Iterative time-stepping for the Met Office's semi-implicit, semi-Lagrangian non-hydrostatic deep atmosphere model

07th International SRNWP-Workshop

on Non-Hydrostatic Modelling

Bad Orb, 05 - 07 November 2007

Michail Diamantakis, Terry Davies and Nigel Wood

DYNAMICS RESEARCH, MET OFFICE, EXETER

michail.diamantakis@metoffice.gov.uk

Talk outline

1. Semi-implicit semi-Lagrangian techniques and the non-hydrostatic Met Office Unified Model (MetUM)
2. Summary of iterative time-stepping for the MetUM Semi-implicit semi-Lagrangian (SISL) scheme
3. High resolution flow over orography case and iterative time stepping
4. Concluding remarks and acknowledgements

Semi-implicit semi-Lagrangian schemes

SISL technique in non-hydrostatic NWP models:

- Semi-Lagrangian: unconditionally stable & accurate treatment of advection
- Semi-Implicit: unconditionally stable, second order (Crank-Nicholson) treatment of fast modes (gravity and acoustic)

SISL approach: more expensive per time step, however, allows very large timesteps.

- Cost-effective for Global weather/climate modelling and for Unified modelling environments.

In practice weak instabilities may remain due to approximations used in the handling of implicit and advection terms.

- Iterative techniques for the temporal discretization can be used to eliminate these instabilities (Côté et all 1998, Cullen QJRMS 2001 & 2003, Bénard MWR 2003).

Weaknesses of MetUM numerical formulation

MetUM SISL scheme operates efficiently and accurately for a very wide range of time and space scales. There are, however, two weaknesses in the formulation which we would like to address:

- Time extrapolating departure point calculation: weak instability (Cordero et al QJRMS, 2005).
- Explicit handling of the deep atmosphere Coriolis terms in momentum equations

Iterative time-stepping (Diamantakis et al QJRMS 2007) addresses these points demonstrating improvements.

MetUM predictor-corrector time scheme

Prognostic equation:

$$\frac{D\mathbf{X}}{Dt} = \mathbf{L}(\mathbf{x}, t, \mathbf{X}) + \mathbf{N}(\mathbf{x}, t, \mathbf{X}) + \mathbf{S}(\mathbf{x}, \mathbf{t}, \mathbf{X}) + \mathbf{F}(\mathbf{x}, \mathbf{t}, \mathbf{X})$$

is solved via a predictor-corrector (P-C) approach, Davies et al QJRMS 2005:

$$\begin{aligned}
\mathbf{X}^{(1)} &= \mathbf{X}_d^n + (1-\alpha)\Delta t(\mathbf{L}+\mathbf{N})_d^n + \Delta t\,(\mathbf{S})_d^n + \alpha\Delta t(\mathbf{L}+\mathbf{N})^n \\
\mathbf{X}^{(2)} &= \mathbf{X}^{(1)} + \Delta t\mathbf{F}(\mathbf{X}^n, \mathbf{X}^{(1)}, \mathbf{X}^{(2)}) \\
\mathbf{X}^{(3)} - \alpha\Delta t\mathbf{L}^{(3)} &= \mathbf{X}^{(2)} + \alpha\Delta t(\mathbf{N}^* - \mathbf{N}^n - \mathbf{L}^n)
\end{aligned}$$

where, $\mathbf{x}$, $\mathbf{L}$, $\mathbf{N}$ denote position, linear and nonlinear dynamical terms and $\mathbf{S}$, $\mathbf{F}$ slow and fast physics forcing.

$\mathbf{X}^{(1)}$ first predicted value, $\mathbf{X}^{(2)}$ predicted value after fast physics, $\mathbf{X}^{(3)} \equiv \mathbf{X}^{n+1}$ the final estimate and $\mathbf{N}^* \approx \mathbf{N}^{n+1}$. P-C steps equivalent to:

$$\mathbf{X}^{n+1} = \mathbf{X}_d^n + (1-\alpha)\Delta t(\mathbf{L}+\mathbf{N})_d^n + \Delta t\,(\mathbf{S})_d^n + \alpha\Delta t\left(\mathbf{L}^{n+1} + \mathbf{N}^*\right) + \Delta t\mathbf{F}(\mathbf{X}^n, \mathbf{X}^{(1)}, \mathbf{X}^{(2)}).$$

Iterative SISL scheme for the MetUM

Solve, iterating:

$$
\begin{aligned}
\mathbf{X}^{(1)[\ell]} &= \mathbf{X}^n_{d_\ell} + (1-\alpha)\Delta t\,(\mathbf{L}+\mathbf{N})^n_{d_\ell} + \Delta t\,(\mathbf{S})^n_{d_\ell} + \alpha\Delta t\,(\mathbf{L}+\mathbf{N})^{(3)[\ell-1]} \\
\mathbf{X}^{(2)[\ell]} &= \mathbf{X}^{(1)[\ell]} + \Delta t\mathbf{F}(\mathbf{X}^n, \mathbf{X}^{(1)[\ell]}, \mathbf{X}^{(2)[\ell]}) \\
\mathbf{X}^{(3)[\ell]} - \alpha\Delta t\mathbf{L}^{(3)[\ell]} &= \mathbf{X}^{(2)[\ell]} + \alpha\Delta t\left(\mathbf{N}^{*[\ell]} - \mathbf{N}^{(3)[\ell-1]} - \mathbf{L}^{(3)[\ell-1]}\right)
\end{aligned}
$$

where,

$$
\mathbf{L}^{(3)[\ell]} \equiv \mathbf{L}\left(\mathbf{X}^{(3)[\ell]}\right), \quad \mathbf{N}^{(3)[\ell]} \equiv \mathbf{N}(\mathbf{X}^{(3)[\ell]}), \quad \mathbf{L}^{(3)[0]} \equiv \mathbf{L}\left(\mathbf{X}^n\right), \quad \mathbf{N}^{(3)[0]} \equiv \mathbf{N}(\mathbf{X}^n)
$$

- For $\ell = 1$ the original MetUM SISL scheme is obtained
- For $\ell > 1$ more stable and accurate scheme:
  - Sufficient information to use a time-interpolating trajectory scheme, handle implicitly deep Coriolis atmosphere terms, improve VPG term
  - Improved physics-dynamics coupling

Mesoscale case study from T-REX IOP-6 campaign

IOP-6 mesoscale case study, 25/03/06-26/03/06 over the Sierra Nevada mountains (SouthWest USA) and the Owens valley. A challenging case study: rotor flows, hydraulic jumps and wave breaking in the lower stratosphere observed.

- MetUM forecasts on nested grids (Simon Vosper & Peter Sheridan 2006): [40km Global [12km Regional [4km [1km [333 m]]]]]
- 1km and 333m horizontal resolution runs, starting at 12:00 and 14:00 at 25 March 2006:
  - 76 vertical levels, $\Delta z_{min} = 5m$
  - No convection and gravity wave drag parametrization
  - Small off-centring, time-weights as small as $\alpha = 0.55$ can be used giving a stable forecast $\Rightarrow$ approximately 2nd order dynamics.

Iterative time-stepping tests on T-REX

The following 3 versions have been tested:

(a) 2 iterations full scheme: 2 dynamics calls, 2 fast physics calls per timestep; time interpolating scheme for the d.p. calculation, implicit handling of Coriolis and improved vertical pressure gradient terms

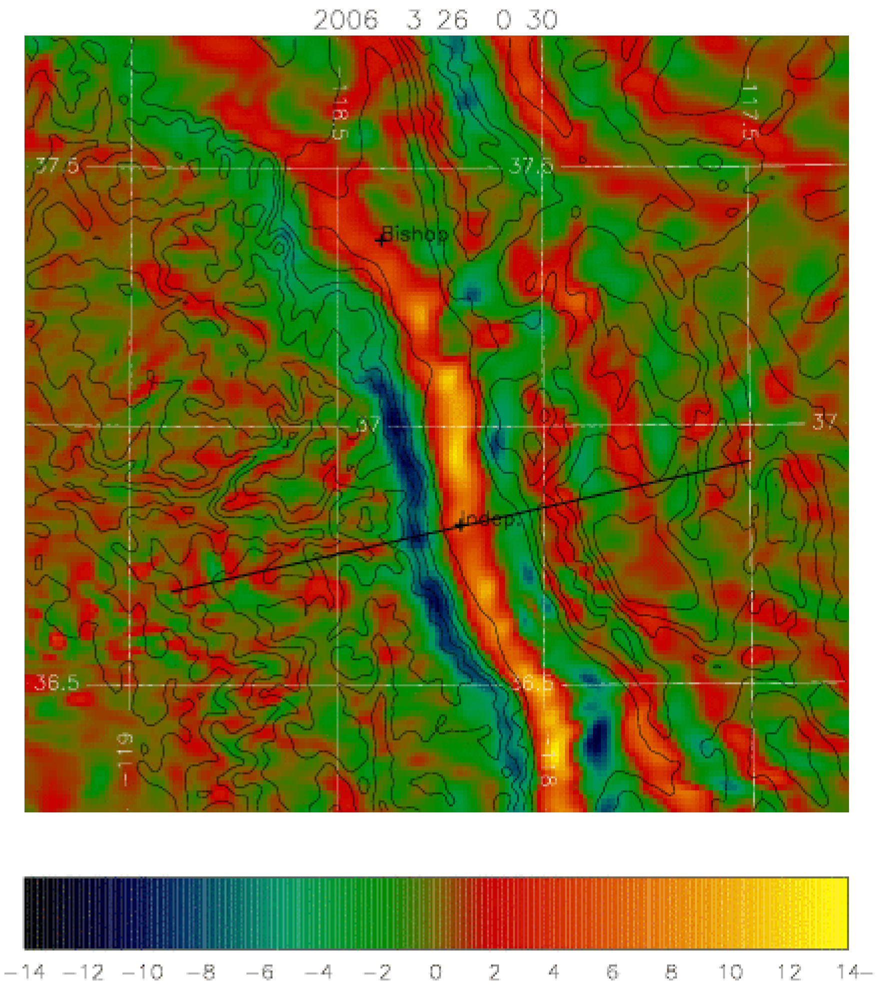
(b) 2 iterations as in (a) but activating only the time interpolating departure point scheme

(c) 2 iterations of dynamics as in (b) but only 1 fast physics call after dynamics

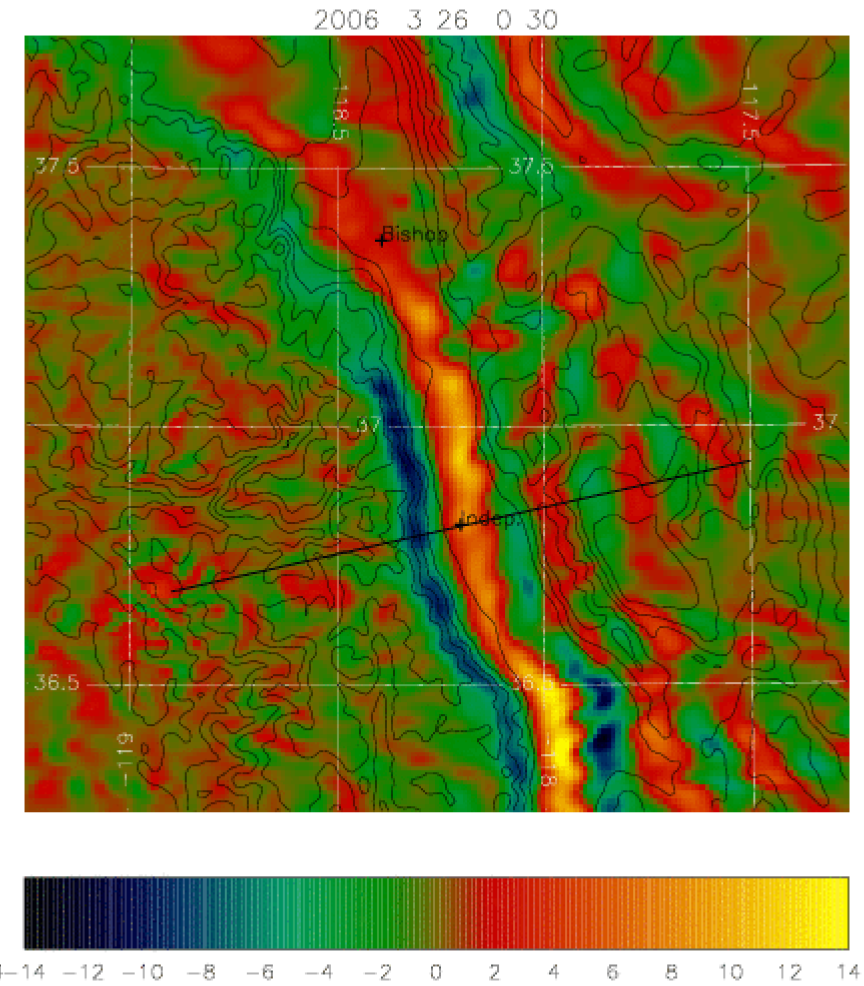
Global model tests with objective verification show (a) and then (b) is the most accurate formulation. However, high resolution tests show that in terms of model stability there is no noticeable difference among (a), (b), (c).

Iterative time-stepping 1km model results (i)

Control, $\Delta t = 12$s
max (adv) CFL: hor=0.72, vert=12



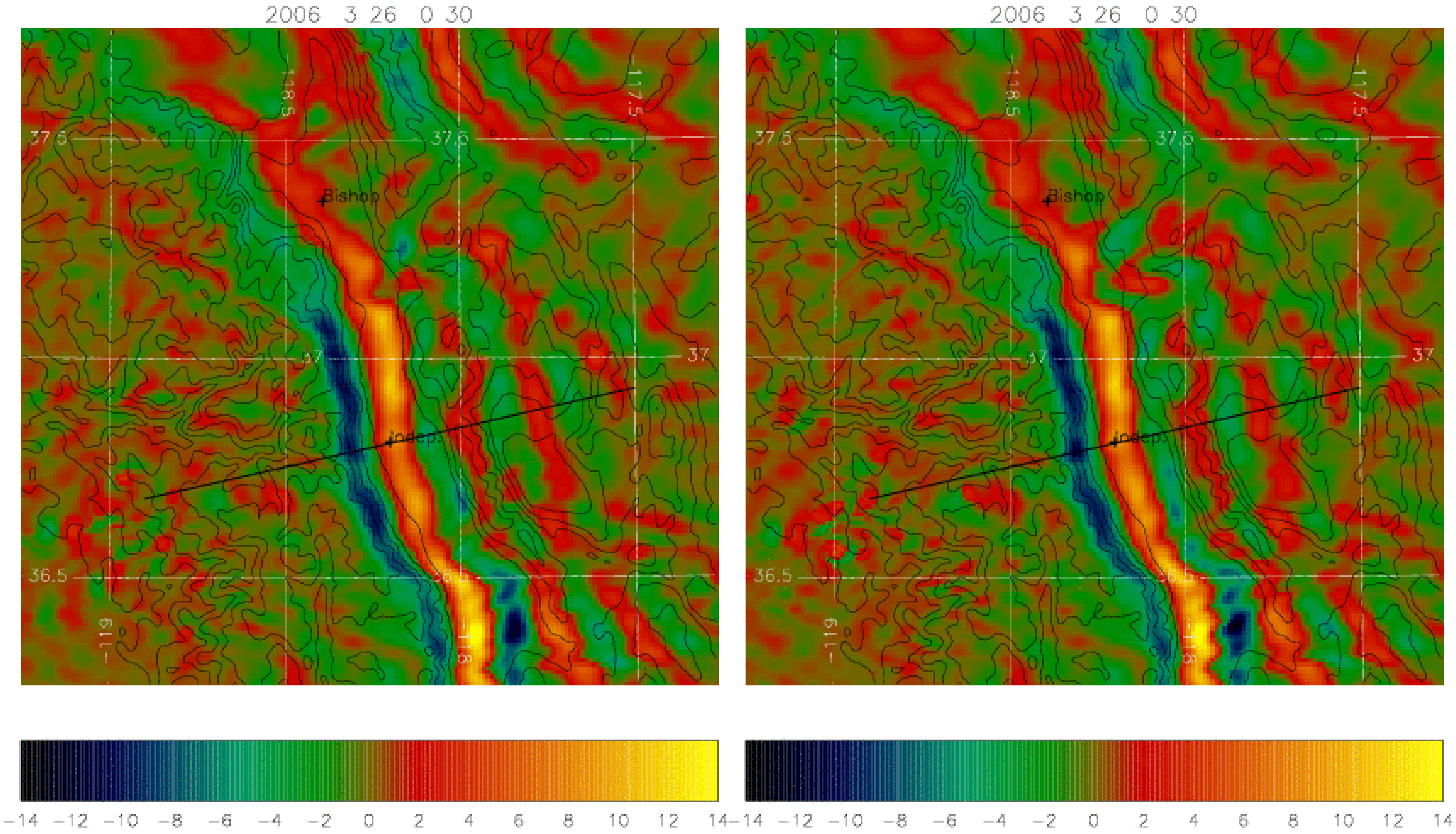
2-iterations, $\Delta t = 24$s
max (adv) CFL: hor=1.45, vert=24



Vertical velocity (W) from 1km run at 5km at 00:30, 26 Mar 06, T+12.5hrs.

ITERATIVE TIME-STEPPING 1KM MODEL RESULTS (II)

(b): 2 dynamics & physics calls (c): 2 dynamics & 1 fast physics call

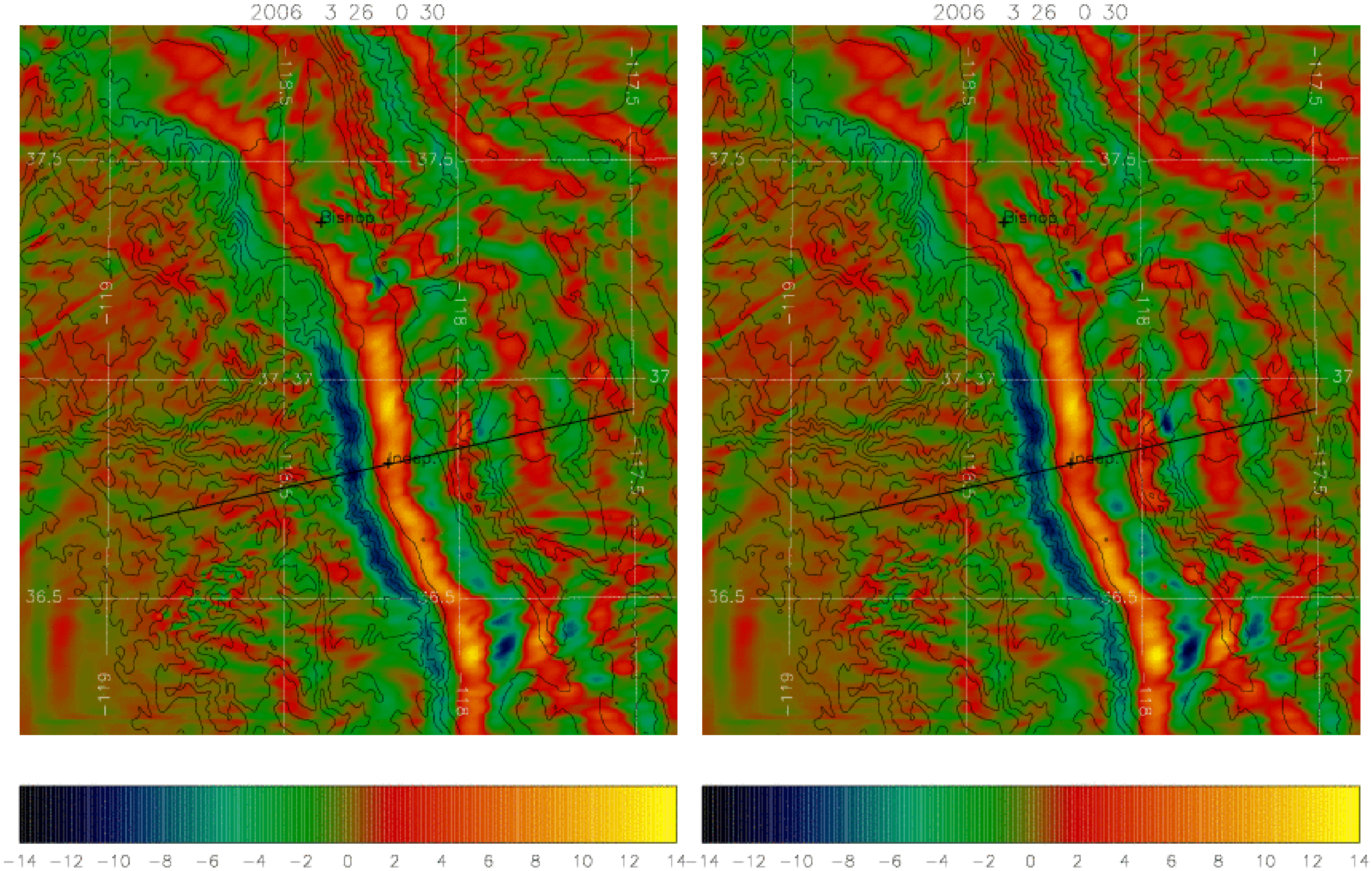


W from 1km, $\Delta t = 24$s run at 5km at 00:30, 26 Mar 06, T+12.5hrs.

ITERATIVE TIME-STEPPING 333M MODEL RESULTS (I)

(b): 2 dynamics & physics calls
max (adv) CFL: hor=0.9, vert=9

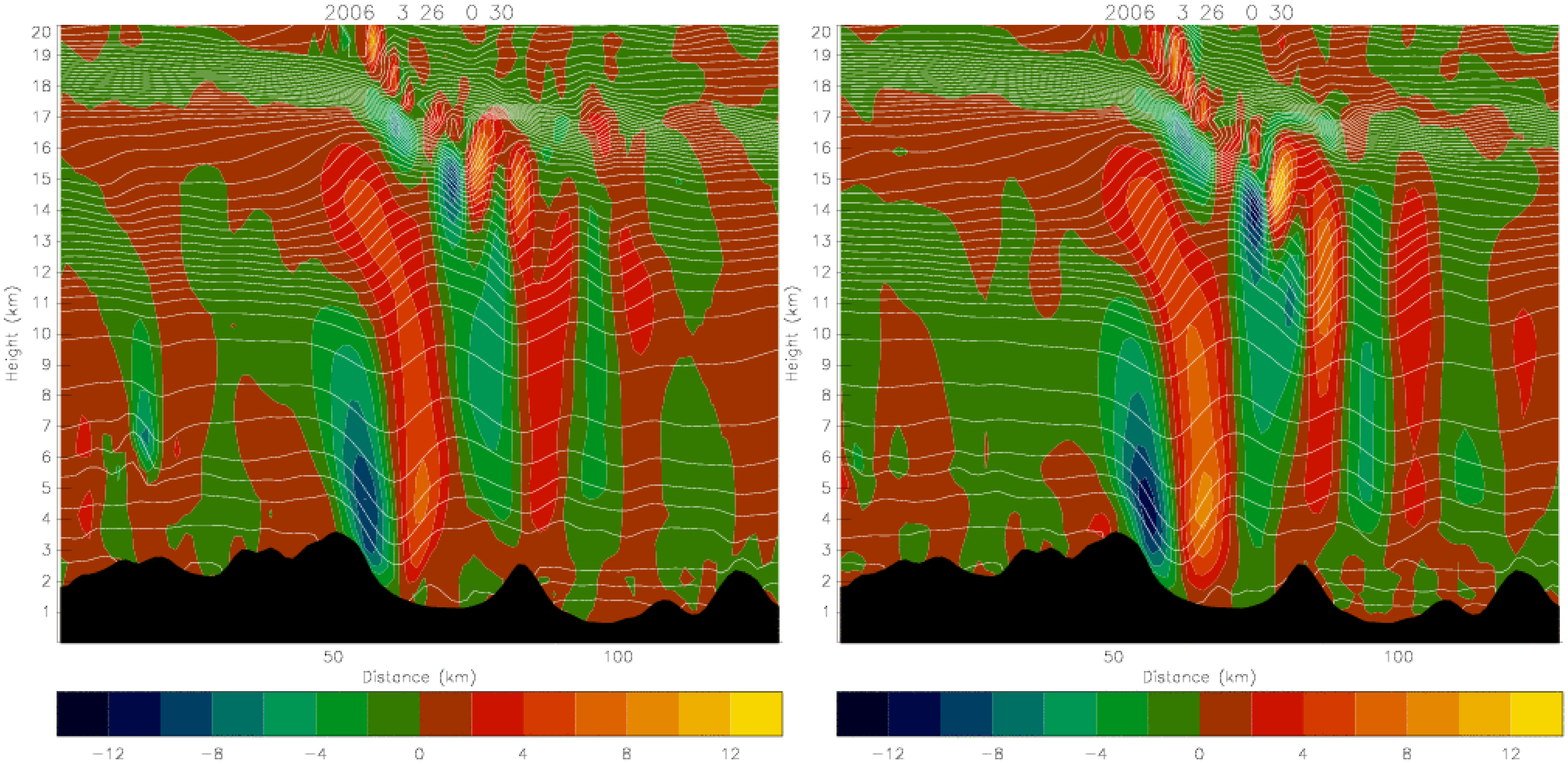
(c): 2 dynamics & 1 fast physics call
max (adv) CFL: hor=0.9, vert=18



W at 5km at 00:30 26 Mar 06.

Iterative time-stepping 1km model results (III)

Control run, $\Delta t = 12$s 2-iteration run (1 physics call), $\Delta t = 24$s

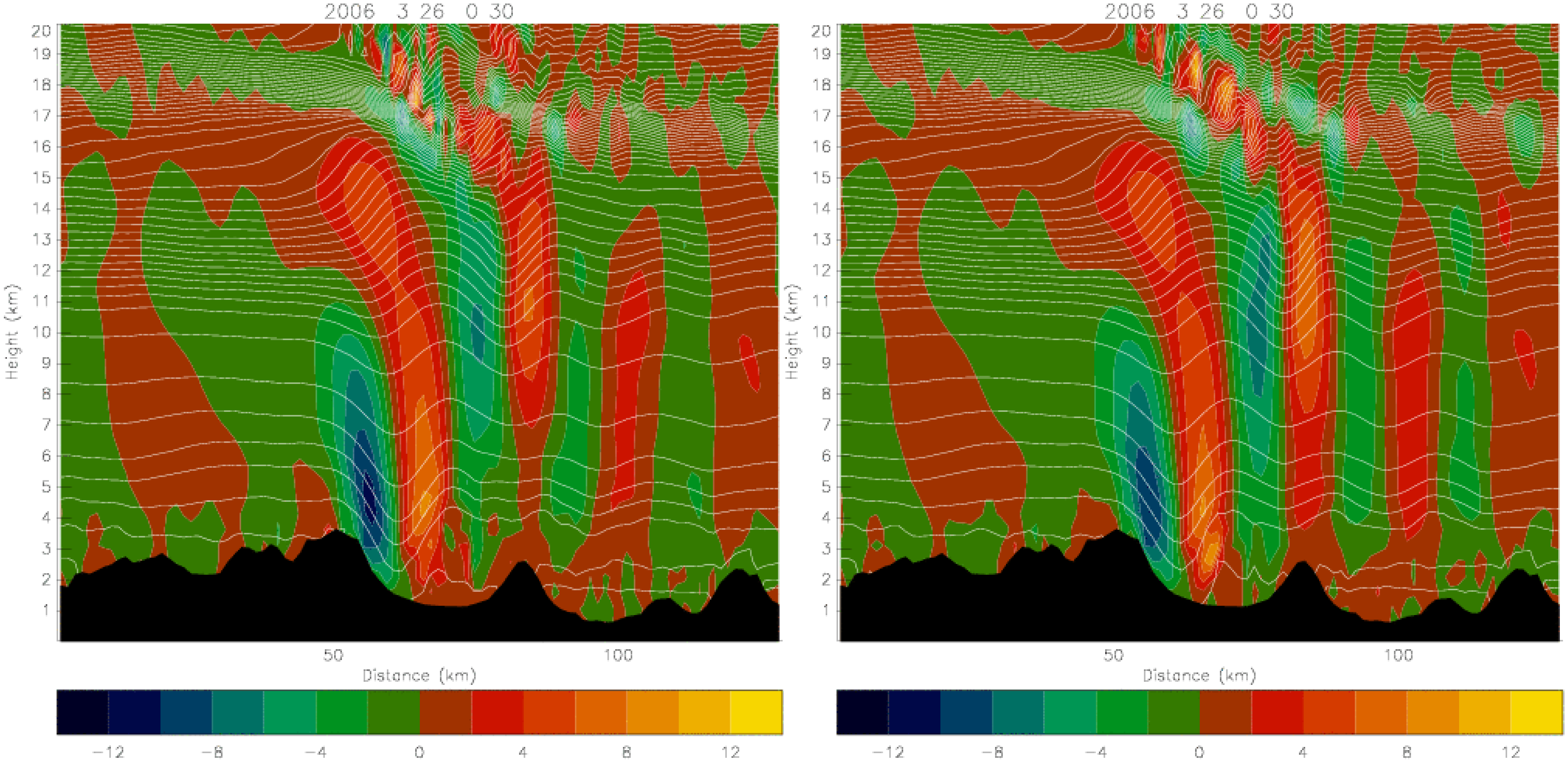


W cross section at 00:30 26 Mar 06. Yellow lines: 4K θ intervals.

ITERATIVE TIME-STEPPING 333M MODEL RESULTS (II)

Control run, $\Delta t = 5$s

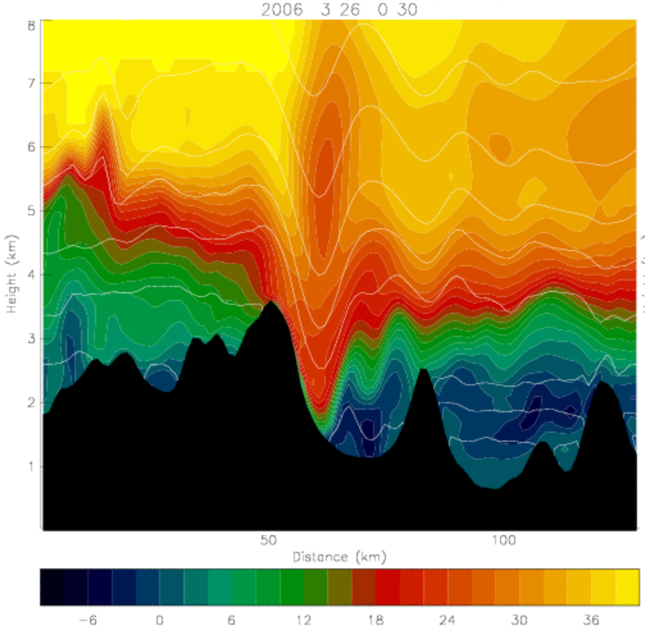
2-iteration run, $\Delta t = 10$s



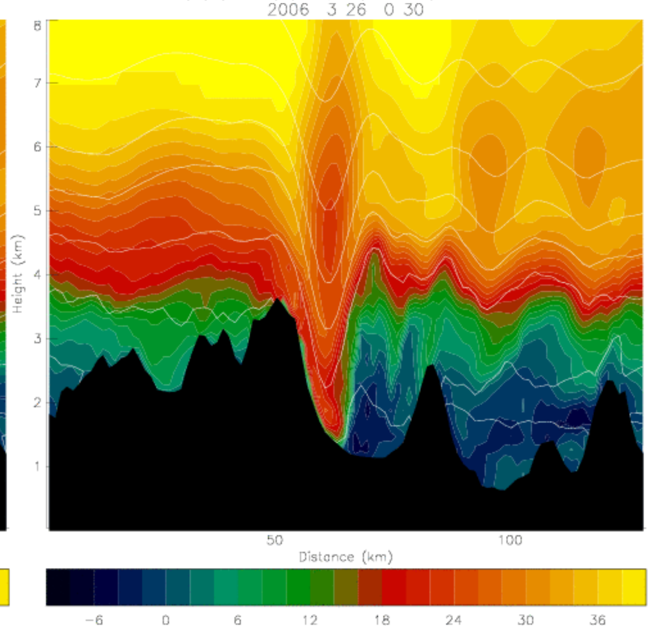
W cross section at 00:30 26 Mar 06.

U-WIND CROSS SECTION (FLOW SEPARATION)

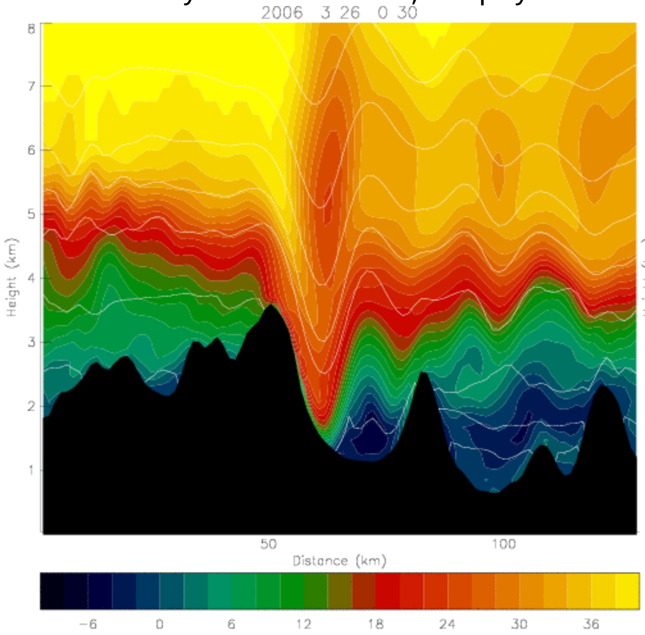
1km Control run



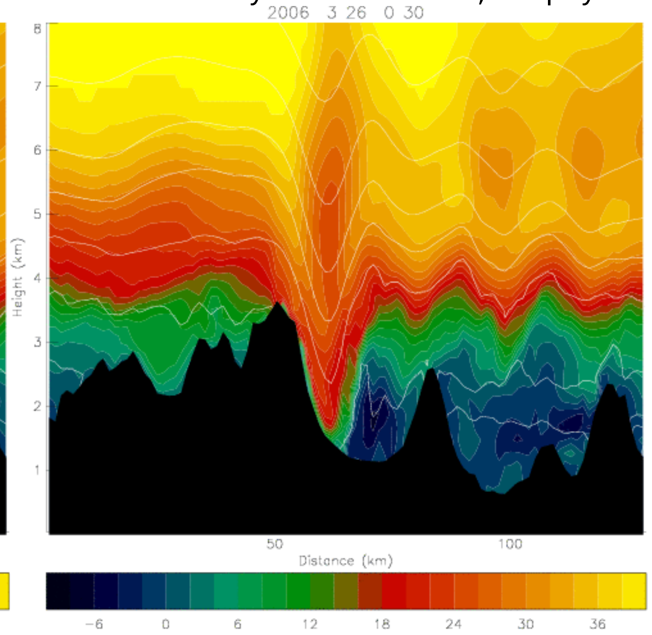
333m Control run



1km: 2 dyn iterations, 1 phys call

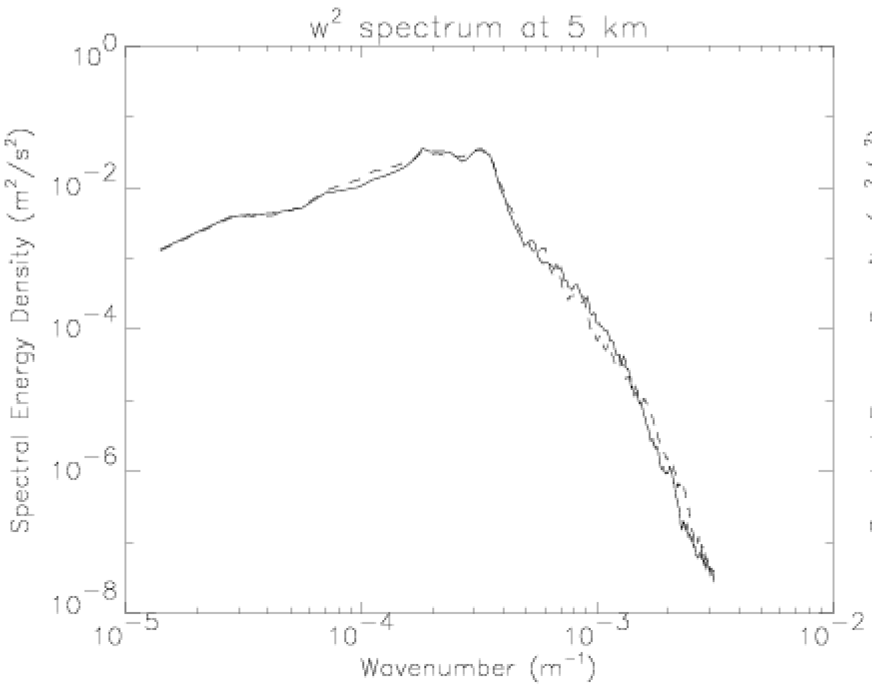


333m: 2 dyn iterations, 1 phys call

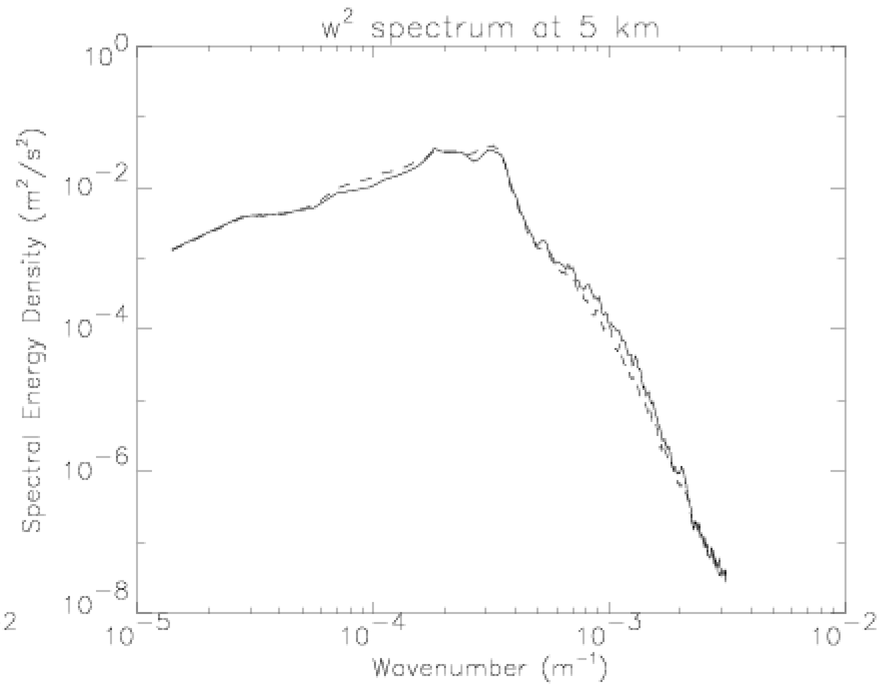


ITERATIVE TIME-STEPPING 1KM MODEL RESULTS (IV)

Control, $\Delta t = 12$s /
2 iterations, $\Delta t = 24$s

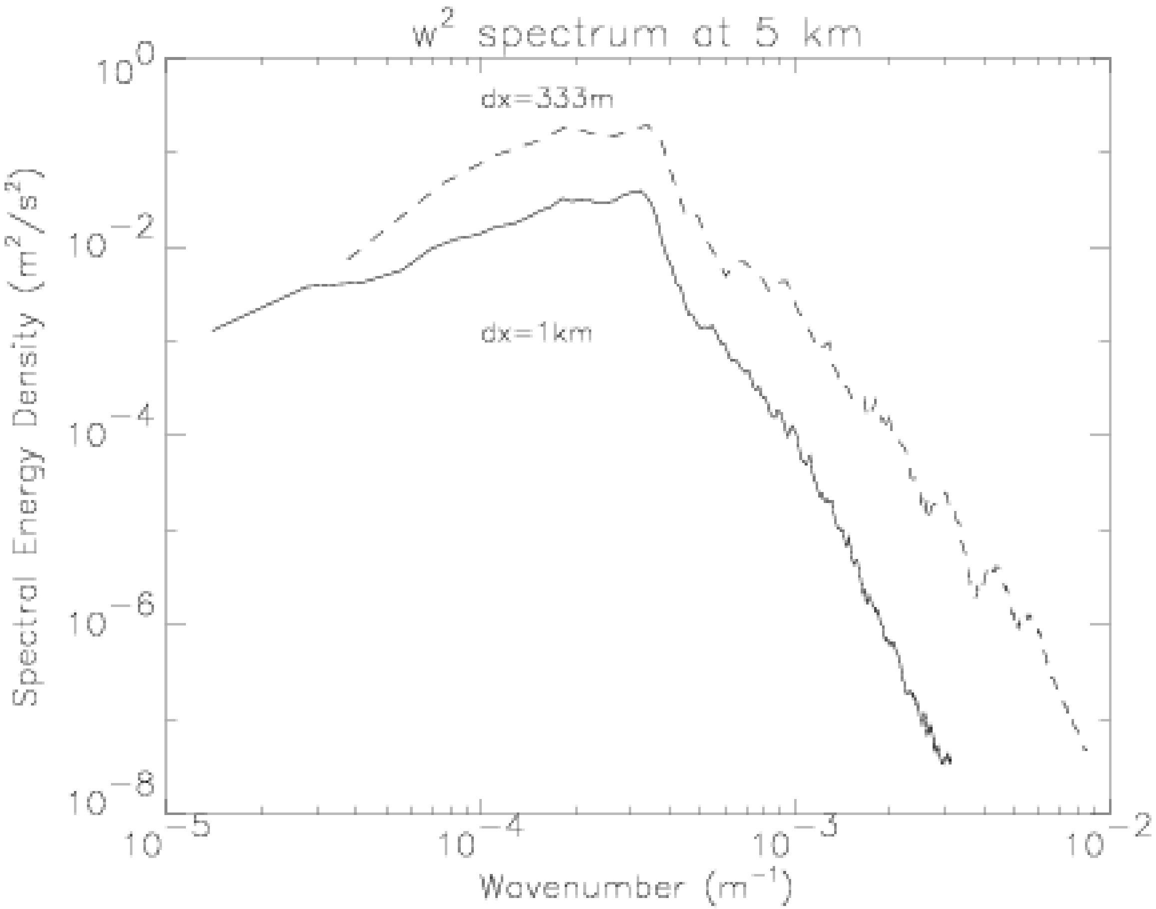


Control, $\Delta t = 12$s /
2 iterations (1 fast phys call), $\Delta t = 24$s

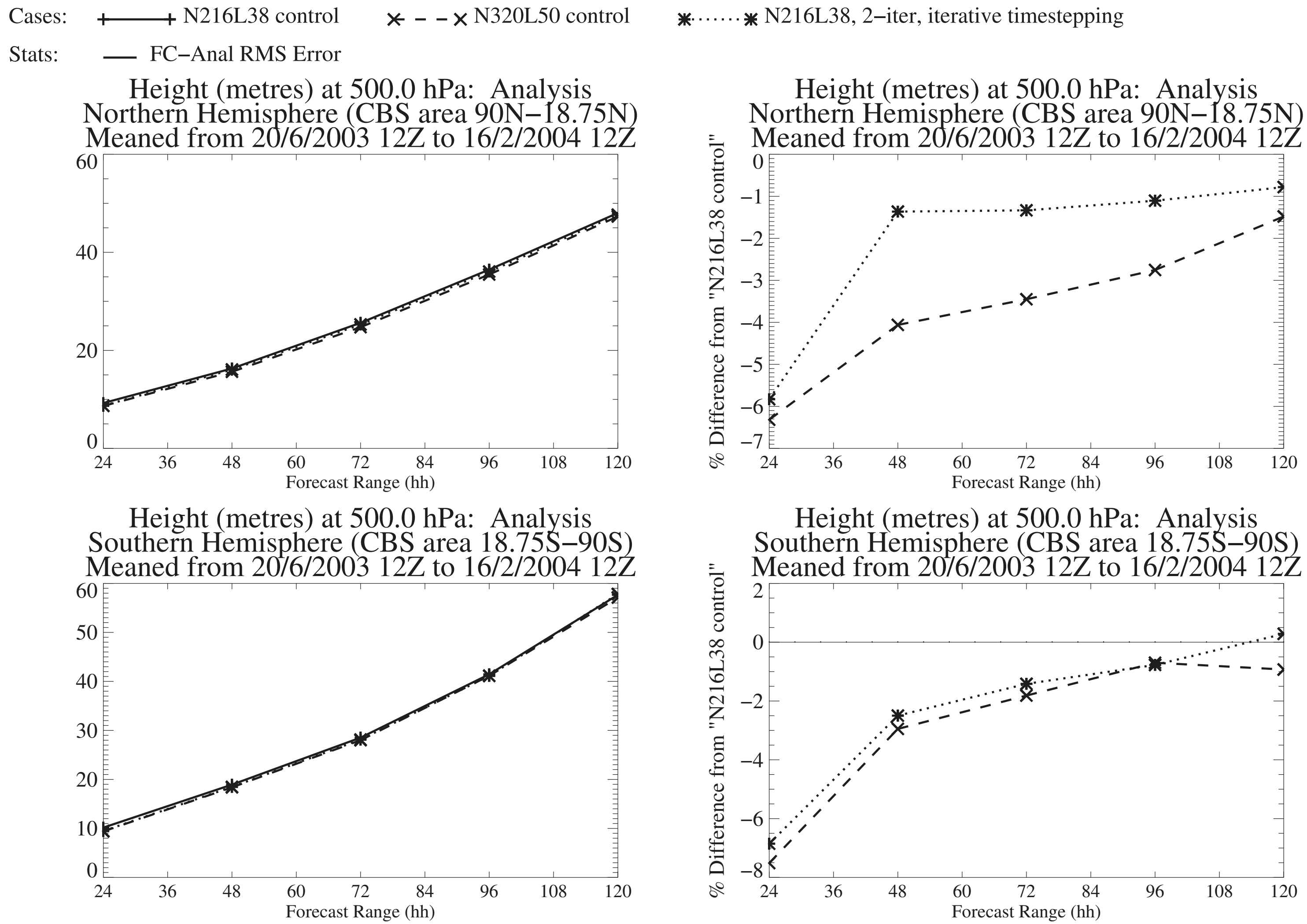


1km model vertical velocity energy spectra at 5km at 00:30 26 Mar 06. Continuous line: control run. Dashed: 2 iteration experiment.

ITERATIVE-TIME-STEPPING 333M MODEL RESULTS (III)



Vertical velocity energy spectra at 5km at 00:30 26 Mar 06 from 2 iteration run (c).



(a) **Actual RMSE and difference in RMSE (against the 60 km res 38 level control) for the extratropical H500 hPa**

Concluding remarks

T-REX IOP-6 case study:

- MetUM develops large amplitude wave response, low-level flow separation and wave breaking in the stratosphere consistent with observations
- MetUM captures correctly the amplitude but not the phase (good agreement in IOP-8 and IOP-10 cases)
- Iterative scheme enhances stability allowing doubling of the timestep and CPU time savings: 117hrs of total CPU time down to 98 hrs
- A very small amount of off-centring is used

In Global cases:

- Iterative timestepping scheme improves noticeably forecasting skill

Because of the benefits demonstrated, iterative framework will form a basis for reformulation of MetUM dynamical core.

Acknowledgements

Thanks to Simon Vosper & Peter Sheridan, Met Office Atmospheric Processes and Parametrization group, who provided the T-REX IOP-6 case study.