Numerical Developments for the Model COSMO-DE of the DWD

Jochen Förstner, Michael Baldauf
Deutscher Wetterdienst, Kaiserleistraße 29/35, 63067 Offenbach am Main
jochen.foerstner@dwd.de

1. Introduction

In the last four years the further development of the COSMO-model – before Local-model (LM) – at the DWD aimed at an expansion of the forecasting system by a model for shortest range forecasts over Germany. This model variant called COSMO-DE is operational since April this year and produces 21 h forecasts every three hours at current resolution of 2.8 km. In particular the explicit simulation of deep convection, i.e. no parameterization of this process, is the main goal of COSMO-DE. The developments – mainly an outcome of the so called 'Aktionsprogramm 2003' – are the assimilation of quality controlled radar data via the latent heat nudging technique, a new 6-category microphysics parameterization including graupel and a new non-hydrostatic dynamical core based on two time levels.

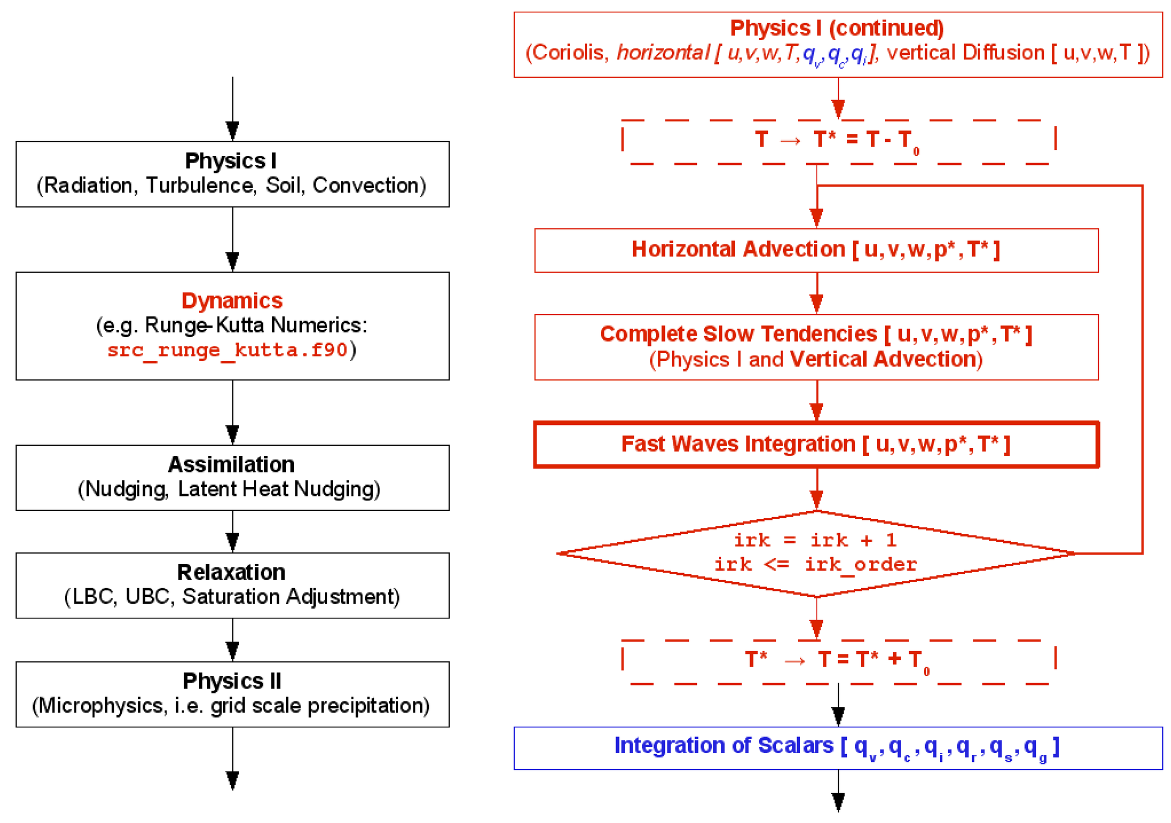
This poster deals with two main developments in the numerical field. First, specifics of the split-explicit integration of the model equations using a 3rd order Runge-Kutta scheme. Here especially the treatment of the prognostic equation for temperature. Second, special aspects of the treatment of the advection of moisture variables. Besides the conservation and stability properties, errors due to the splitting in the different spatial directions are of great importance to establish the overall quality of the transport scheme.

2. Specifics of the New Dynamical Core

The flow chart outlines an integration step of the COSMO-DE. Highlighted in red, the procedure of the dynamical core is given. In the operational setup a three-step Runge-Kutta scheme (`irk_order=3`) is combined with a 5th order upwind scheme for horizontal advection. The integration of the fast waves (box with thicker border) is spilt into several small time steps, while keeping the tendencies due to the slow processes fixed.

During the development it came out, that it is better to switch from an equation for the temperature T to an equation for the temperature perturbation T*, i.e. the deviation from the base state T_0, in the dynamical core. This is indicated via the dashed framed boxes in the flow chart. The old variant will be called p*-T-, the new one p*-T*-Dynamics in the following. Two aspects have to be pointed out here, which are apparent in the equations given on the right side. First, the terms in blue (vertical advection) and in green (horizontal advection) cancel analytically. Errors due to different discretization in the old p*-T-variant are avoided. Second, the remaining red term is now dealt with in the fast waves solver, leading to a better representation of the gravity waves in this part.

The positive effect of the first point is clearly visible in the test case of the atmosphere at rest over a hill (see below). Analytically the atmosphere should stay at rest. However during the simulation significant disturbances develop. The beneficial effects of the second point mainly showed up in long term runs with the climate version of the COSMO-model, which can not be shown here for lack of space.



1. Advection of a Decomposed (Reference State + Deviation) Scalar: $\psi = \psi_0 + \psi^*$

$$\left(\frac{\partial\psi}{\partial t}\right)^{(n)}_{VADV} = -\overline{\dot\zeta^{(n)}\delta_\zeta\psi^{*(n)}}^{\zeta} - \overline{\dot\zeta^{(n)}\delta_\zeta\psi_0}^{\zeta}$$
$$= -\overline{\dot\zeta^{(n)}\delta_\zeta\psi^{*(n)}}^{\zeta} - \overline{w^{(n)}\frac{\partial\psi_0}{\partial z}}^{\zeta} + \overline{\left(\frac{J_\lambda}{a\cos\varphi}\overline{u^{(n)}}^{\lambda,\zeta} + \frac{J_\varphi}{a}\overline{v^{(n)}}^{\varphi,\zeta}\right)\frac{\partial\psi_0}{\partial z}}^{\zeta}$$

$$\left(\frac{\partial\psi}{\partial t}\right)^{(n)}_{HADV} = -\frac{1}{a\cos\varphi}\left(\overline{u^{(n)}\delta_\lambda\psi^{*(n)}}^{\lambda} + \cos\varphi\,\overline{v^{(n)}\delta_\varphi\psi^{*(n)}}^{\varphi}\right) - \overline{\left(\frac{J_\lambda}{a\cos\varphi}\overline{u^{(n)}}^{\lambda,\zeta} + \frac{J_\varphi}{a}\overline{v^{(n)}}^{\varphi,\zeta}\right)\frac{\partial\psi_0}{\partial z}}^{\zeta}$$

2. New Formulation of the Fast Waves Solver

- Vertical Velocity

$$w^{(\nu+1)} = w^{(\nu)} + \left[\frac{1}{\overline{\sqrt{G}}^{\zeta}}\frac{1}{\overline{\rho^{(n)}}^{\zeta}}\left\{\beta^+\delta_\zeta p^{*(\nu+1)} + \beta^-\delta_\zeta p^{*(\nu)}\right\} - g\frac{\overline{\rho_0}^{\zeta}}{\overline{\rho^{(n)}}^{\zeta}}\left\{\overline{\left(\frac{T_0\beta^+}{T^{(n)}p_0}\right)p^{*(\nu+1)}}^{\zeta} + \overline{\left(\frac{T_0\beta^-}{T^{(n)}p_0}\right)p^{*(\nu)}}^{\zeta}\right\} + g\frac{\overline{\rho_0}^{\zeta}}{\overline{\rho^{(n)}}^{\zeta}}\left\{\overline{\left(\frac{\beta^+}{T^{(n)}}\right)T^{*(\nu+1)}}^{\zeta} + \overline{\left(\frac{\beta^-}{T^{(n)}}\right)T^{*(\nu)}}^{\zeta}\right\} + f_w^{(n)}\right]\Delta\tau$$

- Pressure Perturbation

$$p^{*(\nu+1)} = p^{*(\nu)} + \left[g\rho_0\left\{\beta^+\overline{w^{(\nu+1)}}^{\zeta} + \beta^-\overline{w^{(\nu)}}^{\zeta}\right\} + \frac{1}{\sqrt{G}}\frac{p^{(n)}c_{pd}}{c_{vd}}\left\{\beta^+\delta_\zeta w^{(\nu+1)} + \beta^-\delta_\zeta w^{(\nu)}\right\} - \frac{p^{(n)}c_{pd}}{c_{vd}}D_h^{(\nu+1)} + f_{p^*}^{(n)}\right]\Delta\tau$$

- Temperature Perturbation

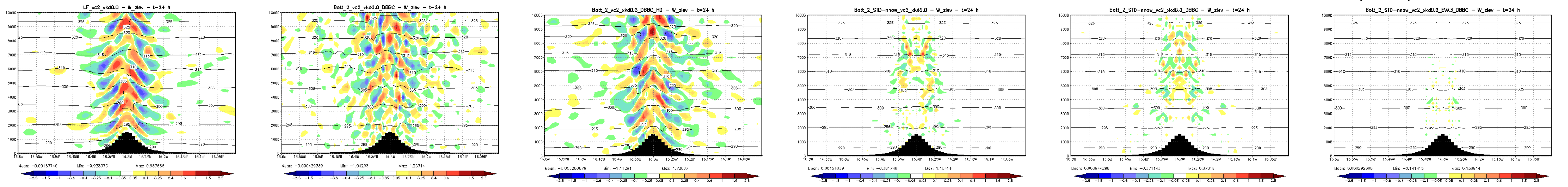
$$T^{*(\nu+1)} = T^{*(\nu)} + \left[-\frac{dT_0}{dz}\left\{\beta^+\overline{w^{(\nu+1)}}^{\zeta} + \beta^-\overline{w^{(\nu)}}^{\zeta}\right\} + \frac{1}{\sqrt{G}}\frac{p^{(n)}}{\rho^{(n)}c_{vd}}\left\{\beta^+\delta_\zeta w^{(\nu+1)} + \beta^-\delta_\zeta w^{(\nu)}\right\} - \frac{p^{(n)}}{\rho^{(n)}c_{vd}}D_h^{(\nu+1)} + f_{T^*}^{(n)}\right]\Delta\tau$$

Idealized Test Case: Atmosphere at Rest

- T(x,y,z,t=0) ≠ $T_0(z)$, dT/dz = -6,5 K/km
- q_v(x,z,z,t=0) = 0 g/kg
- 2D - Simulation at a resolution of 1 km, h_{max} = 1500 m, a = 5 km.
- Physics: only turbulence parameterization.

Contours: vertical velocity   Isolines: potential temperature

Leapfrog | Runge-Kutta old p*-T-Dynamics | Runge-Kutta + HD old p*-T-Dynamics | Runge-Kutta new p*-T*-Dynamics old lower BC | Runge-Kutta new p*-T*-Dynamics | Runge-Kutta new p*-T*-Dynamics explicit Upwind-3rd order VA



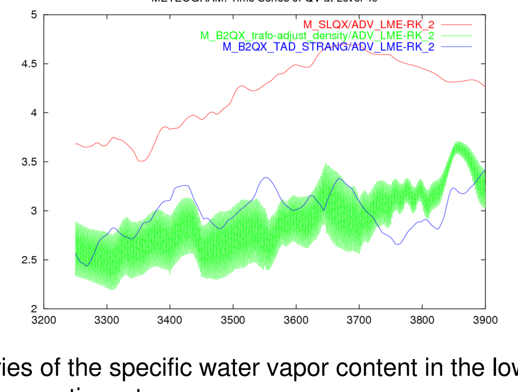
3. Advection of Moisture Variables

After the calculations of the dynamical core the treatment of the scalar quantities follows (blue box in the flow chart). To be able to deal with the advection in a single step, different Courant number independent schemes were implemented. On the one hand a 3d semi-Lagrangian scheme (using tri-cubic interpolation), on the other hand different Eulerian schemes (among others the positive definite scheme of Bott (MWR 1989)). For the latter of them different points summarized in a paper of Skamarock (MWR 2006), are realized, i.e. a Courant number independant formulation, Strang-splitting as well as a mass consistent treatment of the transport in conservation form. The main aspects here are (see equations on the left side below) the change to the densities or back to the specific quantities respectively (in black), simultaneous calculation of the continuity equation (in blue) and the treatment of the moisture advection in conservation form itself (in red).

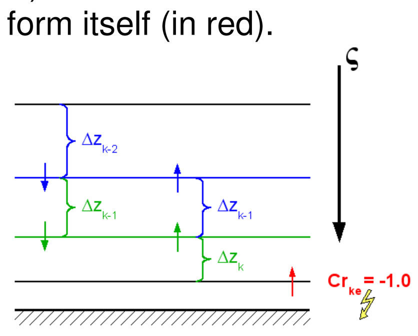
$$(\rho q_x)^n = \rho^n q_x^n$$
$$\rho^* = \rho^n - \Delta t\,\frac{1}{\sqrt{G}}\,F_x\left(\sqrt{G}u^{n+\frac{1}{2}},\rho^n\right)$$
$$(\rho q_x)^* = (\rho q_x)^n - \Delta t\,\frac{1}{\sqrt{G}}\,F_x\left(\sqrt{G}u^{n+\frac{1}{2}},\rho^n q_x^n\right)$$
$$q_x^* = (\rho q_x)^*/\rho^*$$
$$\rho^{**} = \rho^* - \Delta t\,\frac{1}{\sqrt{G}}\,F_y\left(\sqrt{G}v^{n+\frac{1}{2}},\rho^n\right)$$
$$(\rho q_x)^{**} = (\rho q_x)^* - \Delta t\,\frac{1}{\sqrt{G}}\,F_y\left(\sqrt{G}v^{n+\frac{1}{2}},\rho^n q_x^*\right)$$
$$q_x^{**} = (\rho q_x)^{**}/\rho^{**}$$
$$\rho^{n+1} = \rho^{**} - \Delta t\,\frac{1}{\sqrt{G}}\,F_\zeta\left(\sqrt{G}\dot\zeta^{n+\frac{1}{2}},\rho^n\right)$$
$$(\rho q_x)^{n+1} = (\rho q_x)^{**} - \Delta t\,\frac{1}{\sqrt{G}}\,F_\zeta\left(\sqrt{G}\dot\zeta^{n+\frac{1}{2}},\rho^n q_x^{**}\right)$$
$$q_x^{n+1} = (\rho q_x)^{n+1}/\rho^{n+1}$$

$$q_x^{t+\Delta t} = (I + A_z)(I + A_y)(I + A_x)\; q_x^t$$
$$q_x^{t+2\Delta t} = (I + A_x)(I + A_y)(I + A_z)\; q_x^{t+\Delta t}$$
$$q_x^{t+\Delta t} = (I + \tfrac{1}{2}A_x)(I + \tfrac{1}{2}A_y)(I + A_z)(I + \tfrac{1}{2}A_y)(I + \tfrac{1}{2}A_x)\; q_x^t$$

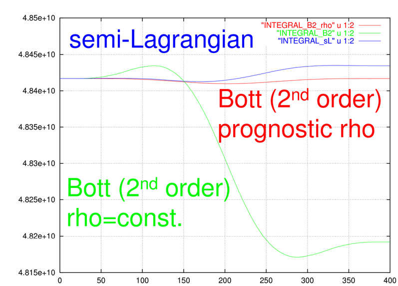
semi-Lagrangian (3d, i.e. no splitting)



Time series of the specific water vapor content in the lowest model layer for every time step.
red: semi-Lagrangianm green and blue: Bott (2nd order) with the given different variant for the Strang-splitting.



Problems arise if the complete mass is transported out of a grid box in a single 1d-substep, e.g. Cr_z = -1.0 in the first layer above ground.



Time series of the volume integral of the tracer.

3.1. Schär et al. (MWR 2002) Test Case

The local and global conservation properties of the different advection schemes in terrain following coordinates show big differences. Apart from the tracer itself, which plays the part of a specific quantity, a exponentially distributed "density" is introduced as a second to be advected field.

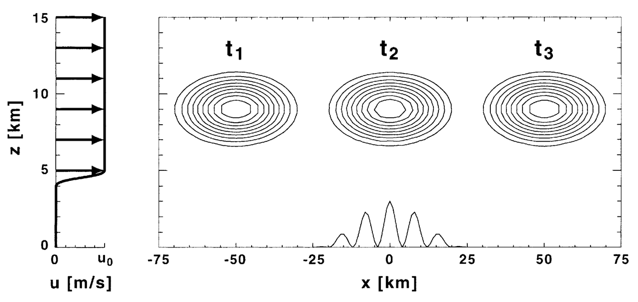
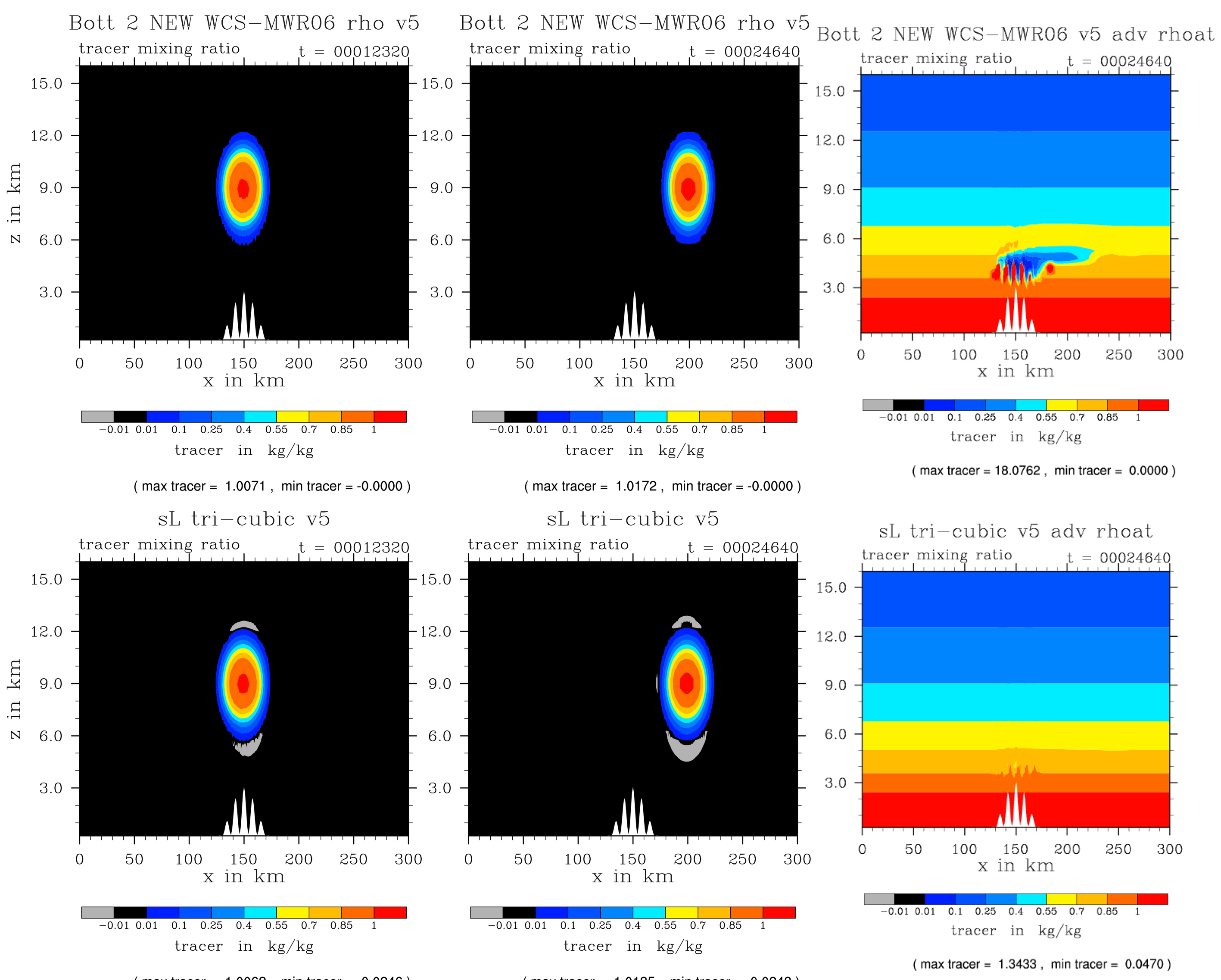


FIG. 4. Vertical cross section of the idealized two-dimensional advection test. The topography is located entirely within a stagnant pool of air, while there is a uniform horizontal velocity aloft. The analytical solution of the advected anomaly is shown at three instances.



upper row: Bott (2nd order) scheme / lower row: semi-Lagrangian (tri-cubic) scheme
left: tracer at time t_2 / middle: tracer at time t_3 / right: "density" at time t_3
It is important to notice, that the simultaneous transport of the artificial density field is only an integral part of the Eulerian treatment, the according calculation with the semi-Lagrangian scheme was done separately.
While the Eulerian treatment shows good global conservation properties (see figure with volume integral of the tracer), serious deficiencies are apparent when it comes to local conservation (especially of the "density" field).

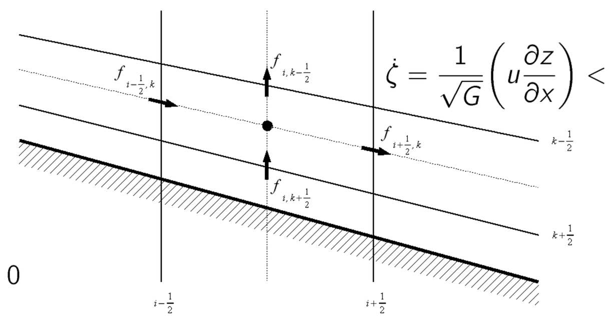
3.2. … Further Simplification of the Test Problem…

… to the upwind 1st order advection of a constant field with the value one. Theoretically no flux divergence should occur. However, inconsistencies arise, due to the averaging of the u-component in the staggered grid during the calculation of the contravariant vertical velocity, in the region of the velocity shear.

$$-\frac{\Delta t}{\Delta x}\left(f_{i+\frac{1}{2},k} - f_{i-\frac{1}{2},k}\right) - \frac{\Delta t}{\Delta\zeta}\left(f_{i,k+\frac{1}{2}} - f_{i,k-\frac{1}{2}}\right) = 0$$

$\phi = 1$; $\Delta\zeta = 1$; Advection: UP(1^{st} order) $\Longrightarrow$

$$-\frac{\Delta t}{\Delta x}\left[u_{i+\frac{1}{2},k}\left(z_{i,k-\frac{1}{2}} - z_{i,k+\frac{1}{2}}\right) - u_{i-\frac{1}{2},k}\left(z_{i-1,k-\frac{1}{2}} - z_{i-1,k+\frac{1}{2}}\right)\right] + \frac{\Delta t}{\Delta x}\left[\overline{u}^{x\zeta}_{i,k+\frac{1}{2}}\left(z_{i-1,k+\frac{1}{2}} - z_{i,k+\frac{1}{2}}\right) - \overline{u}^{x\zeta}_{i,k-\frac{1}{2}}\left(z_{i-1,k-\frac{1}{2}} - z_{i,k-\frac{1}{2}}\right)\right] \neq 0$$



$$\dot\zeta = \frac{1}{\sqrt{G}}\left(u\frac{\partial z}{\partial x}\right) < 0$$