

*Regional Cooperation for  
Limited Area Modeling in Central Europe*



**ACC  RD**

A Consortium for CONvection-scale modelling  
Research and Development

**Dynamics in LACE - towards hectometric scales**

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**ARSO METEO**  
Slovenia

- ❑ Dynamical core in ACCORD
- ❑ SI time scheme
- ❑ Orographic terms in linear model (based on ideas of Fabrice Voitus and Jozef Vivoda)
- ❑ Idealised test cases
- ❑ Real simulations @200m
- ❑ Consecutive domain approach

## Basic equations

- hydrostatic primitive equation system (HPE) or Euler equations (EE); recently implemented quasi elastic equation system (QE)
- prognostic variables  $\vec{v}, T, q_s = \ln(\pi_s)$ , in EE with  $w, \hat{q} = \ln(\frac{p}{\pi})$

## Discretization

- spectral method for horizontal direction
- hybrid vertical coordinate  $\eta$  based on hydrostatic pressure  
 $\pi(\eta) = A(\eta) + B(\eta)\pi_s$ ;  
 $A(top) = B(top) = 0, A(bottom) = 0, B(bottom) = 1$
- finite differences or finite elements for vertical direction
- semi-implicit or iterative centred implicit scheme for time
- semi-Lagrangian advection

## System evolution

$$\frac{dX}{dt} = \mathcal{M}X$$

## Linearization

$$X = \mathbf{X}^* + \mathbf{X}', \quad \frac{\partial}{\partial t} \mathcal{M} \rightarrow \mathcal{L}^*$$

Using linear model  $\mathcal{L}^*$  we divide

$$\frac{dX}{dt} = \mathcal{L}^* \overline{[X]}^t + (\mathcal{M} - \mathcal{L}^*)X$$

and discretize in time to obtain

## Semi-implicit scheme

$$\frac{\mathbf{X}^+ - \mathbf{X}^0}{\Delta t} = \mathcal{L}^* \left( \frac{\mathbf{X}^+ + \mathbf{X}^0}{2} \right) + (\mathcal{M} - \mathcal{L}^*)\mathbf{X}^{+\frac{1}{2}}$$

## Iterative centered implicit scheme

or

$$\frac{\mathbf{X}^{+(n)} - \mathbf{X}^0}{\Delta t} = \frac{\mathcal{L}^* \mathbf{X}^{+(n)} + \mathcal{L}^* \mathbf{X}^0}{2} + \frac{(\mathcal{M} - \mathcal{L}^*)\mathbf{X}^{+(n-1)} + (\mathcal{M} - \mathcal{L}^*)\mathbf{X}^0}{2}$$

We know that both can be second order accurate in time when some care is taken (averaging along semi-Lagrangian trajectory).

## Temperature

$$\frac{dT}{dt} = \frac{\kappa T}{\kappa - 1} (D + d)$$

## Horizontal momentum

$$\frac{d\vec{v}}{dt} = -RT \frac{\nabla \pi}{\pi} - \nabla \phi - RT \nabla \hat{q} - \frac{1}{m} \frac{\partial(p - \pi)}{\partial \eta} \nabla \phi$$

## Vertical momentum

$$\frac{dw}{dt} = \frac{g}{m} \frac{\partial(p - \pi)}{\partial \eta}$$

## Pressure departure

$$\frac{d\hat{q}}{dt} = \frac{1}{\kappa - 1} (D + d) - \frac{1}{\pi} \frac{d\pi}{dt}$$

## Surface pressure

$$\frac{dq_s}{dt} = -\frac{1}{\pi_s} \int_0^1 \nabla \cdot (m\vec{v}) d\eta$$

## Diagnostic relations

$$\begin{aligned} \frac{d\pi}{dt} &= \vec{v} \cdot \nabla \pi - \int_0^\eta \nabla \cdot (m\vec{v}) d\eta' \\ \phi &= \phi_s - \int_\eta^1 \frac{mRT}{p} d\eta' \\ d &= \frac{p}{mRT} \left( \nabla \phi \frac{\partial \vec{v}}{\partial \eta} - g \frac{\partial w}{\partial \eta} \right) \end{aligned}$$

## Definitions

$$\begin{aligned} D &= \nabla \cdot \vec{v} \\ \kappa &= \frac{c_p}{R} \\ m &= \frac{\partial \pi}{\partial \eta} \end{aligned}$$

## Temperature

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## Diagnostic relations

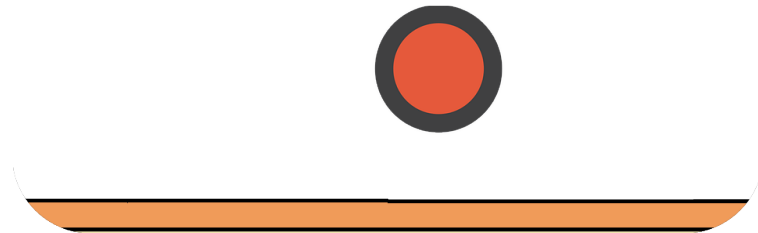
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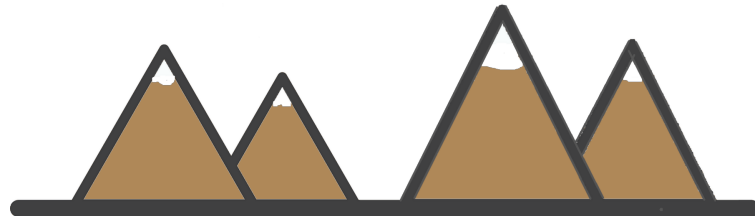
## Current:

- stationary
- resting
- hydrostatically balanced ( $\pi_s^*$ )
- dry
- isothermal ( $T^*$ )
- with constant orography ( $(\nabla\phi)^* = 0$ )



New:

- stationary
- resting
- hydrostatically balanced ( $\pi_s^*$ )
- dry
- isothermal ( $T^*$ )
- with constant orographic slope (in absolute value,  $|\nabla\phi^*| \neq 0$ )





## Temperature

$$\frac{\partial T}{\partial t} = \frac{\kappa T^*}{\kappa - 1} (D + d)$$

## Horizontal momentum

$$\frac{\partial \vec{v}}{\partial t} = -RT^* \frac{\nabla \pi}{\pi^*} - \nabla \phi - RT^* \nabla \hat{q} - \frac{1}{m^*} \frac{\partial \pi^* \hat{q}}{\partial \eta} (\nabla \phi)^*$$

## Vertical momentum

$$\frac{\partial w}{\partial t} = \frac{g}{m^*} \frac{\partial \pi^* \hat{q}}{\partial \eta}$$

## Pressure departure

$$\frac{\partial \hat{q}}{\partial t} = \frac{1}{\kappa - 1} (D + d) + \frac{1}{\pi^*} \int_0^\eta m^* D d\eta'$$

## Surface pressure

$$\frac{\partial q_s}{\partial t} = -\frac{1}{\pi_s^*} \int_0^1 m^* D d\eta$$

## Diagnostic relations

$$\begin{aligned} \nabla \phi &= \nabla \phi_s - \int_\eta^1 \nabla \left( \frac{mRT}{p} \right) d\eta' \\ (\nabla \phi)^* &= g \Lambda^* S^*(\eta) \\ d &= \frac{\pi^*}{m^* RT^*} \left( (\nabla \phi)^* \frac{\partial \vec{v}}{\partial \eta} - g \frac{\partial w}{\partial \eta} \right) \end{aligned}$$

## Definitions

$$\begin{aligned} \Lambda^* &= \frac{1}{g} \|\nabla \phi_s\|^* \\ S^*(\eta) &= \frac{B \pi_s^*}{\pi^*} \\ m^* &= \frac{\partial \pi^*}{\partial \eta} \end{aligned}$$

## Modified vertical divergence

$$d = \frac{1}{RT^*} ((\nabla\phi)^* \partial^* \vec{v} - g \partial^* w)$$

where

$$\partial^* X = \frac{\pi^*}{m^*} \frac{\partial X}{\partial \eta}$$

## Time evolution in linear model

$$\frac{\partial \vec{v}}{\partial t} = A - \Lambda^* S^* (\eta) B$$

$$\frac{\partial w}{\partial t} = B$$

$$\frac{\partial d}{\partial t} = \frac{1}{RT^*} \left[ g \Lambda^* S^* (\eta) \partial^* \left( \frac{\partial \vec{v}}{\partial t} \right) - g \partial^* \left( \frac{\partial w}{\partial t} \right) \right]$$

$$= \frac{1}{RT^*} \left[ g \Lambda^* S^* (\eta) \partial^* A - g \Lambda^{*2} S^* (\eta) (S^* \partial^* B + B \partial^* S^*) - g \partial^* B \right]$$

We omit the first order terms in  $\Lambda^*$  and then  $\frac{\partial \vec{v}}{\partial t}$  is unchanged and all operators of the RHS of  $\frac{\partial d}{\partial t}$  apply on  $\hat{q}$ .

Time evolution in linear model

$$\begin{aligned}\frac{\partial \vec{v}}{\partial t} &= A - \cancel{\Lambda^* S^* (\eta) B} \\ \frac{\partial d}{\partial t} &= \frac{1}{RT^*} \left[ \cancel{g \Lambda^* S^* (\eta) \partial^* A} - g \Lambda^{*2} S^* (\eta) (S^* \partial^* B + B \partial^* S^*) - g \partial^* B \right]\end{aligned}$$

Finally, since  $B = g(\partial^* + 1)\hat{q}$

Time evolution in linear model

$$\begin{aligned}\frac{\partial \vec{v}}{\partial t} = A &\rightsquigarrow \frac{\partial D}{\partial t} = \frac{\partial(\nabla \cdot \vec{v})}{\partial t} = \nabla \cdot A \\ \frac{\partial d}{\partial t} &= \mathcal{L}_{new}^* \hat{q}\end{aligned}$$

We can define

New vertical Laplacian operator

$$\mathcal{L}_{new}^* = \alpha \partial^* (\partial^* + 1) + \beta (\partial^* + 1)$$

$$\alpha = (1 + \Lambda^{*2} S^2)$$

$$\beta = \Lambda^{*2} (S \partial^* S)$$

$$\Lambda^* = 0 : \mathcal{L}_{new}^* \rightarrow \mathcal{L}_v^* = \partial^* (\partial^* + 1)$$

How to discretize the proposed solution?

New discretized vertical Laplacian operator

$$[\partial^* (\partial^* + 1) X]_l = \dots$$

$$[(\partial^* + 1) X]_l = \dots$$

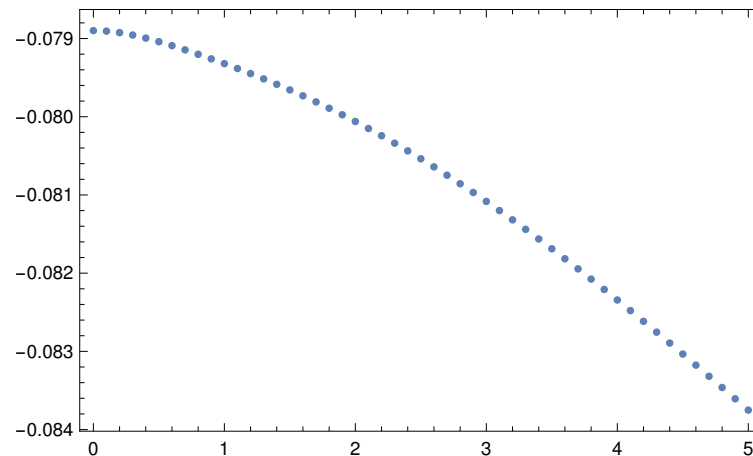
$$S(\eta_l) = \dots$$

$$\partial^* S(\eta_l) = \dots$$

How to set boundary conditions?

Does  $\mathcal{L}_{new}^*$  have only real and negative eigenvalues?

For an example of 87 vertical levels used in Czech operations we are safe.



(courtesy of N.Kastelec)

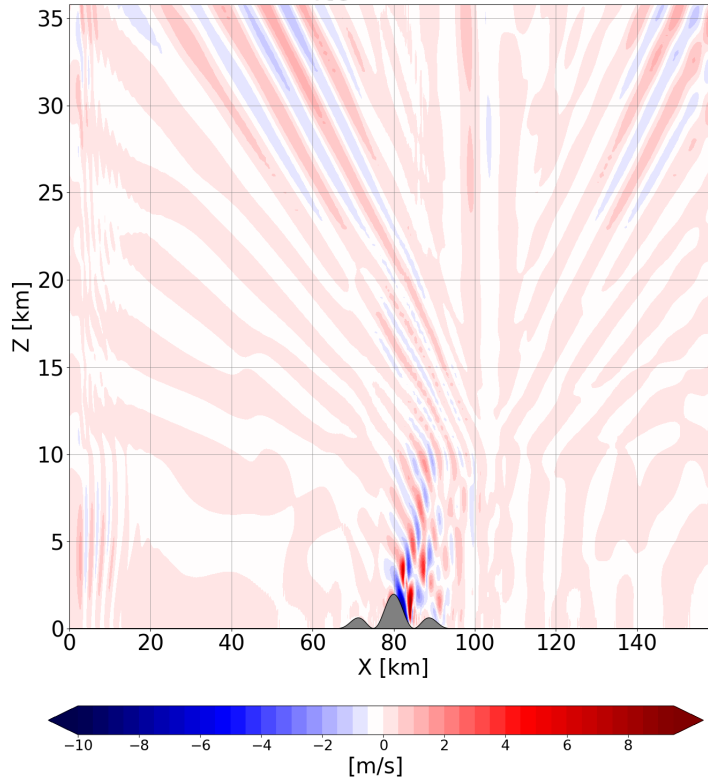
Max value of eigen values depending on  $\Lambda$

Then we can eliminate the discretized equations up to horizontal divergence  $D$  and solve the Helmholtz equation for  $D$ .

Vertical velocity for the Schär mountain case depending on the slope  $\Lambda^*$ .

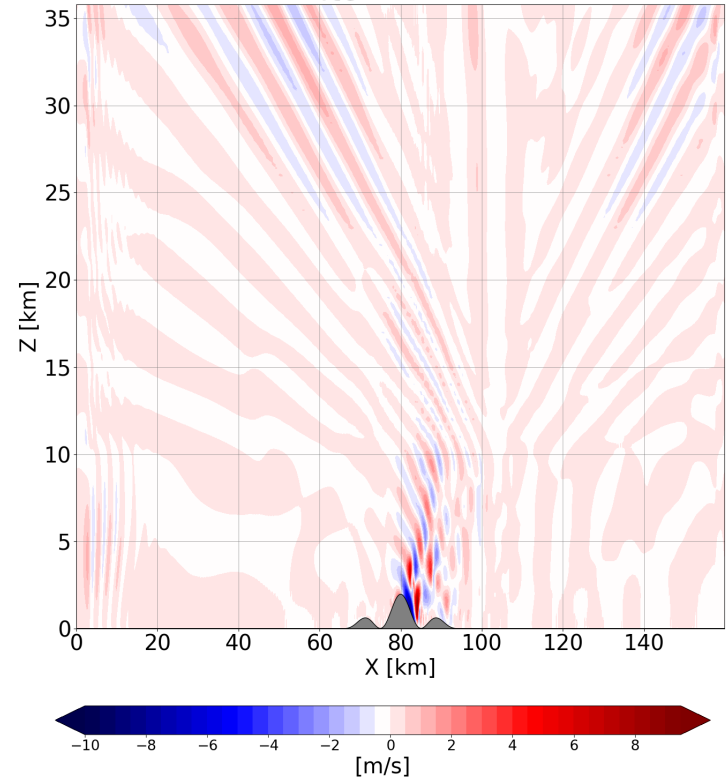
$$\Lambda^* = 0$$

After 1.5h



$$\Lambda^* = 0.23$$

After 1.5h



(courtesy of N.Kastelec)

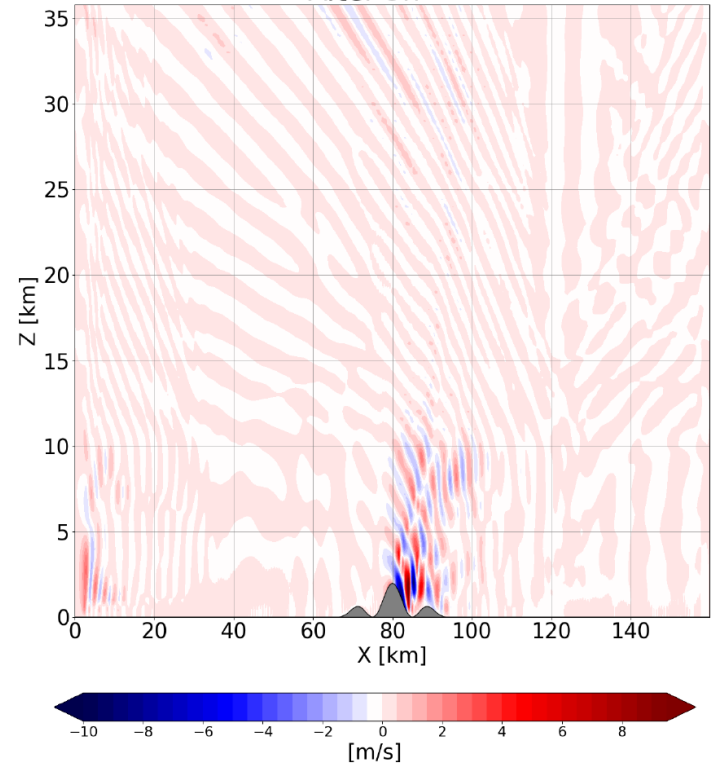
Vertical velocity for the Schär mountain case depending on the slope  $\Lambda^*$ .

$$\Lambda^* = 0$$



$$\Lambda^* = 0.23$$

After 3h



(courtesy of N.Kastelec)

## Conclusions

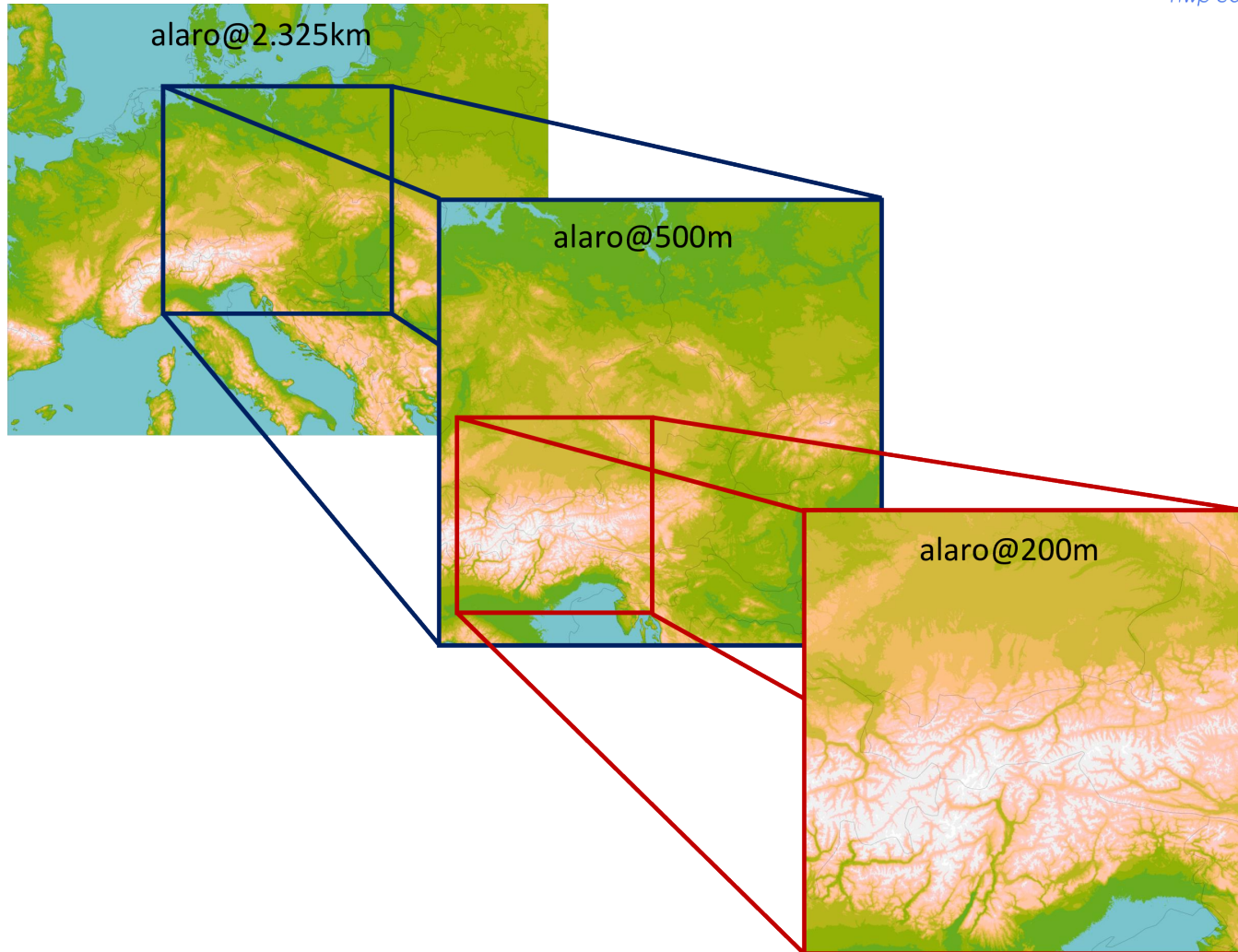
- ❑ We must continue our efforts.
- ❑ We plan to test various possible discretizations.
- ❑ We plan to test various possible boundary conditions.
- ❑ We plan to make further idealised tests and real simulations.





# PROJECT DE\_330

# Real simulations @200m



The basic algorithmic choices for ALARO configurations are:

## Dynamical core

- ❑ semi-Lagrangian advection scheme with 4 iterations for trajectory calculation
- ❑ PC time scheme with one iteration, cheap variant (SL trajectories are not recalculated in corrector)
- ❑ modified vertical divergence d4 for vertical motion, transformation to vertical velocity  $w$  in the non-linear model
- ❑ reference values of the linear model: SITR=300K, SITRA=100K, SIPR=900hPa
- ❑ no decentering
- ❑ semi-Lagrangian horizontal diffusion applied on all model variables + TKE, TTE, hydrometeors
- ❑ linear truncation for all spectral fields except orography; quadratic truncation of orography

## ALARO physics

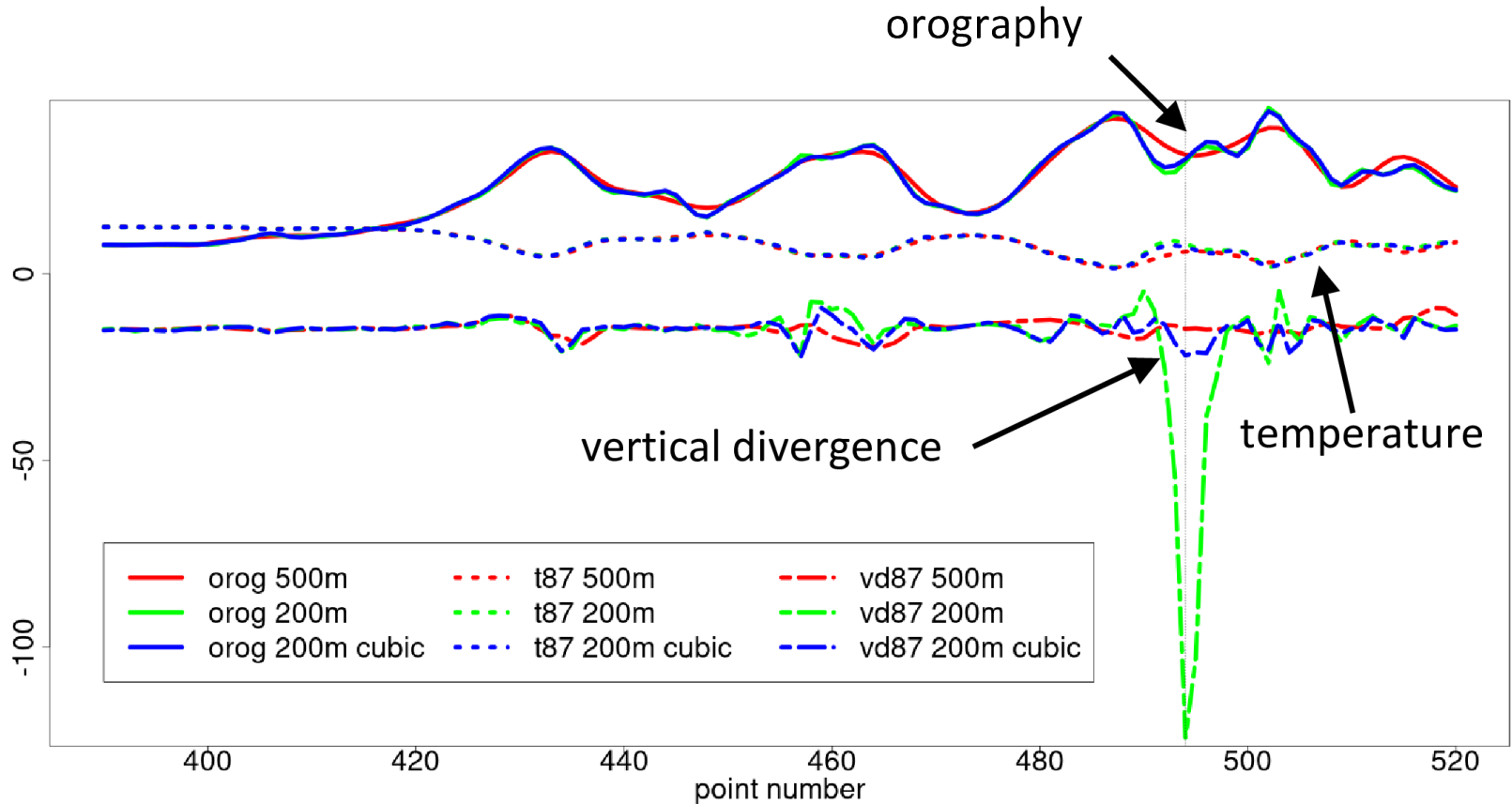
- ❑ radiation scheme ACRANEB2
- ❑ turbulence and shallow convection scheme TOUCANS, model 2
- ❑ scale aware deep convection and microphysics scheme 3MT

## Initialization

- ❑ initialization with 3DVAR + surface DA (canari) for 2.325km run; dynamical adaptation + DFI for 500m and 200m runs

## Particular choices for ALARO@200m:

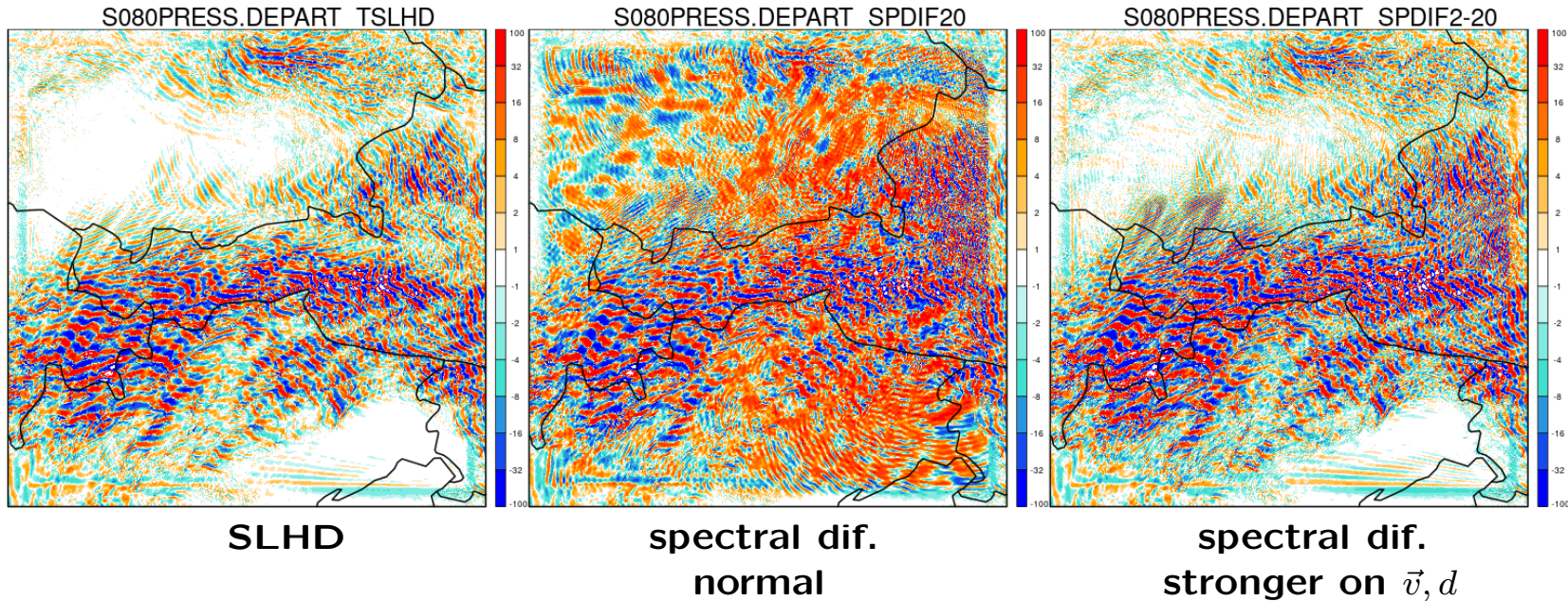
- ❑ cubic truncation of orography
- ❑ SITRA=50K
- ❑ no 3MT (deep convection), only STRAPRO (stratiform precipitation)



**Initialization with DFI using cubic or quadratic orography truncation: small difference in orography, but big difference in vertical divergence.**

## Sensitivity to horizontal diffusion setting

- ❑ SLHD (flow dependent semi-Lagrangian horizontal diffusion, Váňa)  
+ additional spectral diffusion of  $6^{th}$  order + spectral diffusion of  $2^{nd}$  order close to the model top
- ❑  $4^{th}$  order spectral diffusion similar as used in lower resolutions
- ❑  $4^{th}$  order spectral diffusion stronger for motion variables ( $\vec{v}$ ,  $d$ )

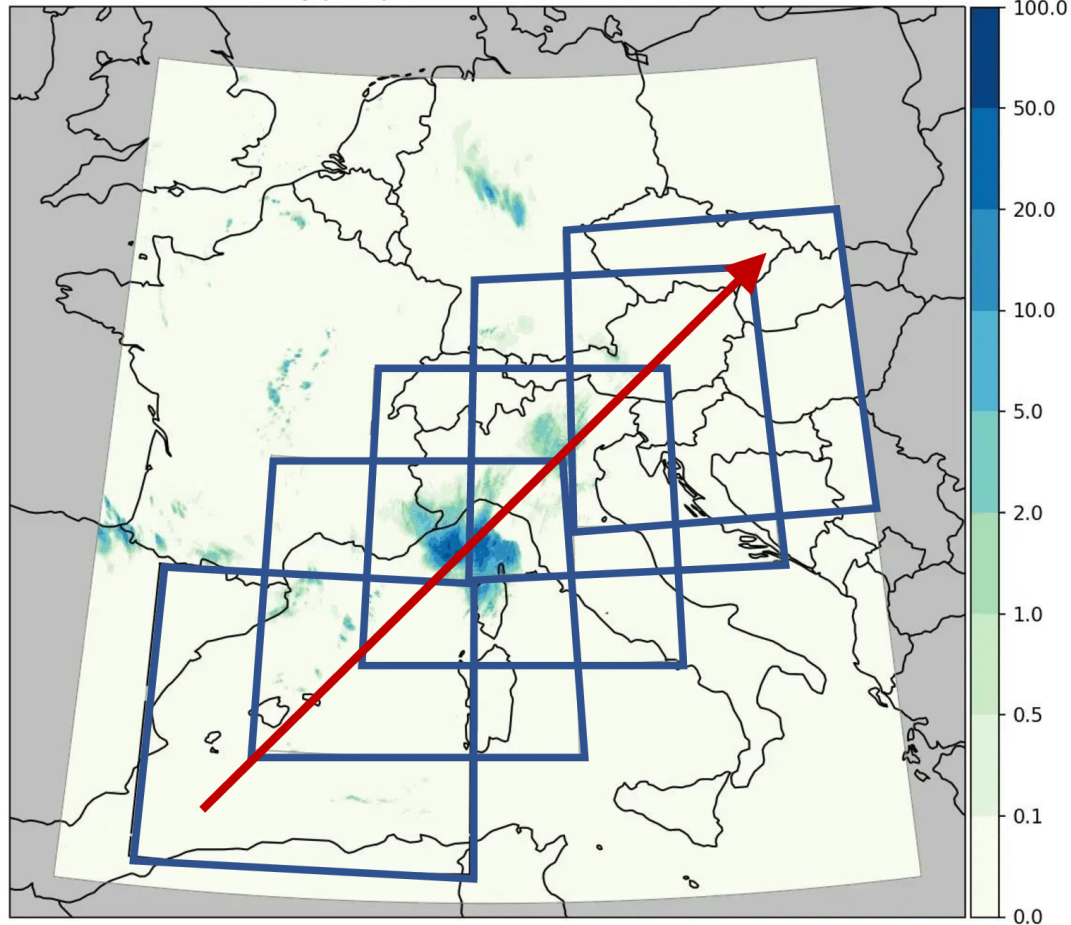


Several AROME runs (2.5km, 1.25km, 500m) were prepared at Geosphere Austria for the case of 18/08/2022 0UTC + 24h in the so called "consecutive domain" approach, i.e. run a big domain on 1.25km coupled to IFS HRES and then run AROME on several 500m domains nested inside (partly overlapping) coupled to the AROME 1.25km. The position of these domains follows the high impact phenomena we are interested in.

Stability issues were observed for the 500m runs which could not be solved by simple namelist parameters changes.

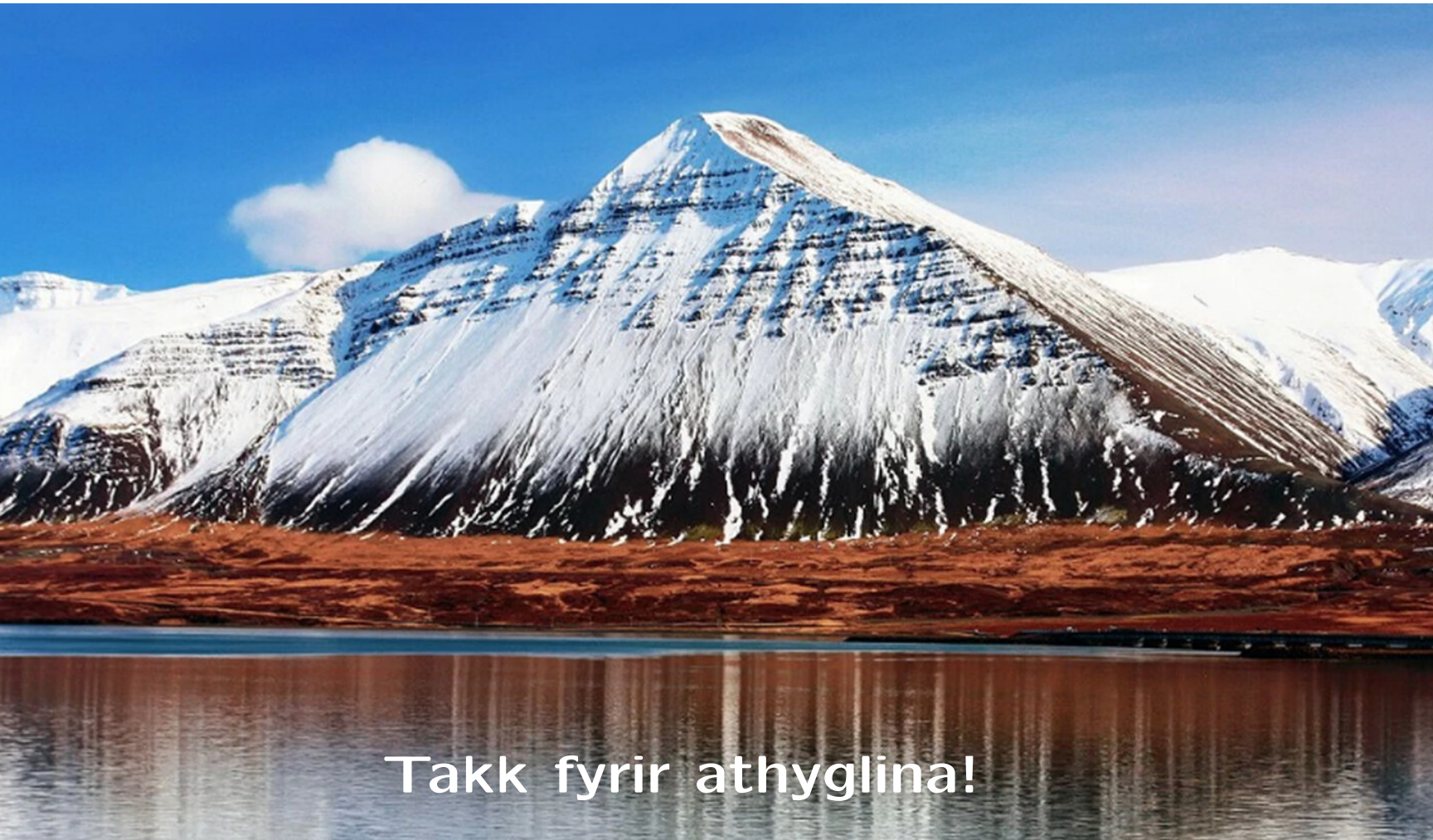
# Consecutive domain approach

Hourly precipitation at 2022-17-08 09 UTC



(courtesy of Ch.Wittman)





Takk fyrir athyglina!