Fractional timestepping of fast physics processes in the Met Office model

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Motivation

Dynamics and physics timestep are the same in the Met Office Unified Model (UM).

- Due to the unconditional stability of the semi-Lagrangian (SL) advection method used this is often too large for fast physics processes:
  - Grid point storms may occur.
  - Numerical instabilities and inaccuracies in the boundary layer.

Current strategy:

- Substep convection within a model timestep.
- Overweight numerical scheme for boundary layer vertical diffusion $\Longrightarrow$ improved stability but loss of accuracy. Instabilities still occur.

Mathematical formulation

Physics-dynamics coupling in the UM:

$\boxed{\text{tlevel n}} \rightarrow \boxed{\begin{array}{c}\text{slow phys:}\\ \text{rad+gwd+micphys}\end{array}} \rightarrow \boxed{\text{SL adv}} \rightarrow \boxed{\begin{array}{c}\text{fast phys:}\\ \text{cnv+bl+hyd}\end{array}} \rightarrow \boxed{\text{solver}} \rightarrow \boxed{\text{tlevel n+1}}$

Nonlinear diffusion equation in the boundary layer:

$$\frac{\partial X}{\partial t} = \frac{\partial F}{\partial z} \quad \textbf{where} \quad F(K,X) \equiv K\frac{\partial X}{\partial z}, \quad X = \left\{u,\ v,\ T_l = T - \frac{L}{c_p}q_{cl} - \frac{L_s}{cp}q_{cf},\ q_{tot} = q + q_{cl} + q_{cf}\right\}$$

is discretized in space. Derived ODE problem is:

- **Often stiff, i.e. stable but with multiple (both fast and slow) timescales. Stiffness depends on K magnitude and vertical resolution Δz.**
- **Nonlinear.**

Numerical Scheme

Because of stiffness, an implicit scheme is used:

$$\frac{X^{n+1}-X^{n}}{\Delta t}=\frac{\Delta}{\Delta z}\left[(1-\gamma)F(K^{n},X^{n})+\gamma F(K^{n+1},X^{n+1})\right], \qquad \gamma \geq 1 \tag{1}$$

To avoid expensive Newton iterations approximate $K^{n+1} \approx K^{n}$ and thus obtain a solution by solving a linear system of equations:

$$(I-\Delta t M)\,X^{n+1}=X^{n}+\frac{\Delta t}{\Delta z}\,\Delta\left[(1-\gamma)F(K^{n},X^{n})\right] \tag{2}$$

where,

$$X=(X_k),\ \gamma=(\gamma_k),\ MX^{n+1}=\frac{\Delta}{\Delta z}\left[\gamma F(K^{n},X^{n+1})\right]\equiv\left\{\left[\frac{\Delta}{\Delta z}\left(\gamma_k K_k^{n}\frac{\Delta X_k^{n+1}}{\Delta z_k}\right)\right]_k\right\}$$

for $k=1,2,\ldots,N$. As K^{n} is available M is a banded matrix of constant coefficients which depends on the vertical discretization operator Δ.

Stability of the scheme

Scheme (1) is unconditionally stable. However, its approximation (2) is not (Kalnay-Kanamitsu 1987):

- The region for which this scheme is stable depends on: K magnitude, γ, nonlinearity degree and timestep length.
- Overweighting γ sufficiently (e.g. set $\gamma = \frac{5}{2}$) may remove instabilities. But it introduces too much damping $\Longrightarrow$ forecasting accuracy deteriorates.

Reducing Δt in (2) is one way of improving:

- Generally smaller truncation errors and fewer instabilities.
- Cycle boundary layer/surface processes $\Longrightarrow$ convection and hydrology has also to be cycled (coupled parametrisations).

Experimental fractional timestepping scheme in the UM: BL discretization

Use $\delta t = \Delta t/k$ where k is number of substeps, Δt SL advection timestep. Solve:

$$\frac{X^{n+1} - X^{n^\star}}{\delta t} = \frac{\Delta}{\Delta z}\left[(1-\gamma)F(K^{n^\star}, X^{n^\star}) + \gamma F(K^{n^\star}, X^{n+1})\right] \tag{3}$$

$$\vdots$$

$$\frac{X^{n+k} - X^{n^\star+k-1}}{\delta t} = \frac{\Delta}{\Delta z}\left[(1-\gamma)F(K^{n^\star+k-1}, X^{n^\star+k-1}) + \gamma F(K^{n^\star+k-1}, X^{n+k})\right] \tag{4}$$

where $K^{n^\star+\ell-1} \equiv K(X^{n^\star+\ell-1})$ and $X^{n+\ell},\ \ \ell = 1, 2, \ldots, k$ is calculated as follows:

$$X^n \xrightarrow{slow\ phys} X^{n^*} \underbrace{\xrightarrow{cnv} X^{n^\star} \xrightarrow{bl} X^{n+1}}_{1st\ cycle} \overbrace{\xrightarrow{cnv} X^{n^\star+1} \xrightarrow{bl} X^{n+2}}^{2nd\ cycle} \cdots \underbrace{\xrightarrow{cnv} X^{n^\star+k-1} \xrightarrow{bl} X^{n+k}}_{kth\ cycle}$$

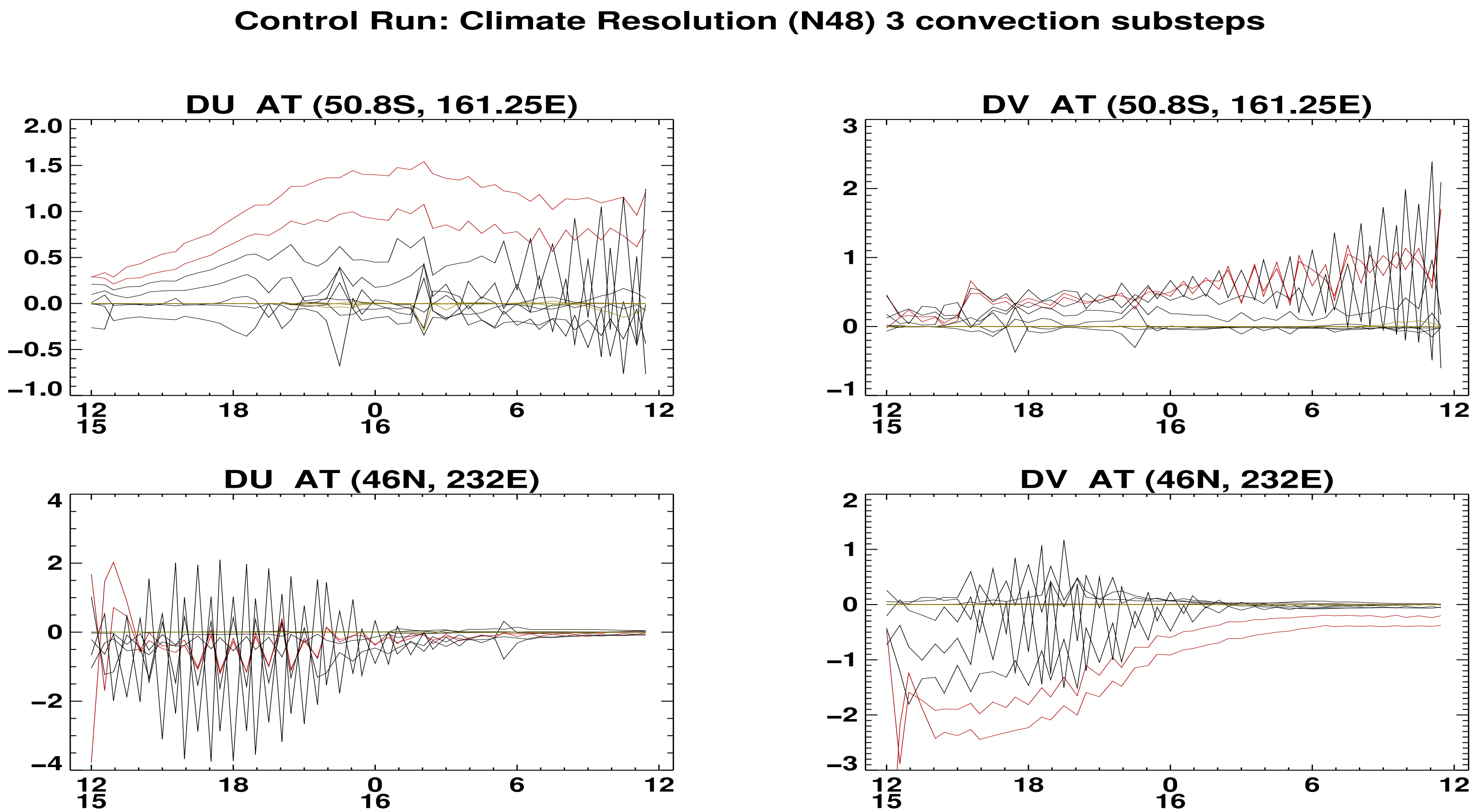

Figure 1: Timeseries of boundary layer increments from Global model at climate resolution for u, v wind components. Each line corresponds to a different BL level: the two bottom levels are coloured in red, the four top in lime and the remaining levels in black.

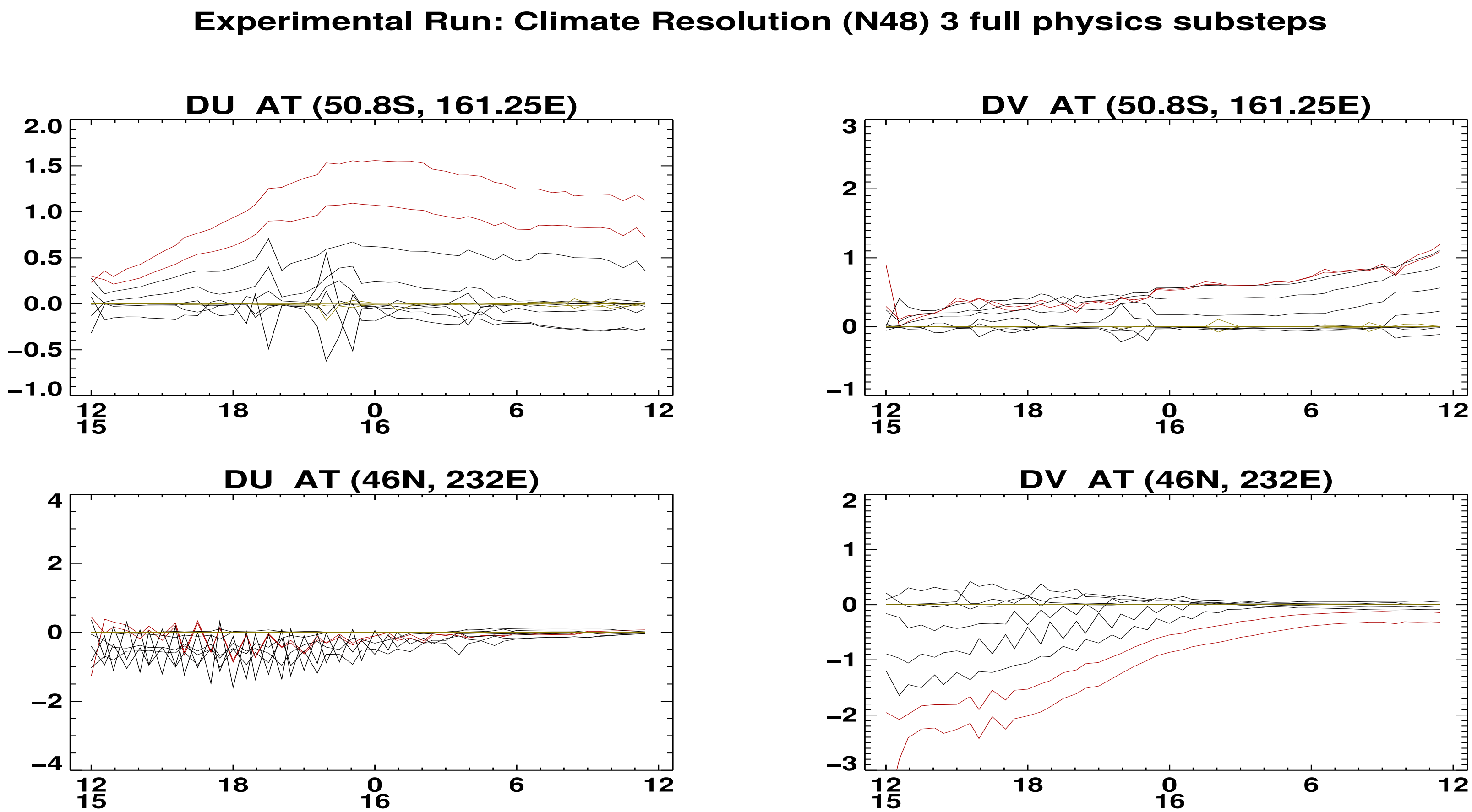

Figure 2: Timeseries of boundary layer increments as previously but with 3 full fast physics substeps.

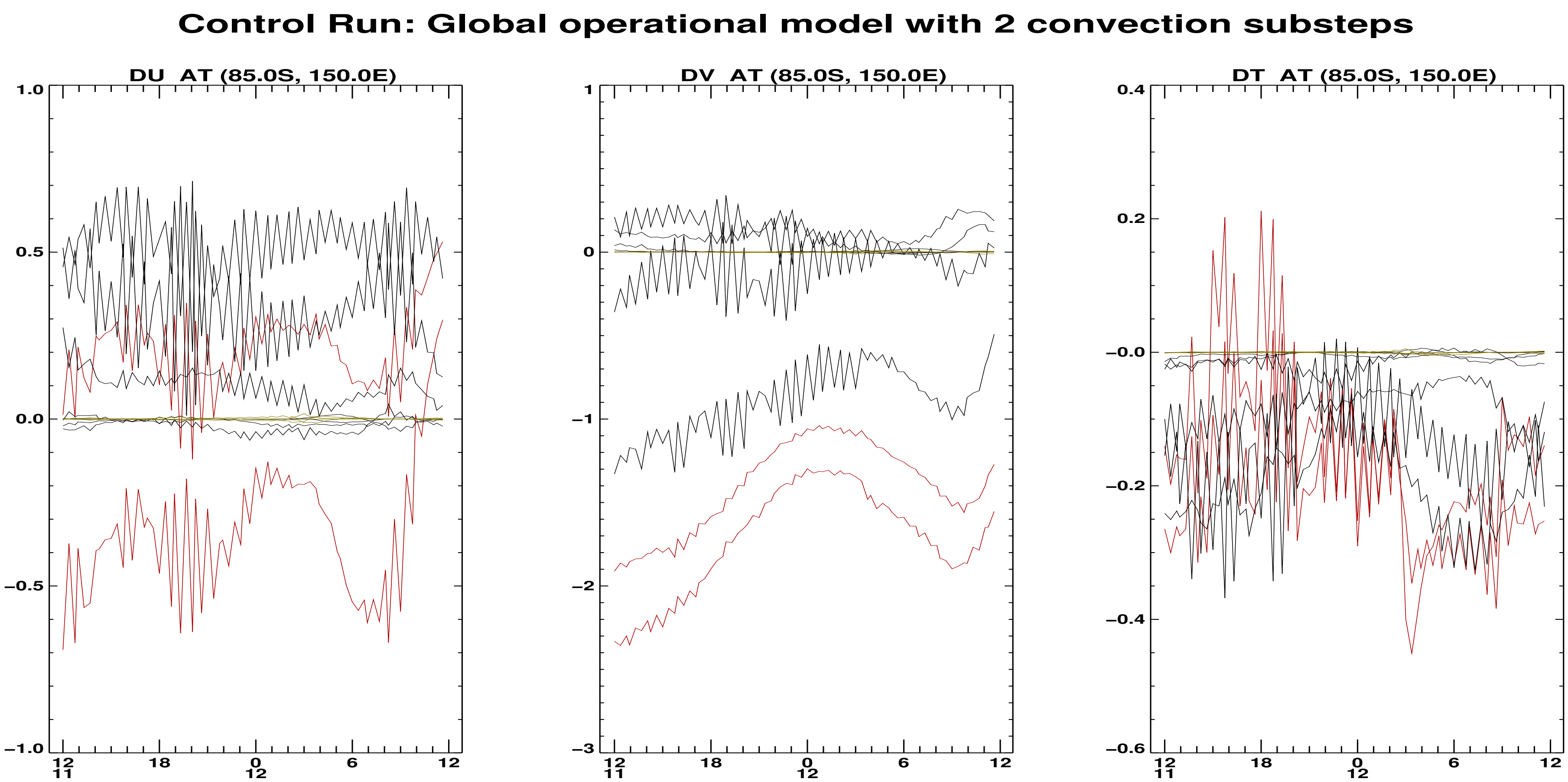

Figure 3: Operational global model boundary layer increments for u, v, T. model. Same colour code as in climate run.

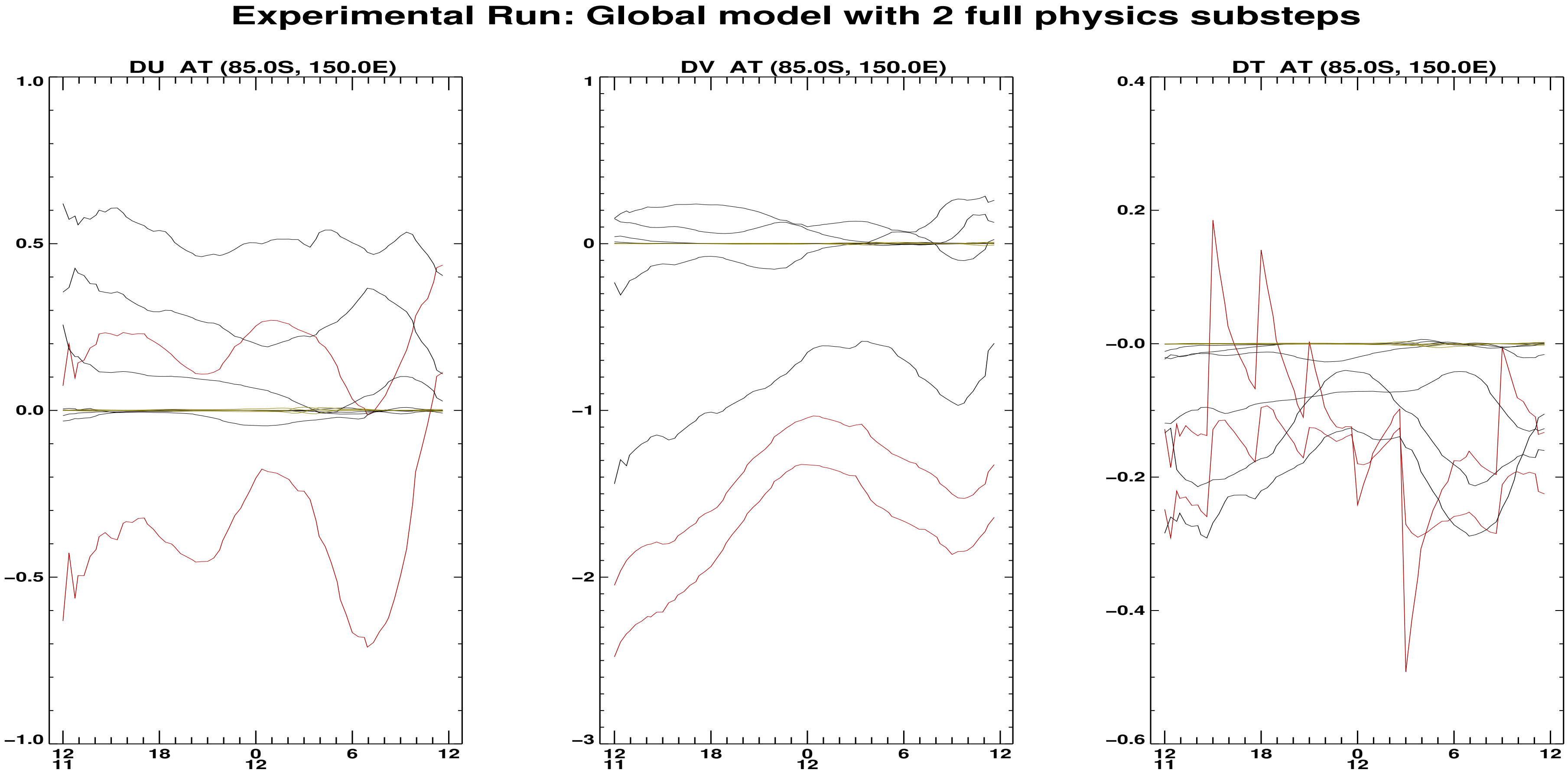

Figure 4: As previously but experimental model with 2 full physics substeps.

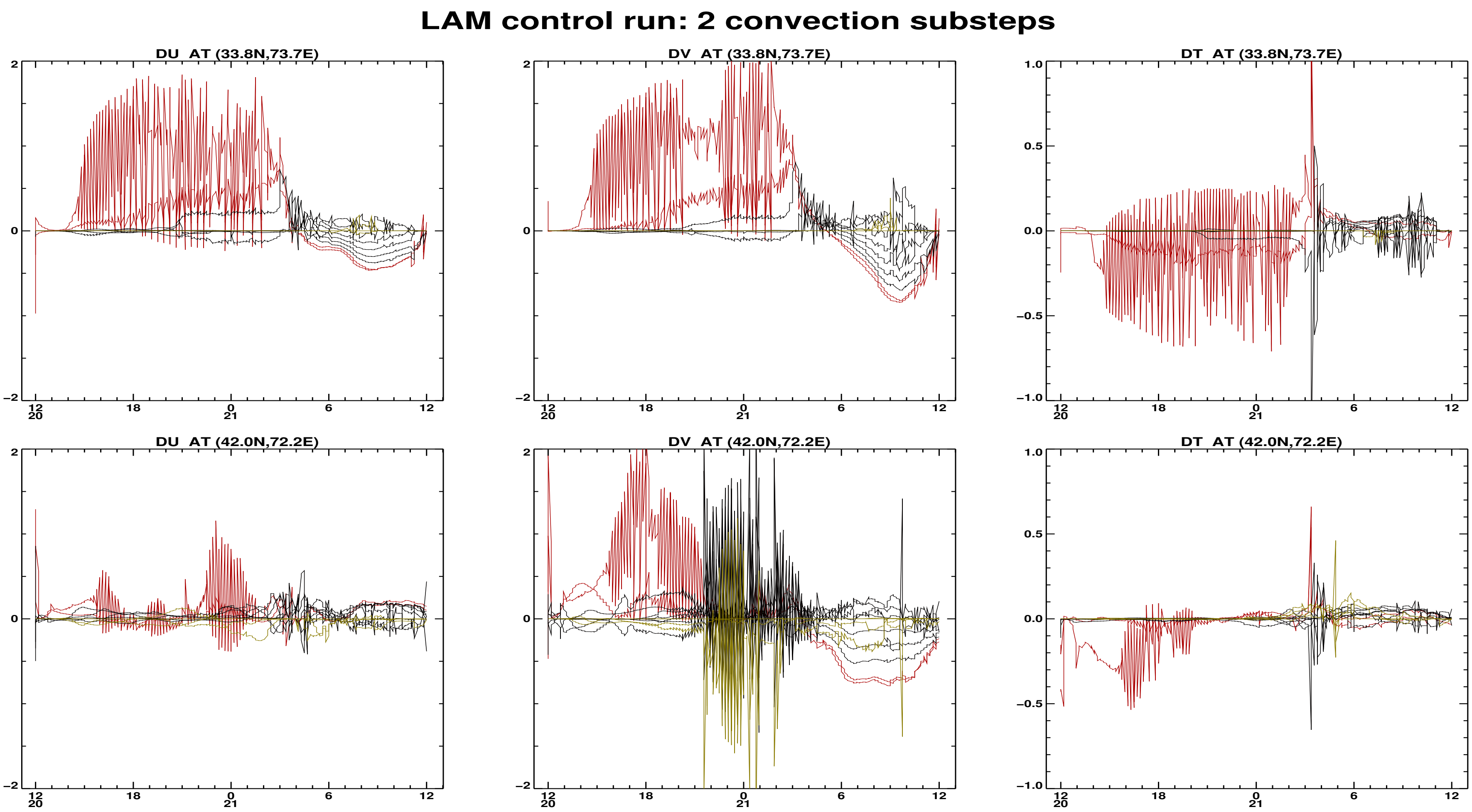

Figure 5: Timeseries of boundary layer increments for u,v,T from a SW Asia LAM test.

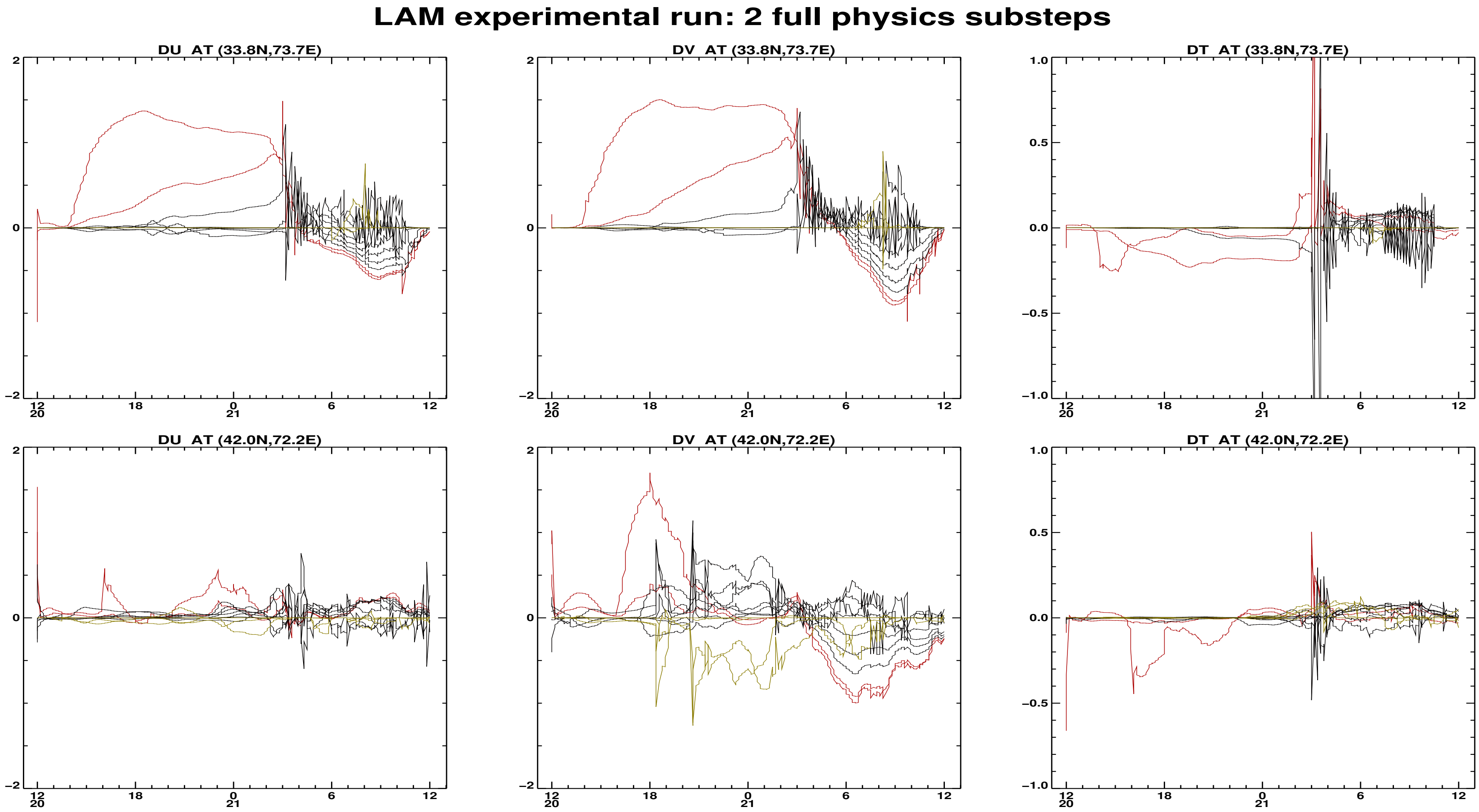

Figure 6: As previously but with 2 full "fast physics" substeps.

Concluding Remarks

The numerical scheme used for the boundary layer vertical diffusion in the UM exhibits unphysical oscillations at isolated points in various resolutions due to numerical instabilities or inaccuracies. Recent investigations with the UM have shown:

- Increasing the scheme's weights is an effective way to eliminate these oscillations but this tends to deteriorate the forecasting accuracy as too much damping is introduced.
- Substepping fast physics processes seems to reduce unwanted oscillations and in some cases eliminate them.
  - However, there is a significant overhead: the substepped global model at operational resolution with 2 full fast physics substeps is about 12% more expensive in CPU time than the control (2 convection steps).
  - A mixture of substepping - overweighting is perhaps a better choice if we can afford the extra CPU cost.

Future Work: Use of implicit exhange coefficients

Approximate $K^{n+1} \approx K^{n+1}_{(P)}$ in (1) where $K^{n+1}_{(P)} \approx K(X^{n+1})$ is a predictor for the exchange coefficients at timelevel $n{+}1$. This should be very close to the unconditionally stable implicit scheme (1). The new algorithm will be part of a modified SL scheme:

- **Dynamics and physics are iterated twice to enable departure point calculations using time interpolated rather than extrapolated winds (current scheme but not ideal as it can lead to instabilities).**
- **Clearly more expensive but it may have similar benefits in forecasting accuracy as increasing resolution. If the interpolated SL scheme is an acceptable approach the extra effort to provide $K^{n+1}_{(P)}$ is small:**

  Set $K^{n+1}_{(P)} \longleftarrow K^{n+1}_{(1)}$ where $K^{n+1}_{(1)} \equiv K(X^{n+1}_{(1)})$ and $X^{n+1}_{(1)}$ is the value of X at timelevel $n+1$ as estimated at the end of first iteration.

Other, more stable, algorithms to replace (2) are also investigated.