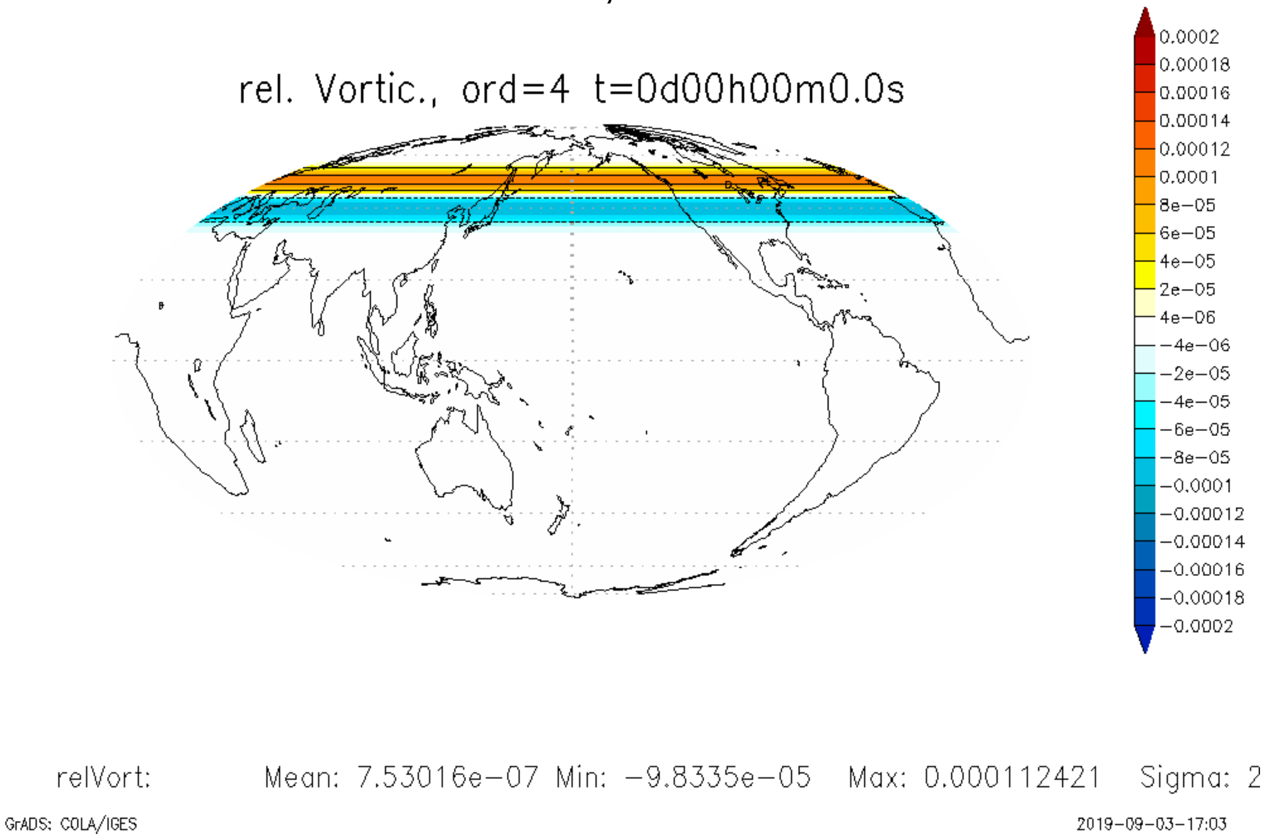
True solution is very smooth →
higher order scheme wins

Barotropic instability test
Galewsky et al. (2004)

4th order DG scheme
without additional diffusion
dx~67 km, dt=15 sec.

simple triangle grid
on the sphere
dx ~ 500km:

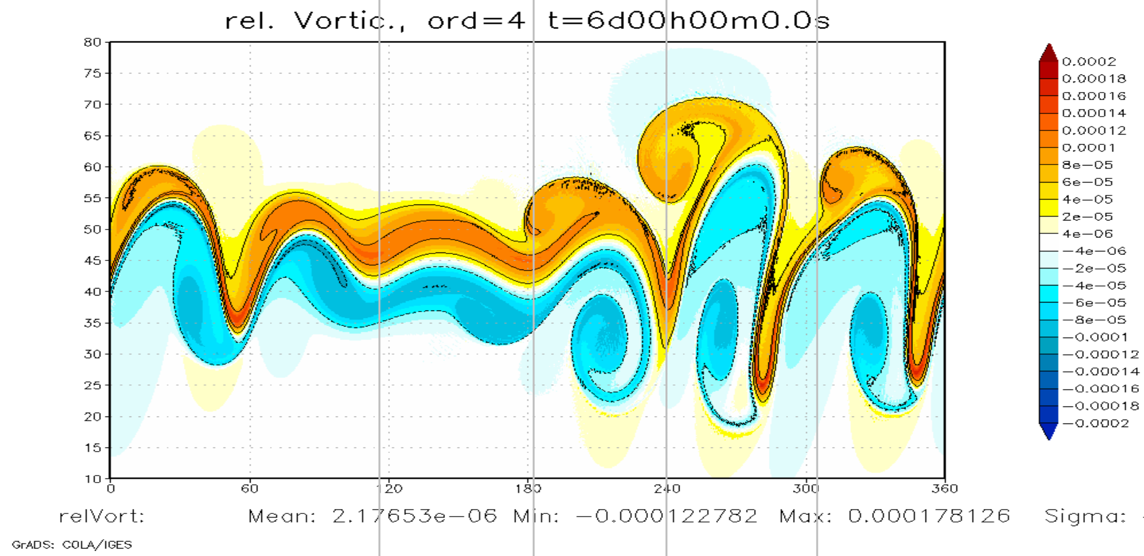




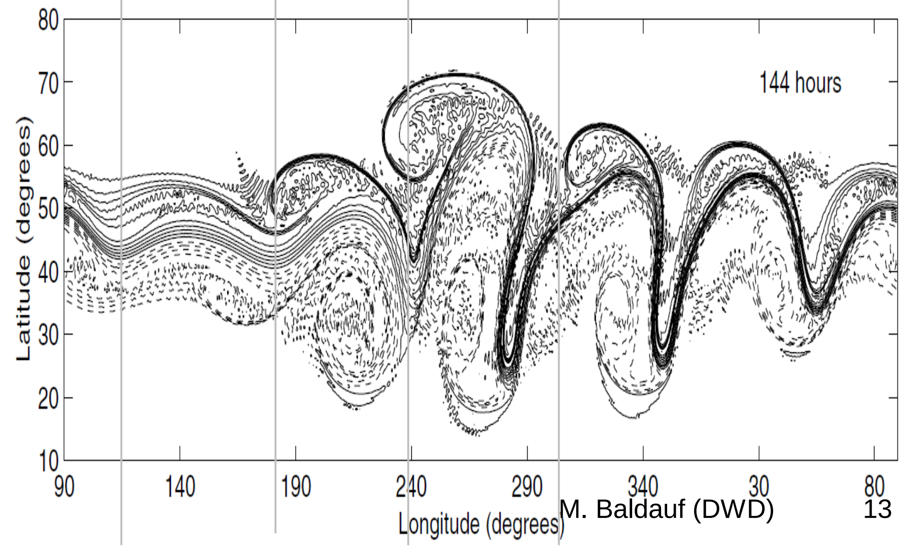
Barotropic instability test

Galewsky et al. (2004)

relative vorticity



4th order DG scheme
without additional diffusion
dx~67 km, dt=15 sec.



FMS-SWM (Geophys. Fl. Dyn. Lab.)
without additional diffusion
dx~60 km (T341), dt=30 sec.

Fig. 4 from *Galewsky et al. (2004)*

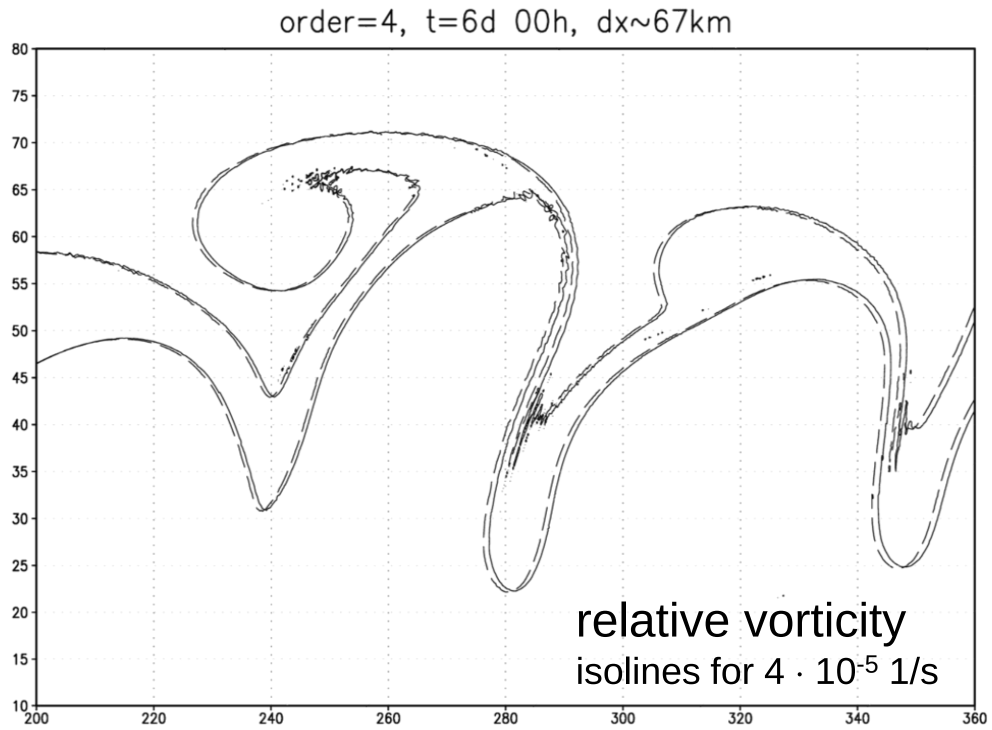
Barotropic instability test
Galewsky et al. (2004)

Comparison between the sphere and the ellipsoid

4th order DG scheme
without additional diffusion
dx~67 km, dt=15 sec.

solid line: sphere
R = 6371.22 km

dashed line: ellipsoid
a = 6378.137 km
c = 6356.752 km
→ numer. excentr. = 0.082



relative vorticity
isolines for $4 \cdot 10^{-5}$ 1/s

→ ellipsoidal solution shows westward phase shift of ~1° after 6 days
→ is in qualitative agreement with *Bénard (2015) QJRMS*

Step 2:

extension for the Euler equations in terrain-following coordinates and a HEVI time integration

Extension to the 3D Euler equations on the sphere together with terrain-following coordinates

Additional metric terms of terrain-following coordinates can destroy numerical local conservation → use *strong conservation form* of the equations, i.e. use both base vectors for a smooth (e.g. spherical) coordinate system K‘ and for the terrain-following system K.

example: strong cons. form of the momentum equation:

$$\frac{\partial}{\partial t}\sqrt{g}\,M^{i'} + \frac{\partial}{\partial x^k}\sqrt{g}\,T^{i'k} = \sqrt{g}\,(S^{i'}_{(M)} - \underset{\uparrow}{\Gamma^{i'}_{k'l'}}T^{l'k'})$$

now: additional metric terms only from the smooth system K‘

momentum flux for Euler eqns.

$$T^{ik} = \frac{1}{\rho}M^i M^k + \tilde{p}g^{ik}$$

for diffusion (D^{ik} = deformation tensor), add

$$T^{ik}_{diff} = -\rho K_a 2D^{ik} - \rho K_b g^{ik}\nabla_l v^l$$

Additionally: Continuity eq. (for ρ) and energy equation (for $\rho\theta$)

Horizontally explicit - vertically implicit (HEVI)-scheme with DG

Motivation: get rid of the **strong time step restriction** by vertical sound wave expansion in **flat grid cells** (in particular near the ground)

$$\frac{\partial q^{(s)}}{\partial t} + \underbrace{\nabla \cdot \mathbf{f}^{(s)}_{slow}}_{\text{explicit}} + \underbrace{\nabla \cdot \mathbf{f}^{(s)}_{fast}}_{\text{implicit}} = \underbrace{S^{(s)}_{slow}}_{\text{explicit}} + \underbrace{S^{(s)}_{fast}}_{\text{implicit}}$$

$$\mathbf{f}^{(s)}_{fast} = f^{(s)}_{z,fast} \mathbf{e}_z$$

$$f^{(s)}_{z,fast} = \sum_{s'} H^{ss'} q^{(s')}$$

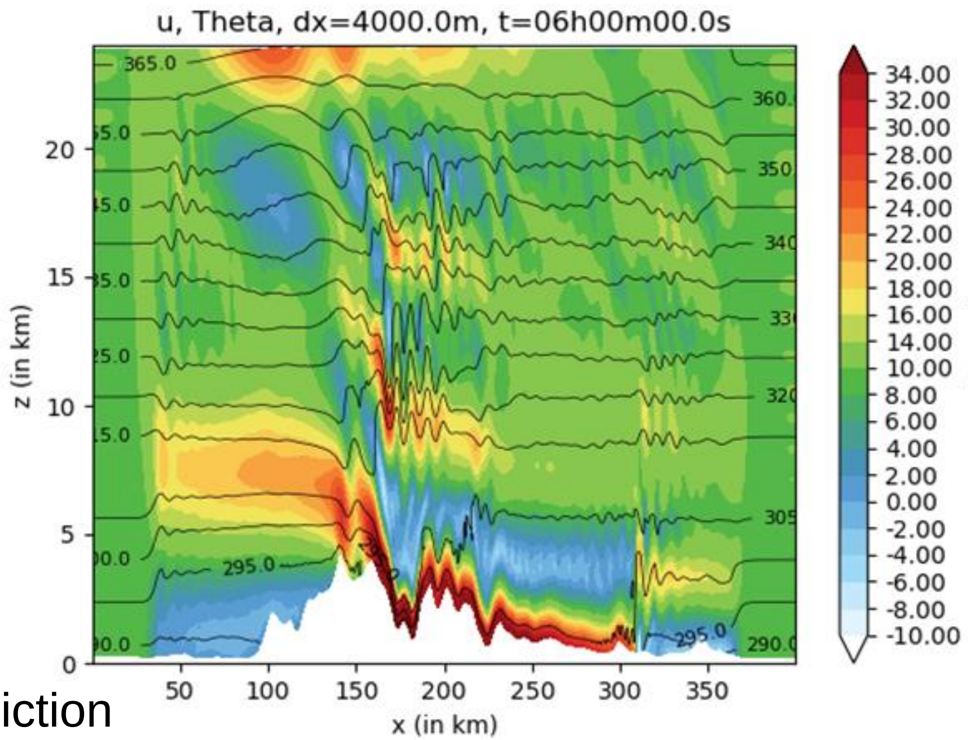
- Use of **IMEX-Runge-Kutta** (SDIRK) schemes: SSP3(3,3,2), SSP3(4,3,3) (*Pareschi, Russo (2005) JSC*)
- The implicit part leads to several block-tridiagonal matrices
  → here a direct solver is used (expensive!)

References:
Giraldo et al. (2010) SIAM JSC: propose a HEVI semi-implicit scheme
Bao, Klöfkorn, Nair (2015) MWR: use of an iterative solver for HEVI-DG
Blaise et al. (2016) IJNMF: use of IMEX-RK schemes in HEVI-DG
Abdi et al. (2019) IJHighPerfCompAppl: use of multi-step or multi-stage IMEX for HEVI-DG

DG 2D toy model: semi-realistic case study

Setup:
U_0= 10 m/s, N=0.01 1/s

2D cross section over the Alps (Monte Rosa region) using orography data on a 0.05° mesh



DG HEVI scheme 4th order,
Smagorinsky model, no surface friction

Δx=4 km; vertical grid stretching: Δz_{min}~46m, Δz_{max}~736m, $z_{lowest\ QP}$~10.3m

Additionally done

- Treatment of diffusion in a HEVI-DG scheme with terrain-following coordinates (by the Bassi, Rebay approach)
- Efficiency improvement of the implicit solver
  (perform expensive LU-decomposition only after several dozen time steps)
- Formulation of boundary conditions for higher order schemes
- Method for consistent use of real orography

Baldauf, M. (2021): *A horizontally explicit, vertically implicit (HEVI) discontinuous Galerkin scheme for the 2-dim. Euler and Navier-Stokes equations using terrain-following coordinates*, J. Comp. Phys. 446

The **BRIDGE** project (**B**asic **R**esearch for **I**CON with **DG E**xtension)

started ~mid 2020
currently: F. Prill, M. Baldauf / joining later: D. Reinert, U. Schättler, S. Borchert, …

Goals:

- develop a prototype for a DG implementation of the 3D Euler equations (‚DG-HEVI on the sphere‘)
- together with a minimal set of physical parameterizations (turbulence, micro physics)
- using ICON infrastructure (parallelisation, I/O, ...)
- more object-orientation and use of standard software (e.g. YAC coupler (DKRZ/MPI-M), YAXT parallel communication (DKRZ), …

as an intermediate step to a full–fledged ICON implementation

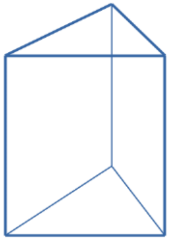
Milestones:

- Shallow-water equations on the sphere ready in Q3/2021
- 3D explicit Euler solver ready in Q4/2021
- 3D HEVI Euler solver ready in Q1/2022 → decision about prolongation of the project
- Implementation into **ICON** (start ~2024)
    - choose optimal approx. order (currently I favor: p_{horiz} =4, p_{vert} =4, p_{time} =3) and grid spacing
    - Operationally useable version might be ready ~2028

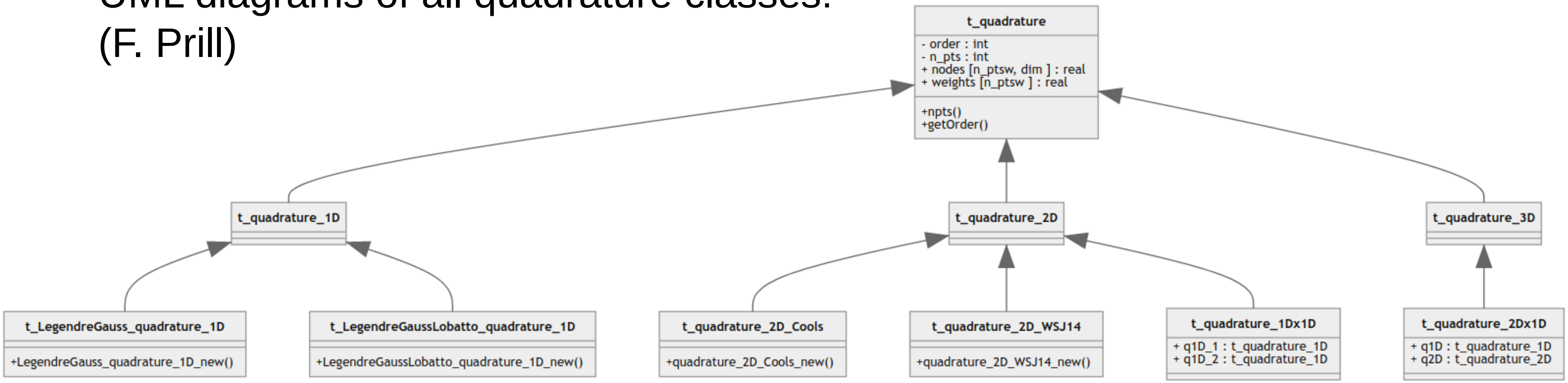
More object orientation with the BRIDGE code

Helps in keeping things as transparent as possible

Example: quadrature classes for the numerical integration over prism volumes or prism faces



UML diagrams of all quadrature classes:
(F. Prill)



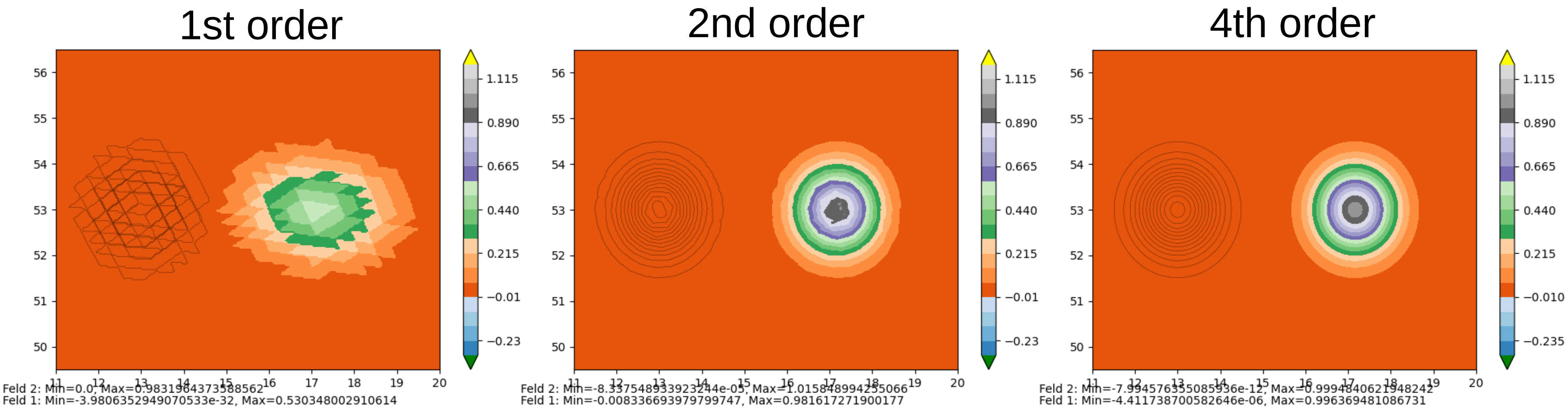
Integration over plane triangles

Integration over lateral faces

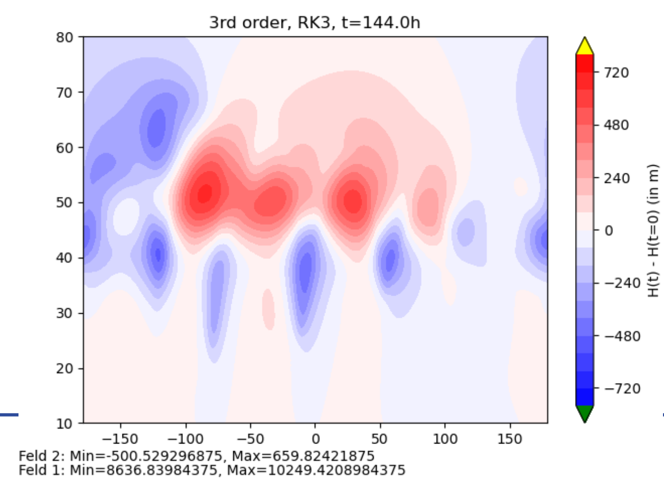
Integration over prism volume

First preliminary results of the BRIDGE code:

1.) Advection by a solid body rotation wind field after 100 time steps (Δt=50 s)



2.) Barotropic instability test on the sphere (Galewsky et al., 2004)
H(t=6d)-H(t=0)



Summary for the DG development

- Basic questions are solved for (by the **2D toy model**)
  - **DG on the sphere** on a triangle grid possible by the use of local coordinates and the covariant formulation of the equations.
  - **HEVI-DG** for *Euler equations* with *terrain-following coordinates* and optionally with *3D diffusion*

Baldauf, M. (2020): *Discontinuous Galerkin solver for the shallow-water equations in covariant form on the sphere and the ellipsoid,* J. Comp. Phys. 410
Baldauf, M. (2021): *A horizontally explicit, vertically implicit (HEVI) Discontinuous Galerkin scheme for the 2-dim. Euler and Navier-Stokes equations using terrain-following coordinates*, J. Comp. Phys. 446

- With respect to the pure dynamical core (=solver for the Euler equations), no show–stopper occured until now. However, total efficiency is still an issue! In particular the vertically implicit solver is still too expensive.
- Further questions must be solved for coupling with parameterizations (time-integration, positive-definiteness, …)
- All this further work is done in the **BRIDGE** project, which is well on the way …

Thank you very much for your attention!

DG – Pros …

- **local conservation** of every prognostic variable
- any **order of approximation (convergence)** possible
- flexible application on **unstructured grids** (also dynamic adaptation is possible, h-/p-adaptivity)
- very good **scalability** on massively-parallel computers (compact data transfer and no extensive halos)
- **separation** between (analytical) equations and numerical implementation
- **boundary conditions** are easily prescribed (fluxes or values in weak form) → coupling with other subcomponents (ocean model, …) should be easy
- higher accuracy helps to **avoid several awkward approaches** of standard 2^{nd} order schemes: staggered grids (on triangles/hexagons, vertically heavily stretched), numerical hydrostatic balancing, grid imprints by pentagon points or along cubed sphere lines, …
- **unified numerical treatment** of all flux terms and source terms
- **explicit** schemes are relatively easy to build and are quite well understood

DG – … and Cons

- **high computational costs** due to
  - (apparently) **small Courant numbers** → small time steps
  - higher number of **degrees of freedom**
    - variables ‚live' both on interior *and* on edge quadrature points
    - this holds additionally for **parabolic problems (diffusion)**
  - HEVI approach leads to **block tridiagonal matrices** with larger blocks

→ All these expenses must be outperformed by:
higher convergence order, better computational intensity, and better parallelization!

- **well-balancing** (hydrostatic, perhaps also geostrophic?) in Euler equations is an issue → can be solved!
- basically ‚only' an **A-grid-method** →however, the ‚spurious pressure mode' is very selectively damped!