Development of the Z-coordinate High-Resolution Non-Hydrostatic Atmospheric Model Using the Shaved Cell Method

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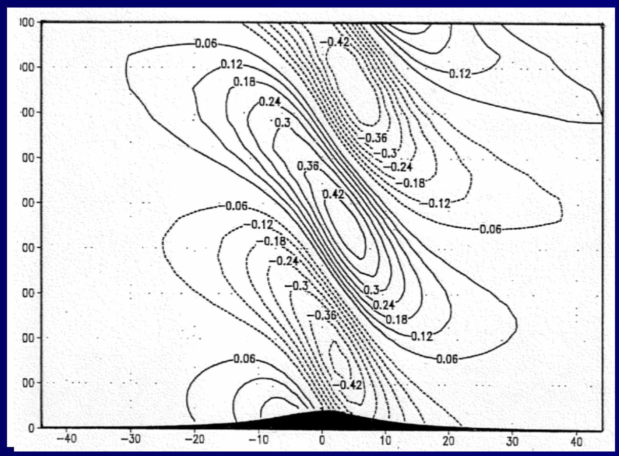
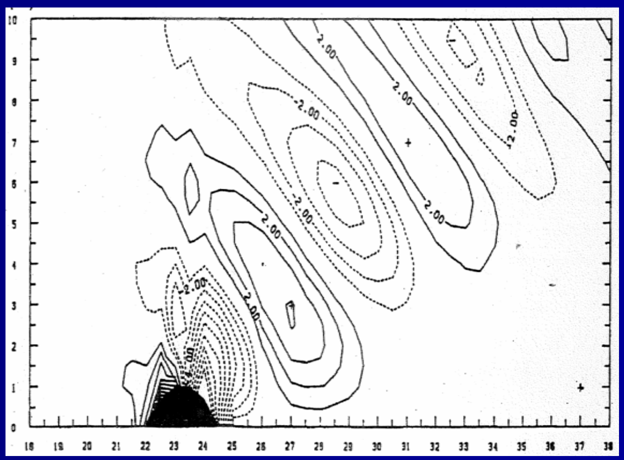
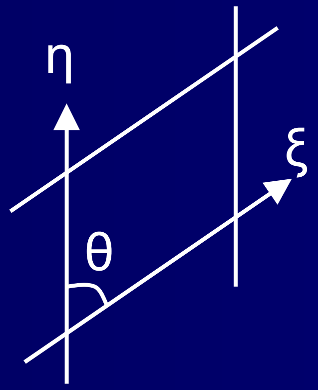
1. Introduction

- For high-resolution simulations, steep slopes are represented over complex terrain.
- The commonly used terrain following representation of topography induces large truncation errors over steep slopes (Thompson et al., 1985) .





(Satomura,1989)



– The steeper the slope, the larger the errors T .

$$T \approx \frac{1}{2}\left\{ -x_{\xi\xi} f_{xx} + \left(y_{\eta\eta} f_{yy} - x_{\xi\xi} f_{xy} \right) \underline{\underline{\cot\theta}} \right\}$$

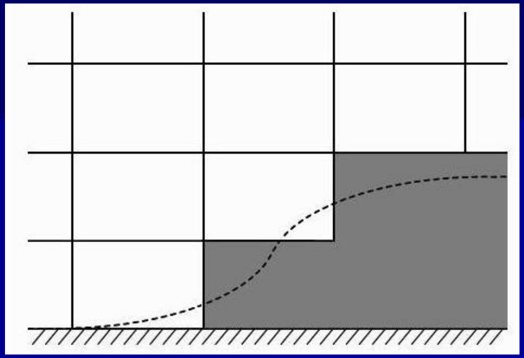
$T \propto \cot\theta$

– This errors will be serious in high-resolution simulations.

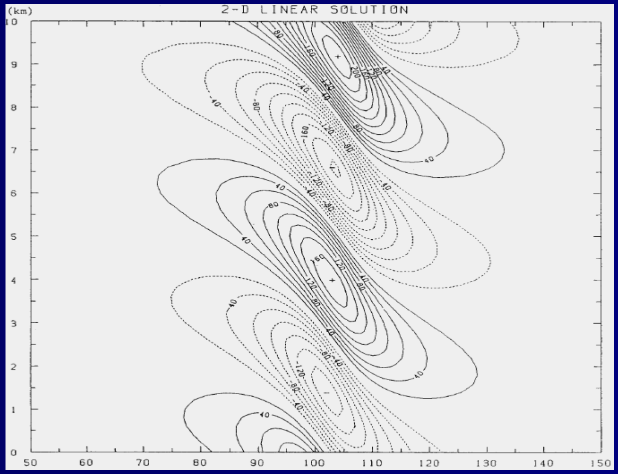
➜ Other representation methods are needed !

box cell method

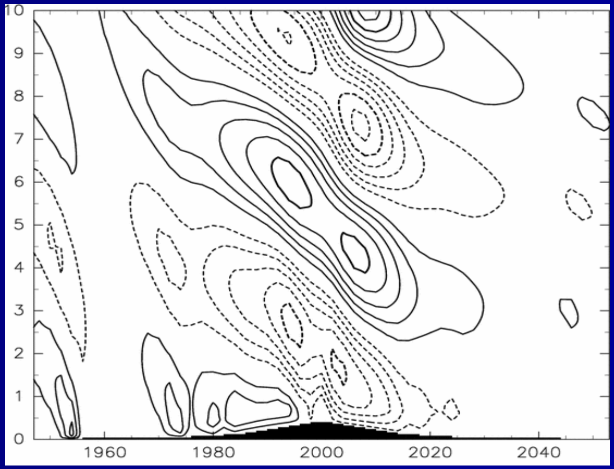
- Approximating topography fitted to models grids.
  - NCEP Eta Model etc.



- Technical issues (Gallus and Klemp,2000)
  - This method represents topography precisely only when the resolutions are very high.
  - In particular, the step-like representation induces large errors over smooth topography.



linear theory



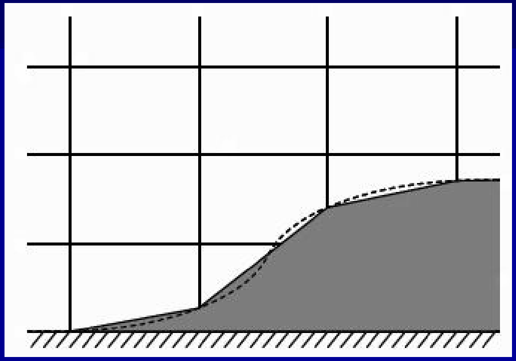
box cell model

➔The box cell method cannot reproduce mountain waves precisely.

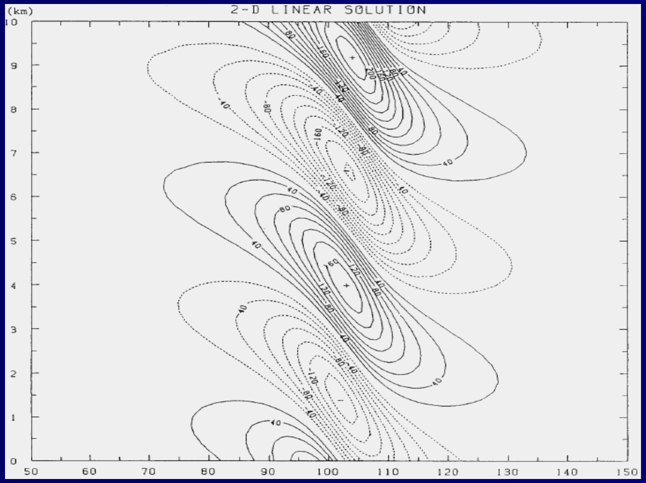
shaved cell method (Adcroft et al., 1997)

- Model cells are cut by piece wise linear topography.
  - Discretized by the Finite Volume Method
  - Small cells require small Δt.

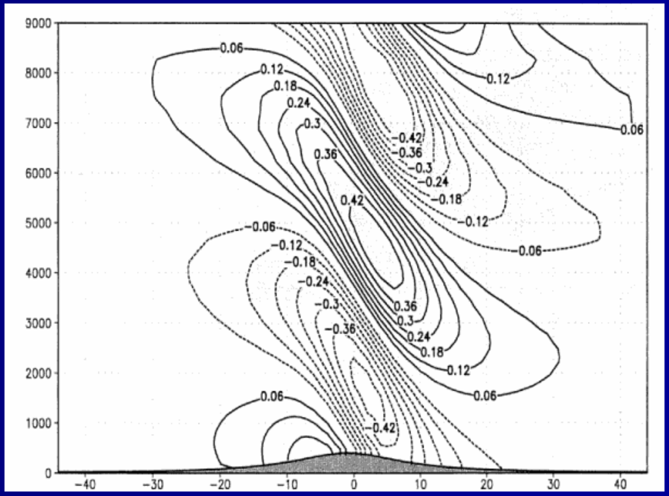
  ➔ Avoiding extremely small Δt is the key point !



- Simulations of flow over a bell-shaped mountain (Steppeler et al., 2002)



linear theory



shaved cell model

➔The shaved cell method can precisely reproduce air flow over smooth topography.

Our viewpoint

- Technical issues of Steppeler et al. (2002)
  1. Advective form equations are used in spite of the FVM.
     ➔ Quasi-flux form equations are employed.
  2. Using thin-wall approximation to avoid extremely small Δt.
     ➔ Small cells are combined to upper cells.

⇩

Simulations of flow over steep slopes were performed.

2. Model Descriptions

- The fully compressible equations in quasi-flux form are employed. (Satomura and Akiba, 2003)

$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho uu)}{\partial x}+\frac{\partial(\rho uw)}{\partial z}=-\frac{\partial p'}{\partial x}+DIF(\rho u)$$

$$\frac{\partial(\rho w)}{\partial t}+\frac{\partial(\rho wu)}{\partial x}+\frac{\partial(\rho ww)}{\partial z}=-\frac{\partial p'}{\partial z}-\rho' g+DIF(\rho w)$$

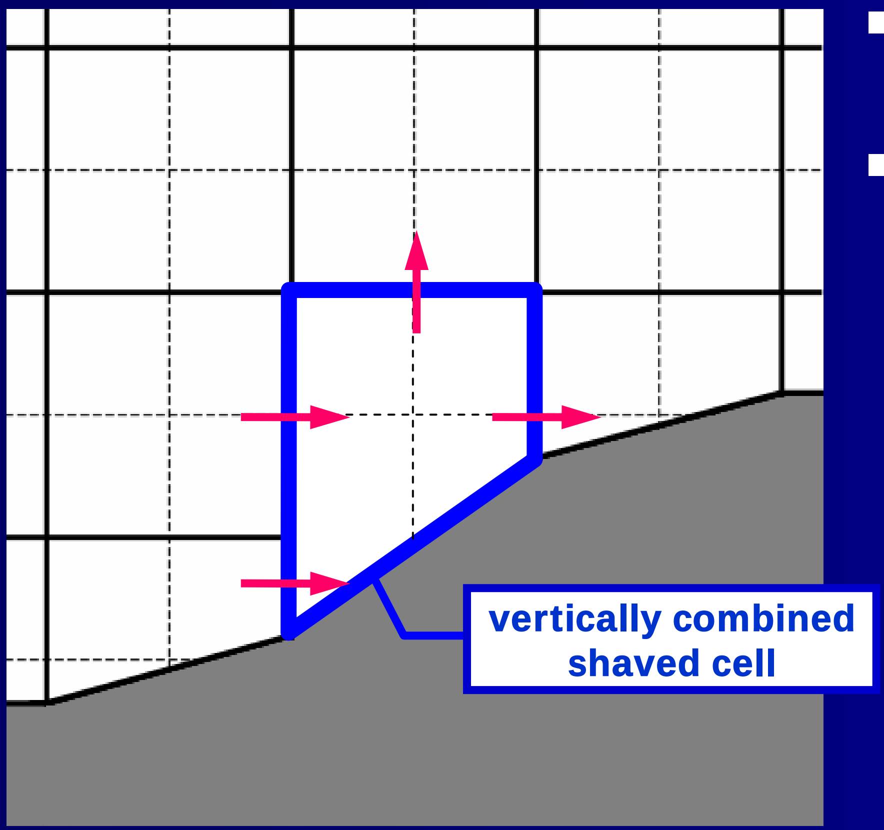
$$\frac{\partial p'}{\partial t}=-\frac{R_d\pi}{1-R_d/C_p}\left\{\frac{\partial(\rho\theta u)}{\partial x}+\frac{\partial(\rho\theta w)}{\partial z}+DIF(\rho\theta)\right\}$$

$$\frac{\partial\rho'}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho w)}{\partial z}=0$$

  - This form does not suffer from the cancellation error due to the subtraction of hydrostatic part ($\overline{p}$ or $\overline{\rho}$) from nearly hydrostatically balanced variable (p or ρ) .
  - Well suited to the Finite Volume Method in the view of conservation characteristics.

➔The combination of the shaved cell method and the quasi-flux form equations is expected to achieve high-resolution and high-precision simulations.

vertical combination of small cells



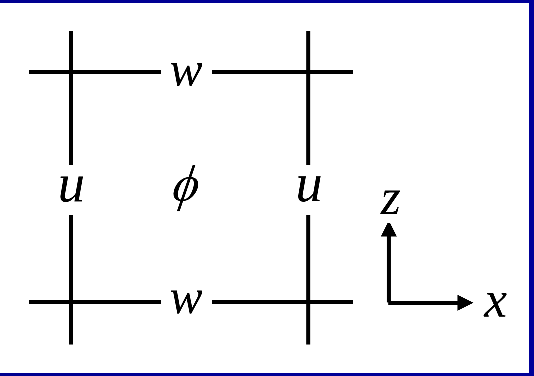
- Equations are descretized by flux exchanges.
- Small cells require small Δt to satisfy the CFL condition.

➔ Combining cells whose areas are smaller than ΔxΔz/2 to upper cells.

Avoiding extremely small Δt and also keeping the conservation characteristics.

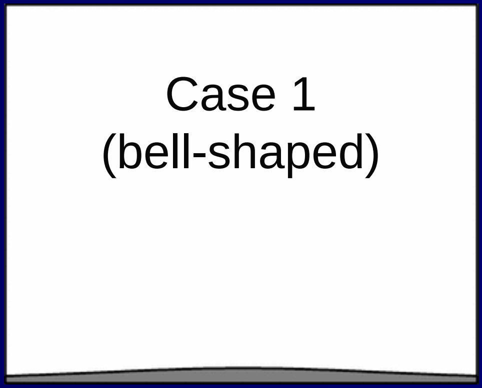
Model descriptions

Dynamics	
Dimension	2-D
Governing equations	Quasi-Flux Form Fully Compressible Non-Hydrostatic System
Spatial discretization Horizontal cell configuration Vertical cell configuration	Finite Volume Method Arakawa-C type Lorenz type
Topography representation	Vertically Combined Shaved Cell Method
Time integration Temporal scheme	Leap-Frog with Asselin Filter All Explicit
Physics	
Subgrid Turbulence Parametarization	1.5 order （Klemp and Wilhelmson,1974）

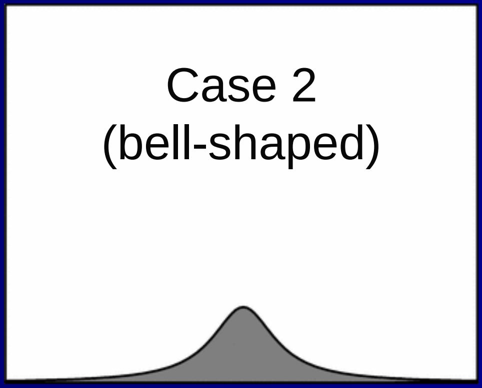


3. Results

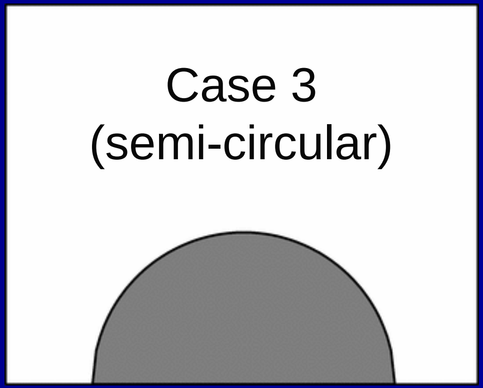
- Two-dimensional numerical simulations of flow over a mountain are performed using the developed model.
  - Constant horizontal velocity and the Brunt-Väisälä frequency are specified for each case.



- height : 100 m
- half width : 5000 m
- average slope angle : 0.57 deg.



- height : 500 m
- half width : 250 m
- average slope angle : 45 deg.

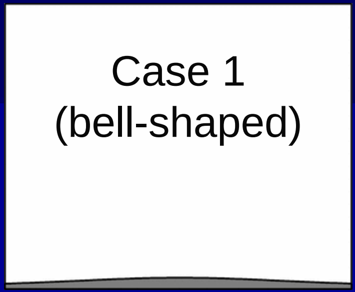


- radius : 1 km
- maximum slope angle : 90 deg.

gentle slope ←·····→ steep slope

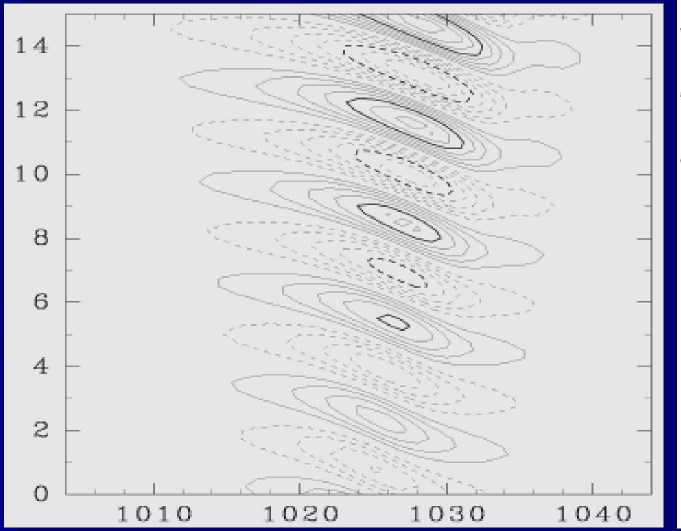
Results

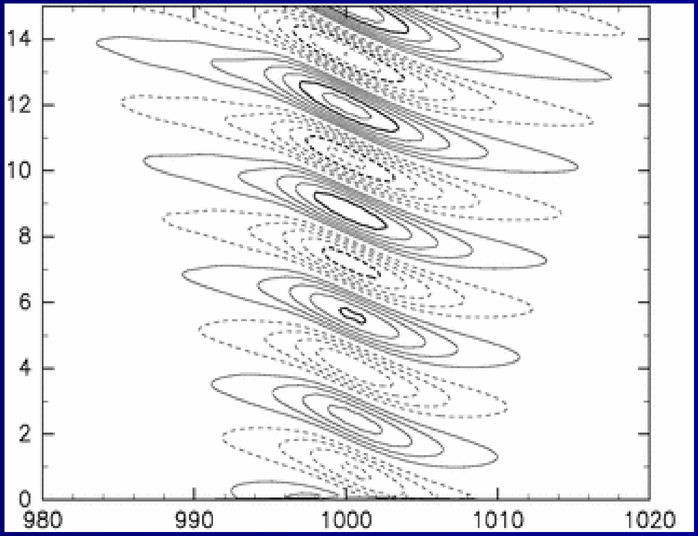
- Case 1 (h=100m, a=5000m, a * N/U=10, t=300min, average slope angle θ= 0.57 deg.)

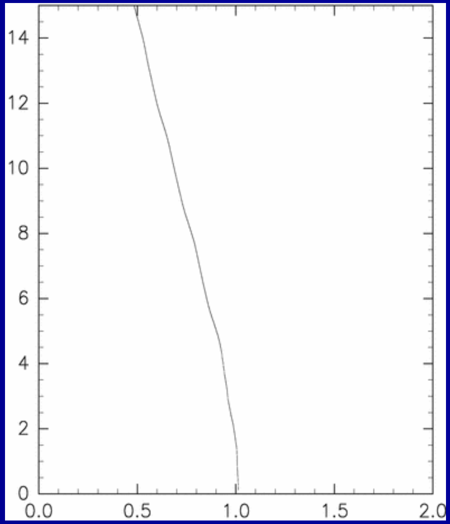


W (contour interval=0.05m/s)

Normalized Flux







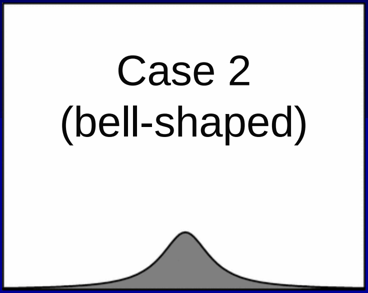
linear theory

vertically combined shaved cell

(Δx = 1000m, Δz = 50m)

Results

- Case 2 (a=500m, h=250m, a * N/U=0.25, t=100min, average slope angle θ= 45 deg.)

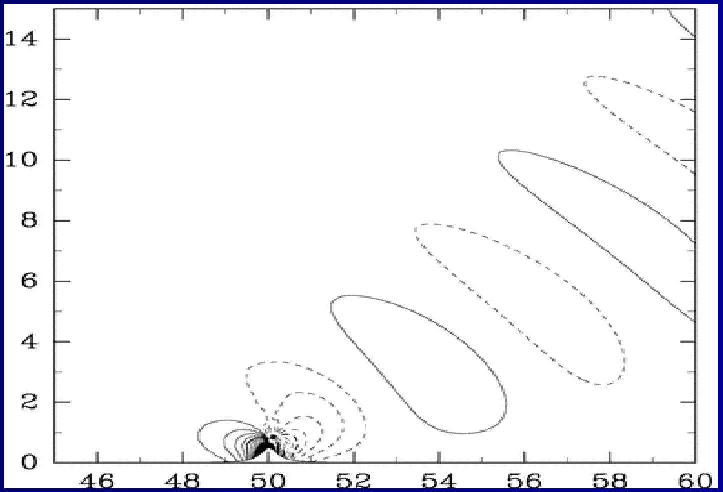


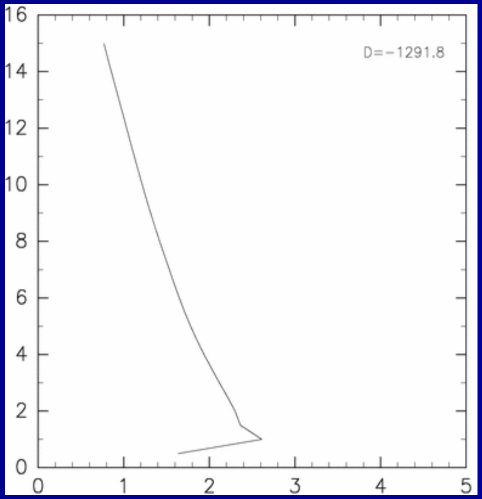
W (contour interval=0.5m/s)

Normalized Flux

z*-coordinate

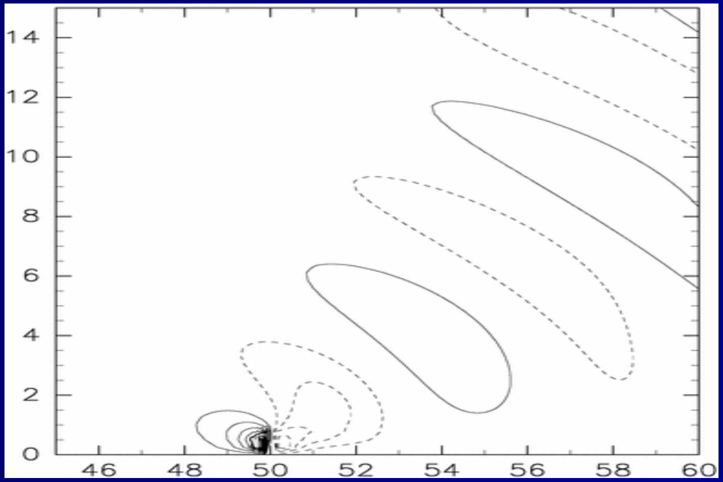
(Δx = 50m, Δz = 250m)

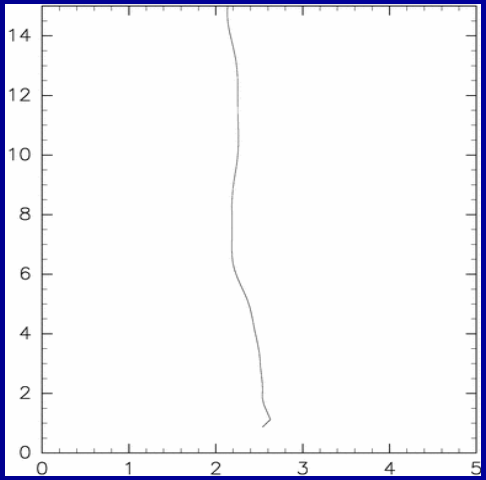




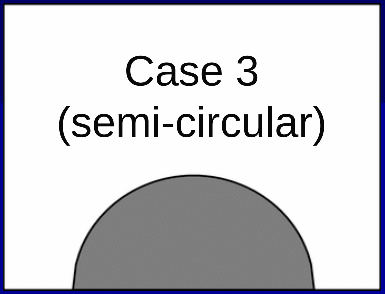
vertically combined shaved cell

(Δx = 50m, Δz = 250m)

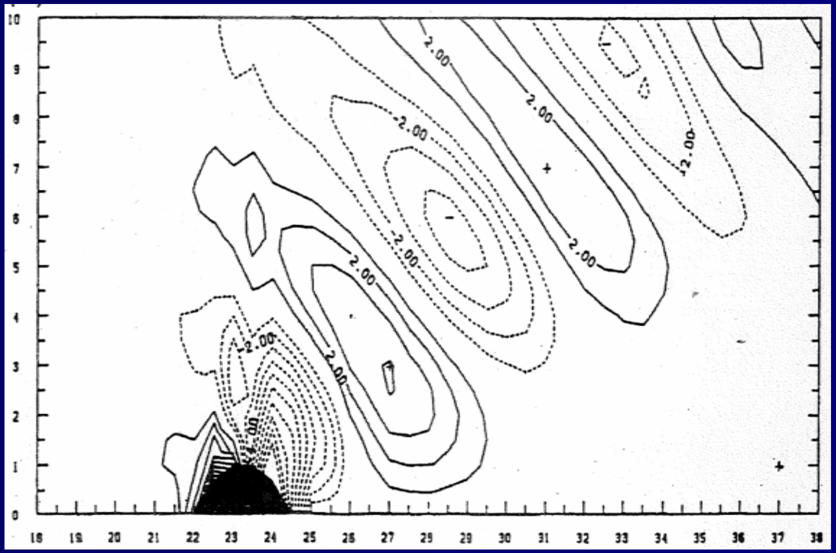


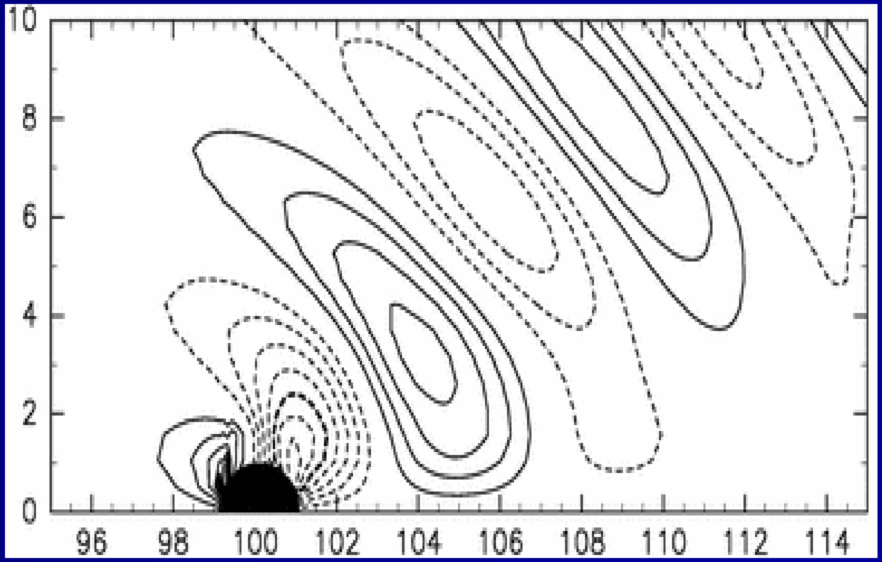


Results



- Case 3 (radius 1km, t=60min,
  maximum slope angle θ= 90 deg.)





z*-coordinate

(Δx = 500m, Δz = 500m)

vertically combined shaved cell

(Δx = 100m, Δz = 500m)

➔ The vertically combined shaved cell method can reproduce smooth and accurate mountain waves over gentle as well as steep slopes !

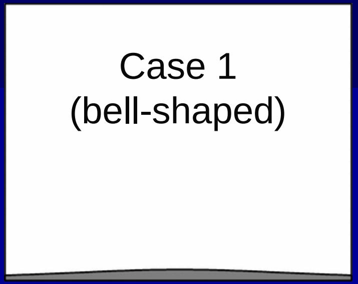
4. Conclusion

- A two-dimensional non-hydrostatic model to simulate air flow over complex terrain including steep slopes was developed.
  - The shaved cell method was implemented.
  - Small cells were combined to upper cells.
  - Quasi-flux form equations were employed.

- Two-dimensional numerical simulations of mountain waves were performed.
  - The vertically combined shaved cell method reproduced smooth and accurate mountain waves over both gentle and steep slopes.

- Future plan
  - speeding up
  - boundary layer
  - physics
  - three-dimensional modelling etc.

Results

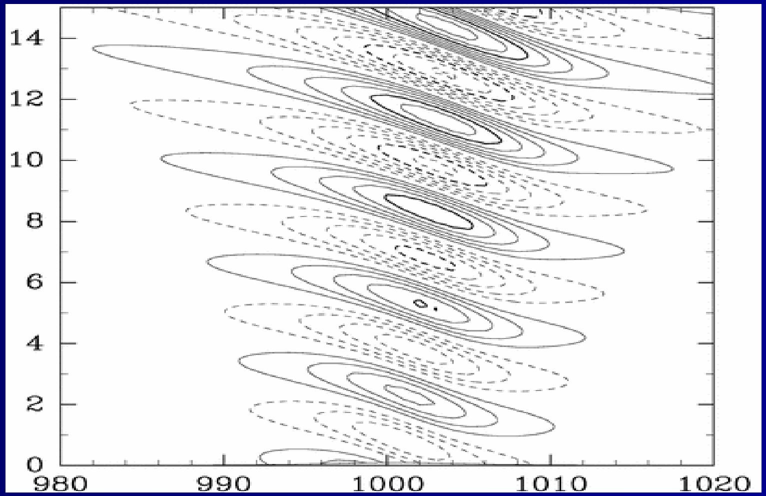


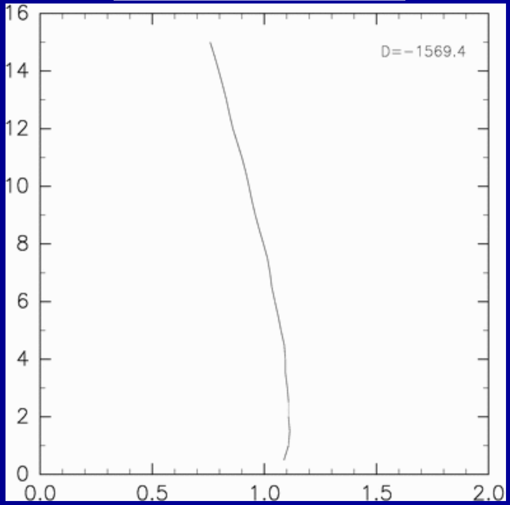
- Case 1 (h=5000m, a=100m, a * l=10, t=300Min)

W (contour interval=0.05m/s)

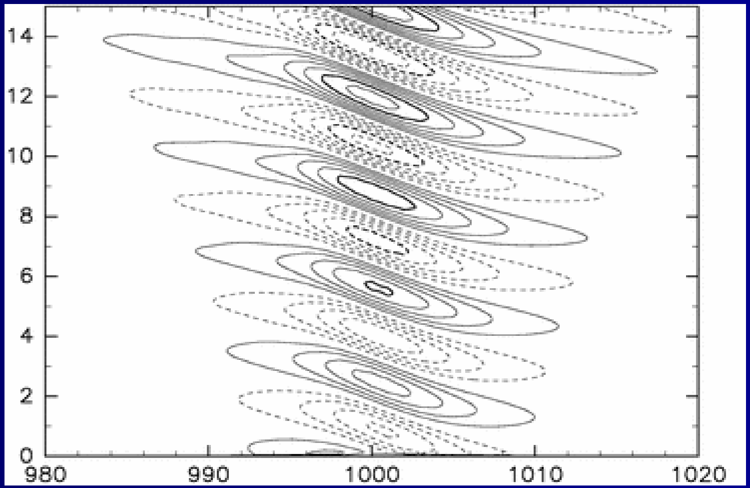
Normalized Flux

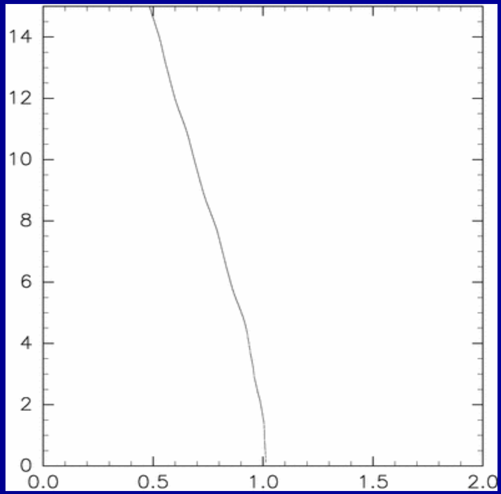
z*-coordinate





vertically combined shaved cell





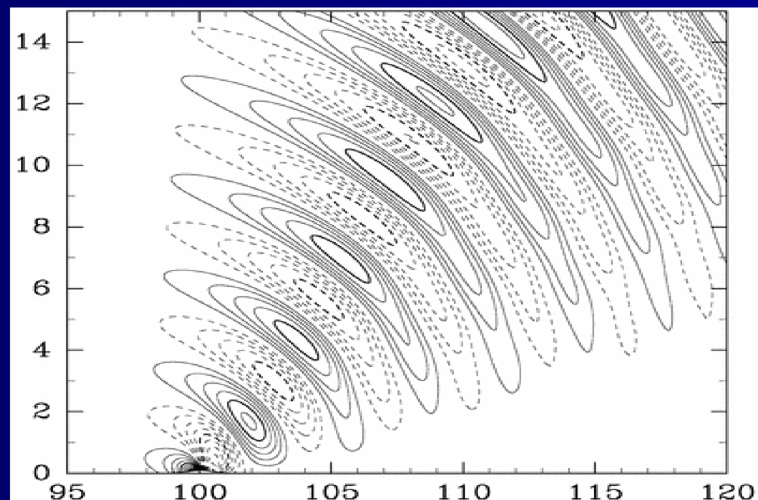
Results

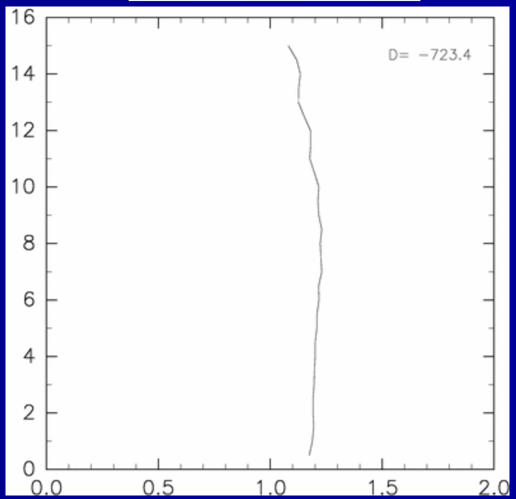
- h=500m, a=100m, a * l=1, t=100Min

W (contour interval=0.1m/s)

Normalized Flux

z*-coordinate





vertically combined shaved cell