The moist compressible atmospheric model ASAM

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Introduction

A dynamic core ASAM (All Sacle Atmospheric Model) of the moist compressible Euler equation in conservative form is presented.

The thermodynamic equations applied here differ slightly from those used in most numerical models. Traditionally, the specific heats of water vapor and liquid water are ignored in numerical models, so that $R_m \approx R$, $c_{pml} \approx c_p$, and $c_{vml} \approx c_v$, yielding the traditional potential temperature equation. Unlike the complex bulk microphysical models typically employed in a majority of mesoscale models, a simple and differentiable parameterization that converts water vapor into total cloud substance was utilized in the model so far. The microphysical model is completed by the addition of the traditional bulk parameterization of the falling of rain. The orography is incorporated in the model through a special grid system, where the orography is represented by cut cells in a Cartesian grid. The time integration is accomplished by a linear implicit method of Rosenbrock type. Because the method is fully implicit, the approach is able to employ time steps that result in Courant-Friedrichs-Lewy (CFL) numbers greater than one for advection, gravity, and sound waves; however, the dynamical time scale of the problem will be respected for accuracy by a dynamic time step procedure.

The dry compressible Euler equation

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{v}) &= 0 \\
\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla(\rho \mathbf{v} \circ \mathbf{v}) &= -\nabla p - \rho \mathbf{g} - 2\Omega \times (\rho \mathbf{v}) \\
\frac{\partial \rho\theta}{\partial t} + \nabla(\rho \mathbf{v}\theta) &= Q_\theta \\
p &= \rho R \theta (p/p_0)^\kappa \quad \text{or} \\
p &= \left(\frac{R\Theta}{p_0^\kappa}\right)^{1/(1-\kappa)}
\end{aligned}
$$

where ρ is the density of dry air, ρu, ρv, ρw are the three components of the mass flux, θ is the potential temperature, and $\kappa = R/c_p$.

Time integration

After spatial discretization an ordinary differential equation

$$y' = F(y)$$

is obtained which we integrate in time by a special Rosenbrock-method.

$$
\begin{aligned}
w^{n+1} &= w^n + \frac{5}{4}k_1 + \frac{3}{4}k_2 \\
Sk_1 &= \tau F(w^n) \\
Sk_2 &= \tau F(w^n + \frac{2}{3}k_1) - \frac{4}{3}k_1 \\
S &= I - \gamma\tau J, \quad J = F'(w^n)
\end{aligned}
$$

with $\gamma = \frac{1}{2} + \frac{1}{6}\sqrt{3}$.

The above described Rosenbrock method allows a simplified solution of the linear systems without loosing the order. When $J = J_A + J_B$ the matrix S can be replaced by $S = (I - \gamma\tau J_A)(I - \gamma\tau J_B)$.A further simplification can be reached by omitting some parts of the Jacobian or replacement of the derivatives by the same derivatives of a simplified operator $\tilde{F}(w^n)$. The structure of the Jacobian

$$
J = \begin{pmatrix}
\frac{\partial F_\rho}{\partial \rho} & \frac{\partial F_\rho}{\partial \mathbf{V}} & 0 \\
\frac{\partial F_V}{\partial \rho} & \frac{\partial F_\mathbf{V}}{\partial \mathbf{V}} & \frac{\partial F_\mathbf{V}}{\partial \Theta} \\
0 & \frac{\partial F_\Theta}{\partial \mathbf{V}} & \frac{\partial F_\Theta}{\partial \Theta}
\end{pmatrix}.
$$

A zero block 0 indicates that this block is not included in the Jacobian or is absent. The derivative with respect to ρ is only taken for the Buoyancy term in the vertical momentum equation. Note that this type of approximation is the standard approach in the derivation of the Boussinesq approximation starting form the compressible Euler equations. The matrix J can decomposed as

$$
J = J_T + J_P = \begin{pmatrix}
\frac{\partial F_\rho}{\partial \rho} & 0 & 0 \\
\frac{\partial F_V}{\partial \rho} & \frac{\partial F_\mathbf{V}}{\partial \mathbf{V}} & 0 \\
0 & 0 & \frac{\partial F_\Theta}{\partial \Theta}
\end{pmatrix}
+
\begin{pmatrix}
0 & \frac{\partial F_\rho}{\partial \mathbf{V}} & 0 \\
0 & 0 & \frac{\partial F_\mathbf{V}}{\partial \Theta} \\
0 & \frac{\partial F_\Theta}{\partial \mathbf{V}} & 0
\end{pmatrix}
$$

or

$$
J = J_T + J_P = \begin{pmatrix}
\frac{\partial F_\rho}{\partial \rho} & 0 & 0 \\
0 & \frac{\partial F_\mathbf{V}}{\partial \mathbf{V}} & 0 \\
0 & 0 & \frac{\partial F_\Theta}{\partial \Theta}
\end{pmatrix}
+
\begin{pmatrix}
0 & \frac{\partial F_\rho}{\partial \mathbf{V}} & 0 \\
\frac{\partial F_V}{\partial \rho} & 0 & \frac{\partial F_\mathbf{V}}{\partial \Theta} \\
0 & \frac{\partial F_\Theta}{\partial \mathbf{V}} & 0
\end{pmatrix}.
$$

The first part of the splitting J_T is called the transport/source part and contains the advection, diffusion and source terms like Coriolis, curvature, Buoyancy, latent heat, and so on. The second matrix is called the pressure part and involves the pressure gradient and the derivative of the divergence with respect to momentum of the density and potential temperature equation. The difference between the two splitting approaches is the insertion of the derivative of the gravity term in the transport or pressure matrix. The first splitting damps sound waves and can be reduced to a Poisson-like equation, whereas the second splitting damps sound and gravity waves but the dimension of the system is doubled.

Dry bubble test case

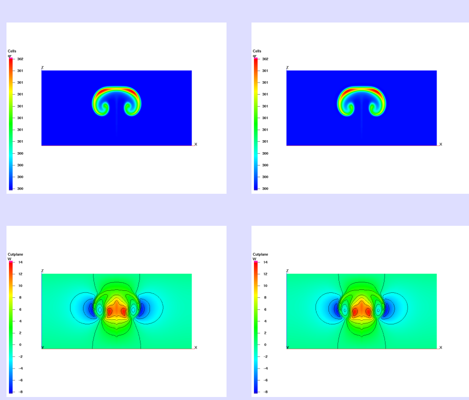
- Two-dimensional with a height of 10 km and a width of 20 km.
- Initial unperturbed conditions is an atmosphere at rest, hydrostatic and neutrally stable. ($\theta_0 = 300$K, $p_0 = 1000$mb).
- A warm perturbation is placed at the center of the domain with

  $$\theta' = 2\cos^2\left(\frac{\pi L}{2}\right)$$

  where

  $$L = \sqrt{\left(\frac{x - x_c}{x_r}\right)^2 + \left(\frac{z - z_c}{z_r}\right)^2}$$

  and $x_c = 10.0\,\mathrm{km}$, $z_c = 2.0\,\mathrm{km}$, and $x_r = z_r = 2.0\,\mathrm{km}$.
- Grid size 100 m, integration time 1000 s.



Dry rising bubble with a time step of 1 s (left) and 10 s (right) with potential temperature (above) and vertical velocity (below).

Outlook

The following model applications and developments are planned:

- Simulation of stratiform clouds and the influence of anthropogenic emissions (Project: EUCAARI).
- Turbulent dispersion of diaspore in complex terrain (joint work with Institute of Meteorology, University of Leipzig)
- Large eddy simulation of clouds and gravity waves (PAKT Antrag, joint work with R. Klein, PIK Potsdam, and U. Achatz, IAP Kühlungsborn).
- Ultrafine particle simulations in the resolved urban boundary layer on the bases of the PURAT experimental data.
- New algorithms for block structured adaptive grids (SPP MetStroem).
- Prandtl layer approximation for cut cell approximation of steep orography.

The moist compressible Euler equation

Some thermodynamic definitions

- Density ρ is now the density of the moist air.
- New densities for water vapor ρ_v and cloud water ρ_l.
- Define mixing ratios $r_V = \frac{\rho_v}{\rho_d}$ and $r_l = \frac{\rho_l}{\rho_d}$.
- Specific heat of moist air at constant pressure $c_{pml} = c_p + c_{pv} r_v + c_{pl} r_l$.
- Specific heat of moist air at constant volume $c_{vml} = c_v + c_{vv} r_v + c_{pl} r_l$.
- Gas constant of moist air $R_m = R + R_v r_v$.
- Latent heat of vaporization $L_v = L_{v0} - (c_{pl} - c_{pv})(T - T_0)$.

Redefined dry potential temperature

$$\tilde{\pi} = \left(\frac{p}{p_0}\right)^{R_m/c_{pml}}, \quad \theta = \frac{T}{\tilde{\pi}}$$

After Bryan and Fritsch the following equation for the new defined dry potential temperature θ can be derived in the moist case

$$\frac{D\ln\theta}{Dt} = -\frac{L_v}{c_{pml}T}\frac{Dr_v}{Dt} - \ln\tilde{\pi}\left(\left(\frac{R_v}{R_m} - \frac{c_{pv}}{c_{pml}}\right)\frac{Dr_v}{Dt} - \frac{c_{pl}}{c_{pml}}\frac{Dr_l}{Dt}\right)$$

together with equations for the mixing ratios r_v and r_l

$$\frac{Dr_v}{Dt} = -\frac{Dr_l}{Dt} = r_{\mathrm{Cond}}.$$

where r_{Cond} is the transfer rate of condensation. With this definition the dry potential temperature is conserved if no phase changes occur. This is not the case for the classical definition of the Exner pressure, where $\pi = \left(\frac{p}{p_0}\right)^{R/c_p}$

Density potential temperature

Reformulated equation of state

$$p = \rho R T \frac{1 + r_v/\epsilon}{1 + r_v + r_l}.$$

Density potential temperature

$$\theta_\rho = \theta\frac{1 + r_v/\epsilon}{1 + r_v + r_l}.$$

With this definition we have

$$p = \rho R\left(\frac{p}{p_0}\right)^{R_m/c_{pml}}\theta_\rho \quad \text{or} \quad p = \left(\frac{R\Theta_\rho}{p_0^{\kappa_m}}\right)^{1/(1-\kappa_m)}$$

with $\kappa_m = R_m/c_{pml}$ and $\Theta_\rho = \rho\theta_\rho$. Use the product rule to derive an equation for θ_ρ.

$$\frac{D\theta_\rho}{Dt} = \frac{D\theta}{Dt}\frac{1 + r_v/\epsilon}{1 + r_v + r_l} + \frac{\theta}{1 + r_v + r_l}\frac{1}{\epsilon}\frac{Dr_v}{Dt} - \theta\frac{1 + r_v/\epsilon}{(1 + r_v + r_l)^2}\left(\frac{Dr_v}{Dt} + \frac{Dr_l}{Dt}\right)$$

and convert to a flux form representation with respect to the full density.

$$
\begin{aligned}
\frac{\partial \rho\theta_\rho}{\partial t} + \nabla(\rho\mathbf{v}\theta_\rho) &= Q_{\theta_\rho} \\
\frac{\partial \rho q_v}{\partial t} + \nabla(\rho\mathbf{v}q_v) &= Q_{q_l} \\
\frac{\partial \rho q_l}{\partial t} + \nabla(\rho\mathbf{v}q_l) &= Q_{q_v}
\end{aligned}
$$

Representation of $\dot{q}_v = -\dot{q}_l$ in the absence of rain.

$$\dot{q}_v = -\Phi_c(q_v - qvs)q_{lmax}\tanh\left(\frac{q_l}{q_{lmax}}\right)$$

where Φ_c is a relaxation factor whose value depends on the grid size, qvs is the saturation ratio and q_{lmax} is a constant (1.e-5).

Changes in the numerics

- More complicated source term in the equation for the "potential temperature".
- Additional equations for the water substances.
- Inclusion of the fall velocity in the advection routine.
- Dependency of the pressure from q_v and q_l is not taken in to account in the Jacobian part of the momentum equation.

Moist bubble test case

How the moist case is constructed.

- Base state is hydrostatic and neutral stable.
- Total water mixing ratio is constant, that is $r_t = r_v + r_l =$ constant.
- The air is saturated everywhere and $r_l > 0$.
- Phase changes are exactly reversible.

Under these assumptions a neutrally stable atmosphere can be characterized by one conservative variable, in our case the wet equivalent potential temperature

$$\theta_e = T\left(\frac{p_d}{p_0}\right)^{-R/(c_p + c_{pl}r_t)}\exp\left[\frac{L_v r_v}{(c_p + c_{pl}r_t)T}\right],$$

where p_d is the partial pressure of dry air.

The moist base case is computed from

$$
\begin{aligned}
\frac{dp}{dz} &= -\rho g \\
r_v &= rvs(T, p) \\
r_t &= r_t^0 \\
p_d &= \frac{\rho}{1 + r_t}RT \\
p &= p_d + \rho r_v R_v T \\
\theta_e(p_d, T, r_v, r_t) &= \theta_e^0.
\end{aligned}
$$

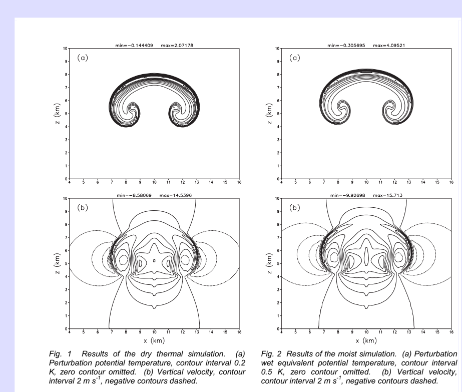
For moist conditions, buoyancy is given by

$$B = g\left(\frac{\theta_\rho}{\theta_{\rho 0}} - 1\right).$$

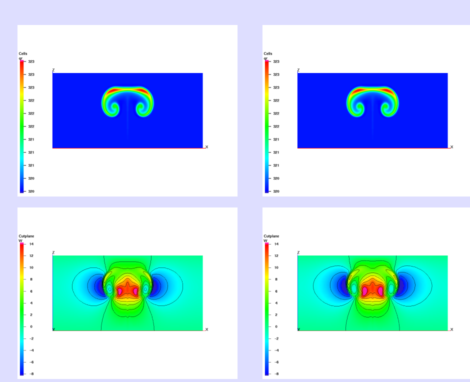
To have the same initial buoyancy as in the dry case

$$\theta_\rho = \theta_{\rho 0}\frac{(1 + r_t)}{(1 + rvs/\epsilon)}\left(\frac{\theta'_d}{\theta_{d0}} + 1\right).$$

In the perturbed bubble a further saturation adjustment is necessary.



Picture from the Fritsch and Bryan paper



Moist rising bubble with a time step of 1 s (left) and 10 s (right) with equivalent potential temperature (above) and vertical velocity (below).