Development of the non-hydrostatic version of the global SL-AV model

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Outline

- SLAV model – what it is?
- Reduced grid in the shallow water version
- Development of the nonhydrostatic dynamical core

SL-AV model

- **Global finite-difference semi-Lagrangian semi-implicit dynamical core of own development + parameterizations of subgrid-scale processes from French model ARPEGE/ALADIN. So far, regular lat-lon grid is used.**

- **Distinct features of dynamical core – vorticity-divergence formulation on the unstaggered grid, wide use of 4th order finite differences, usual and compact (Tolstykh, JCP 2002).**

- **Semi-implicit solver, reconstruction of U and V, horizontal diffusion are carried out in Fourier space in longitude.**

Computationally efficient dynamical core:

- Integration of model dynamics equations with maximum accuracy in minimum time.
- Requires high-order numerical approximations and efficient parallel implementation (MPI+OpenMP).
- It is easier to achieve these goals using a lon-lat grid (provided that its main drawback is removed).

Constant resolution version of SL-AV model

- **Horizontal resolution 0,9° x0,72° lon-lat, 28 vertical levels. Implemented operationally. OI assimilation.**

Variable resolution version of SL-AV model

- **Horizontal resolution 0,5625° in longitude, 26-70 km in latitude, 28 vertical levels. Initial data – interpolated initial data for constant resolution version.**

Potential problems of the regular latitude-longitude grid

- Due to convergence of meridians towards poles, the grid step in longitude is much smaller than the grid step in latitude near the poles.
- This is bad for parameterizations, for calculating the horizontal derivatives.
- Redundant computations.
- Solution – reduced grid: number of points in longitude at each latitude circle can be different.
- Reduced grid is widely used in spectral models

Implementation of the reduced grid in the SLAV model

- A part of calculations is carried out in space of Fourier coefficients in longitude.
- The semi-Lagrangian advection is used (no nonlinear advective terms)

=>

Necessary latitudinal derivatives (i.e. geopotential gradient) can be calculated in Fourier space.

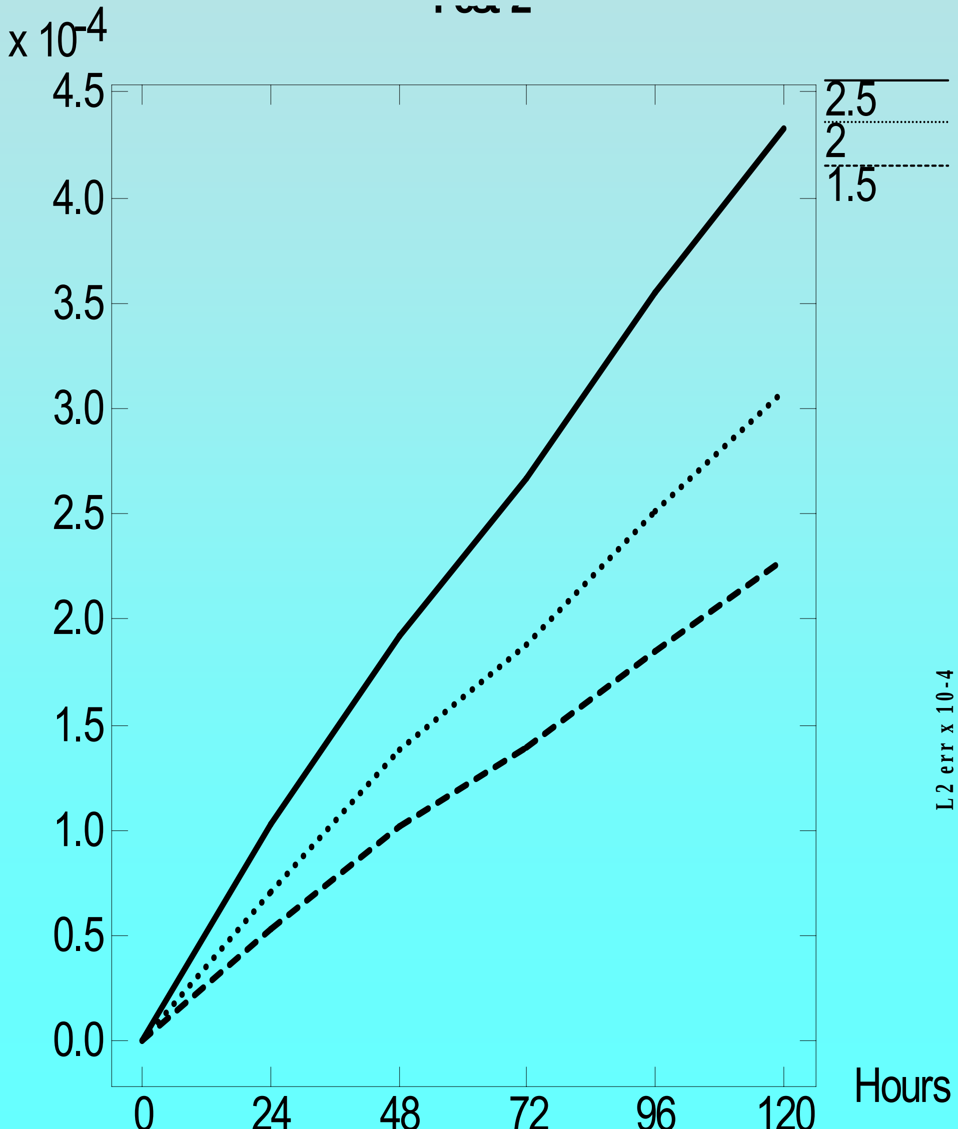
Number of gridpoints at each latitudinal circle should be suitable for FFT.

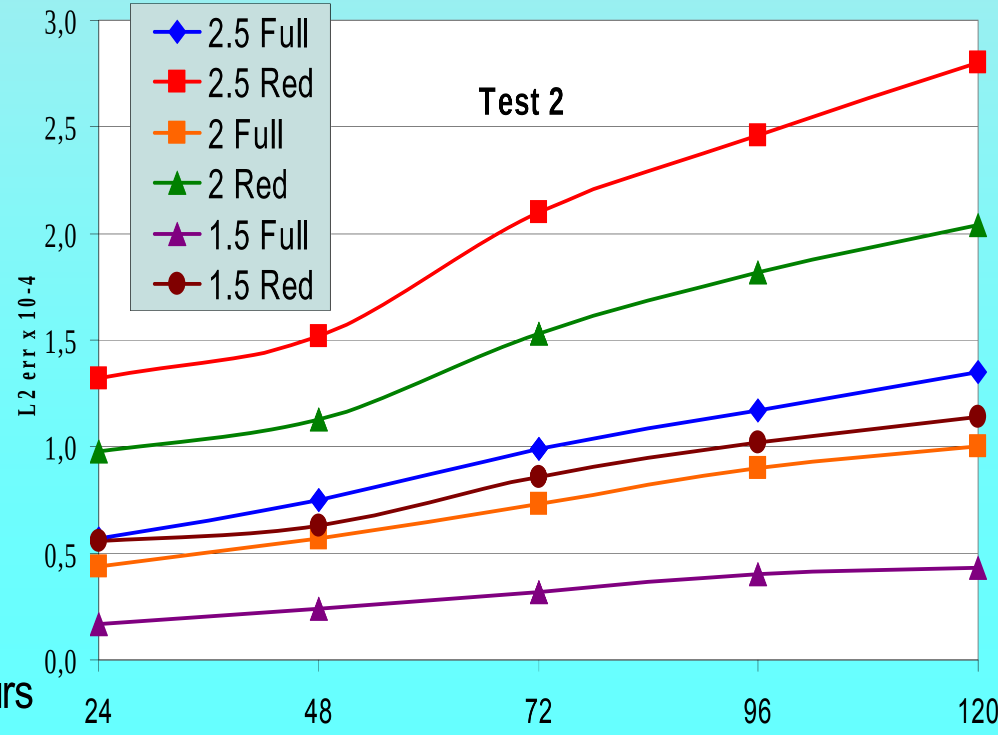
Currently, implemented in the shallow-water version; in the debugging phase for the 3D model.

Reduced grid: Details of implementation

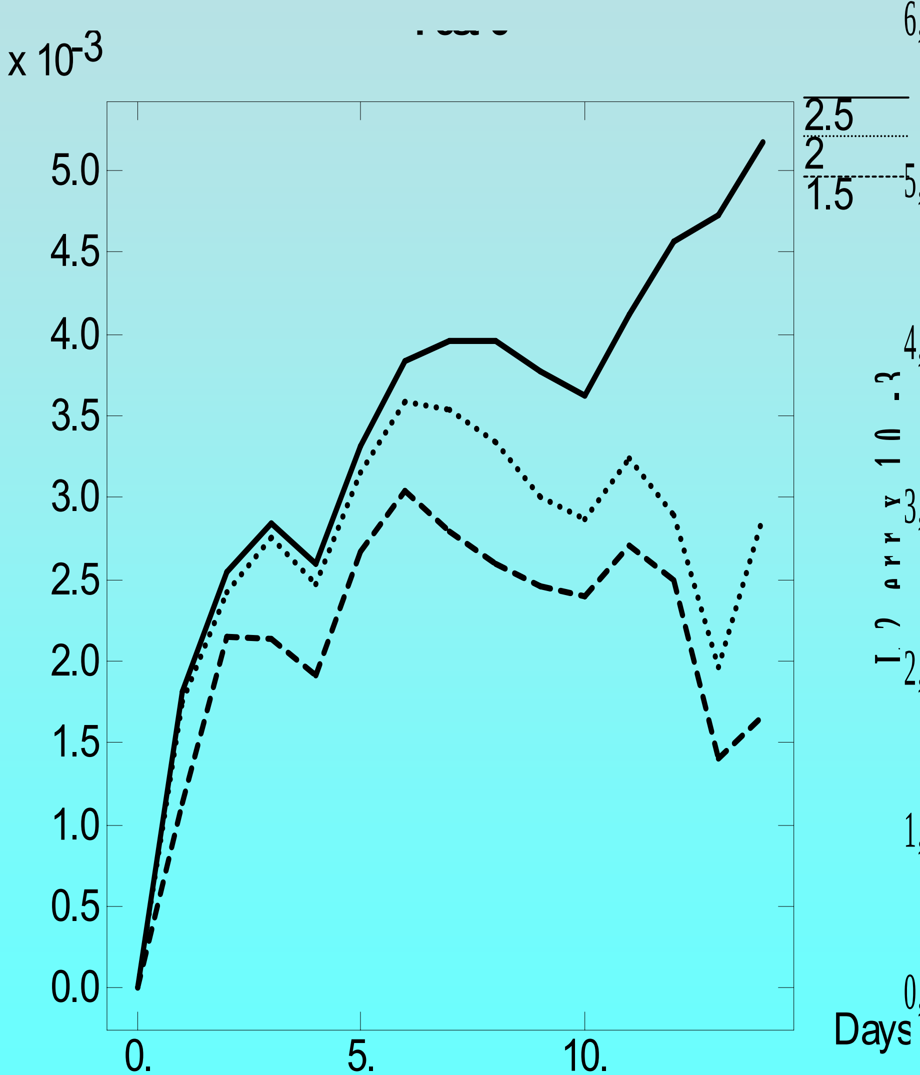
- Explicit terms of dynamics equations and SL advection are calculated on the reduced grid (most expensive parts for 3D model).
- Calculation of nonlinear terms in the grid-point space requires some averaging, which is difficult to calculate on the reduced grid. Nonlinear terms calculated on the full grid are then interpolated to reduced one.
- Calculations in Fourier space are carried out as in the case of the full grid (so far). At the end of these calculations, some variables are restored on the full grid, others – on the reduced one.

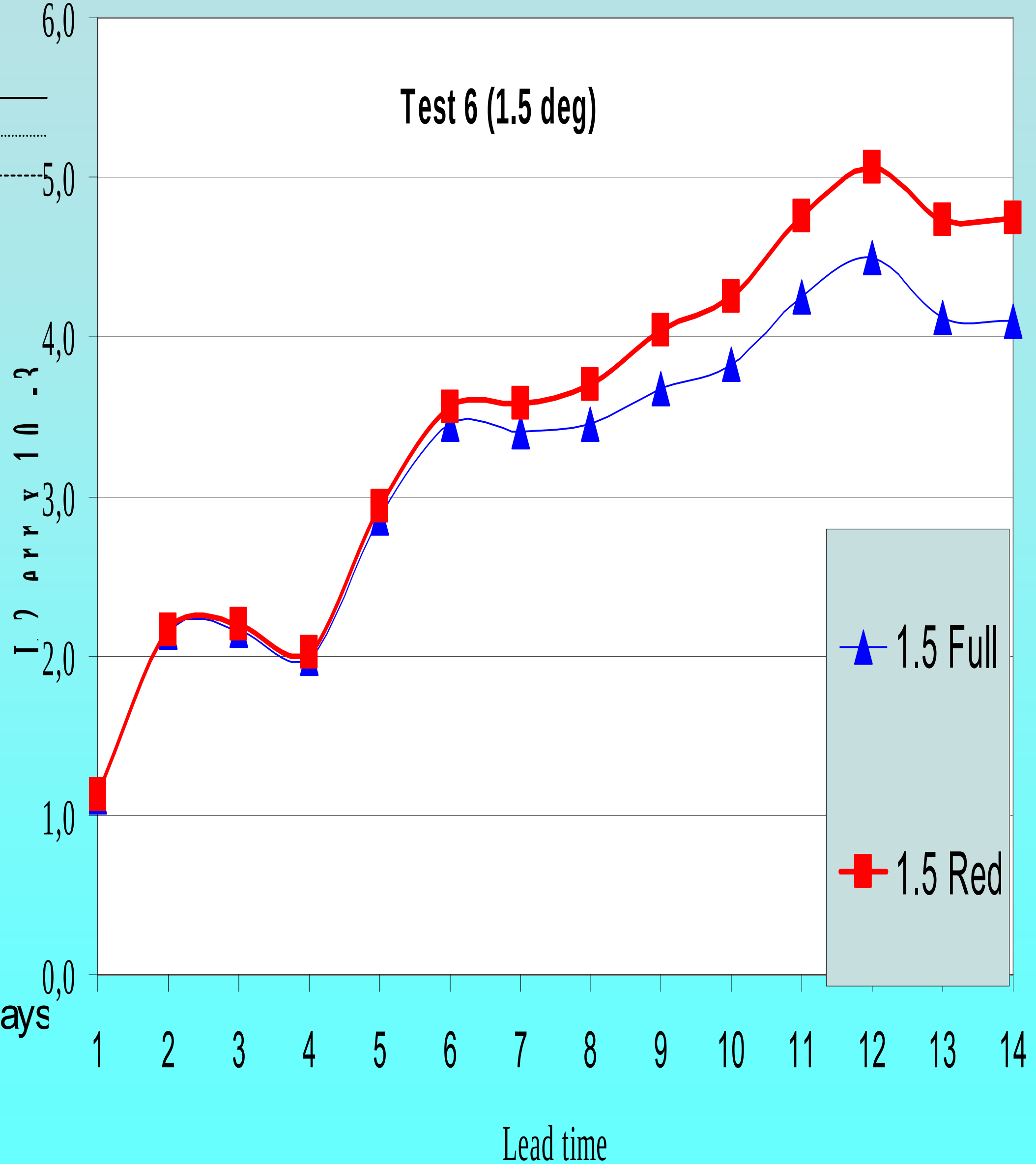
Test 2. Normalized global RMS height error. Results for 2002 version on the full grid (left), updated version on full and reduced grids (right).



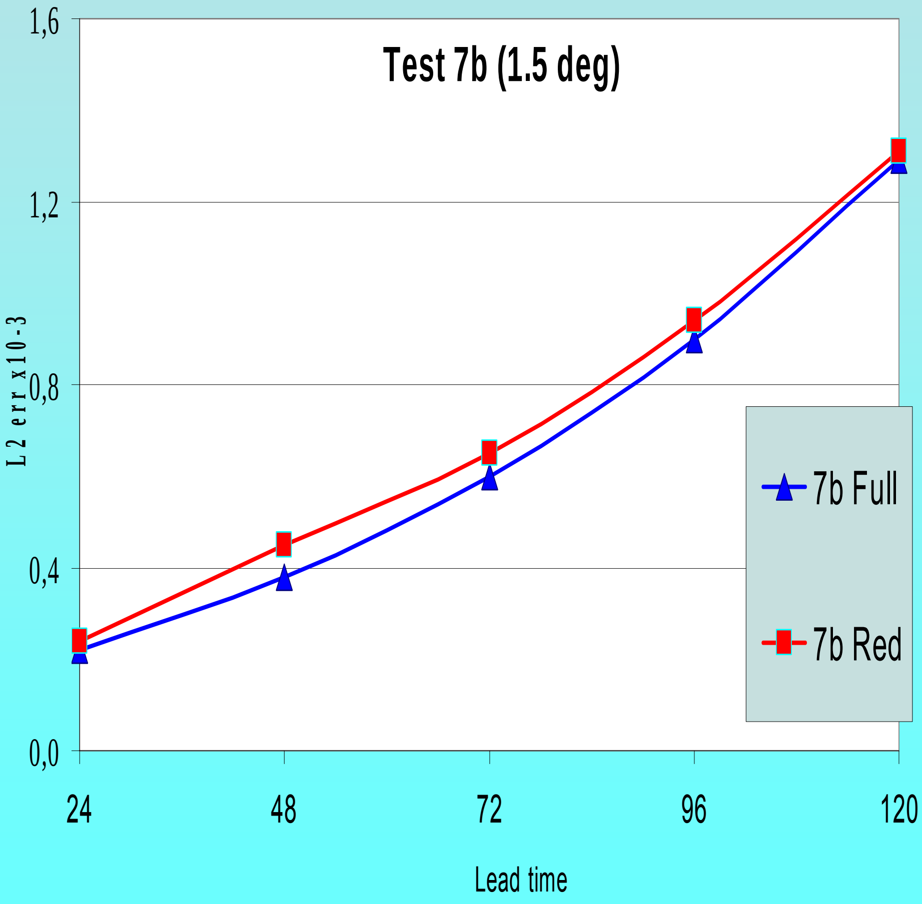


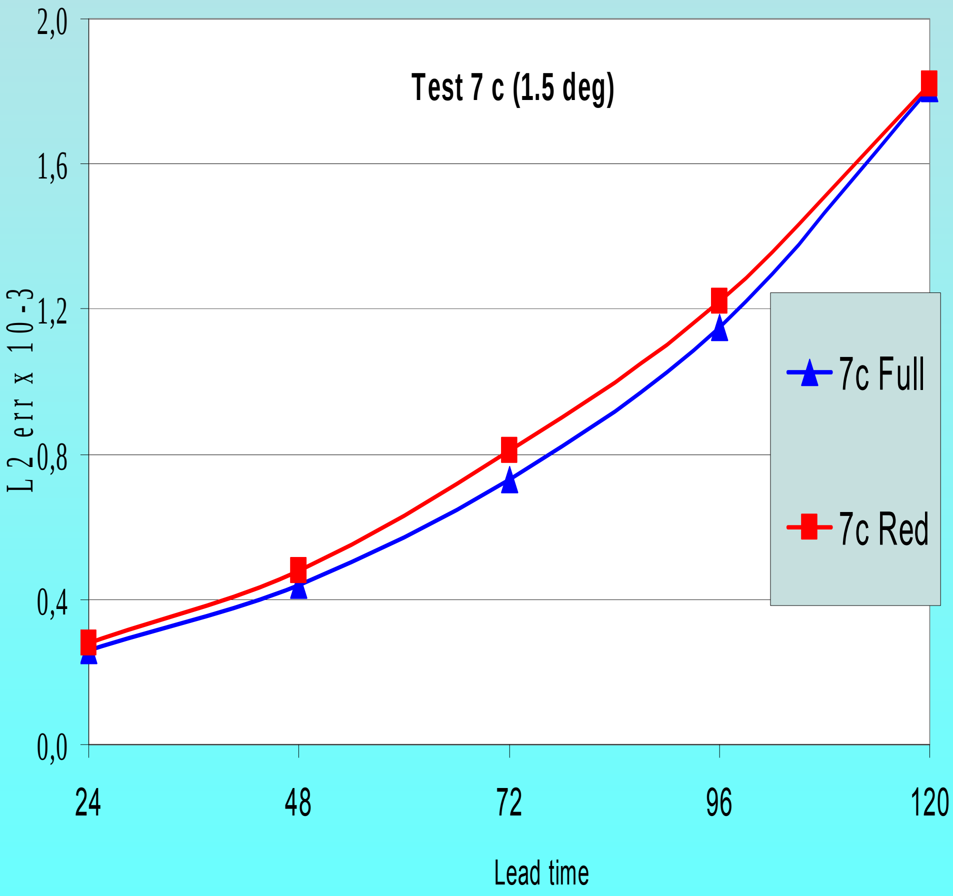
Test 6. Normalized global RMS height error. Results for 2002 version on the full grid (left), updated version on full and reduced grids (right).



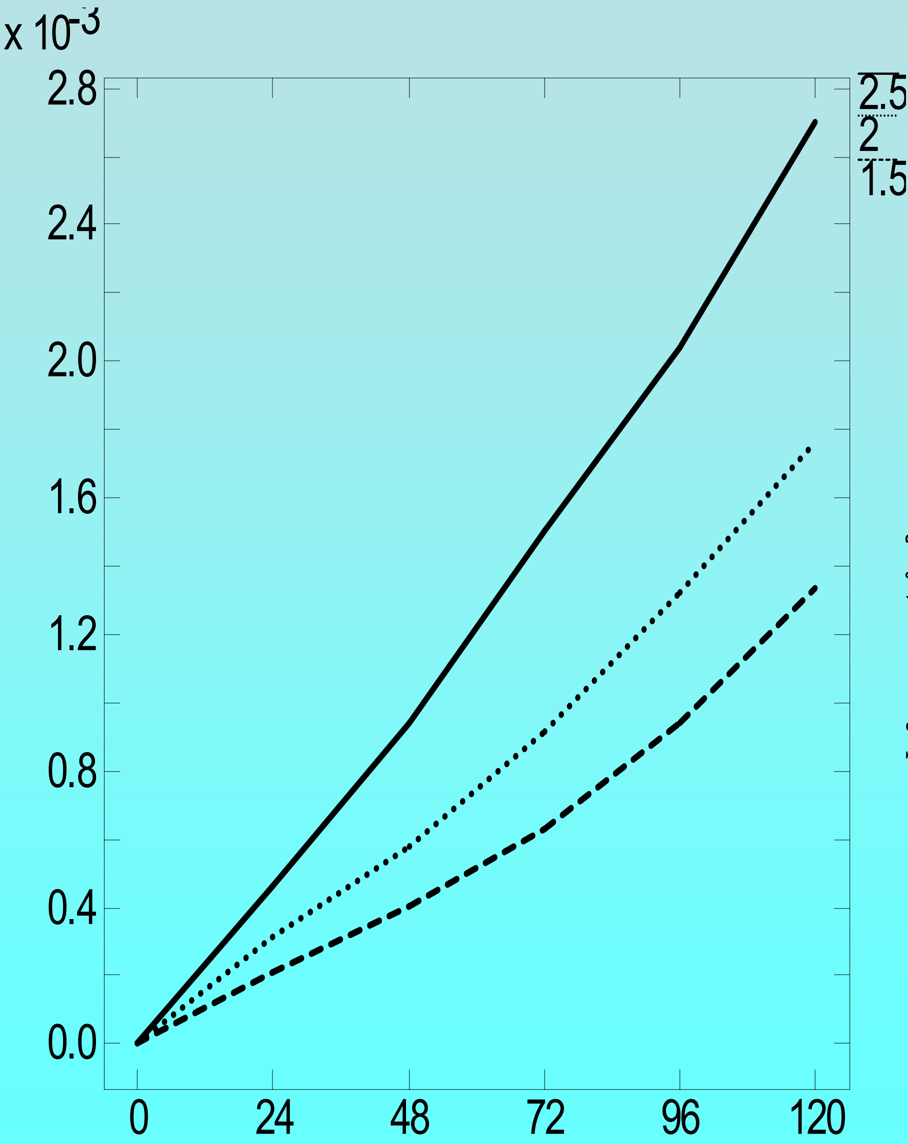


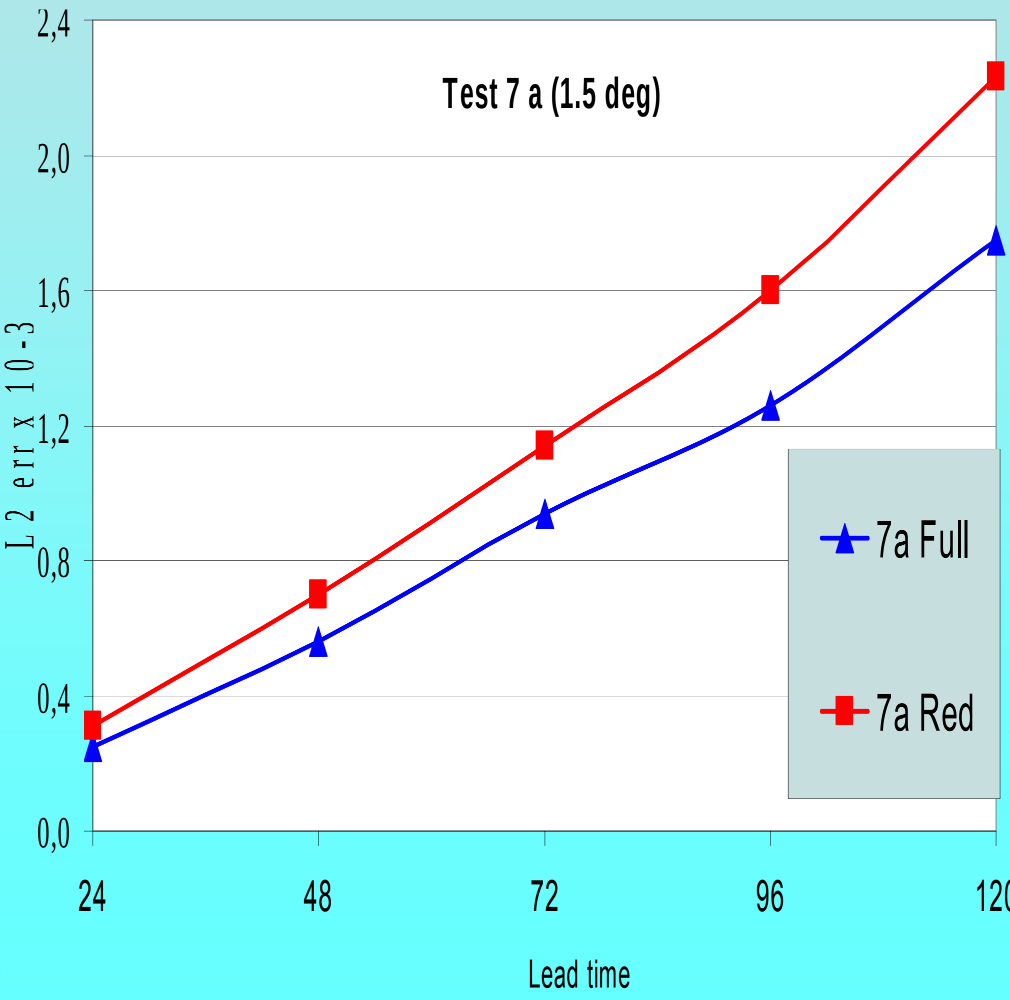
Tests 7b and 7c. Normalized global RMS height error. **Results for updated version on full and reduced grids.**



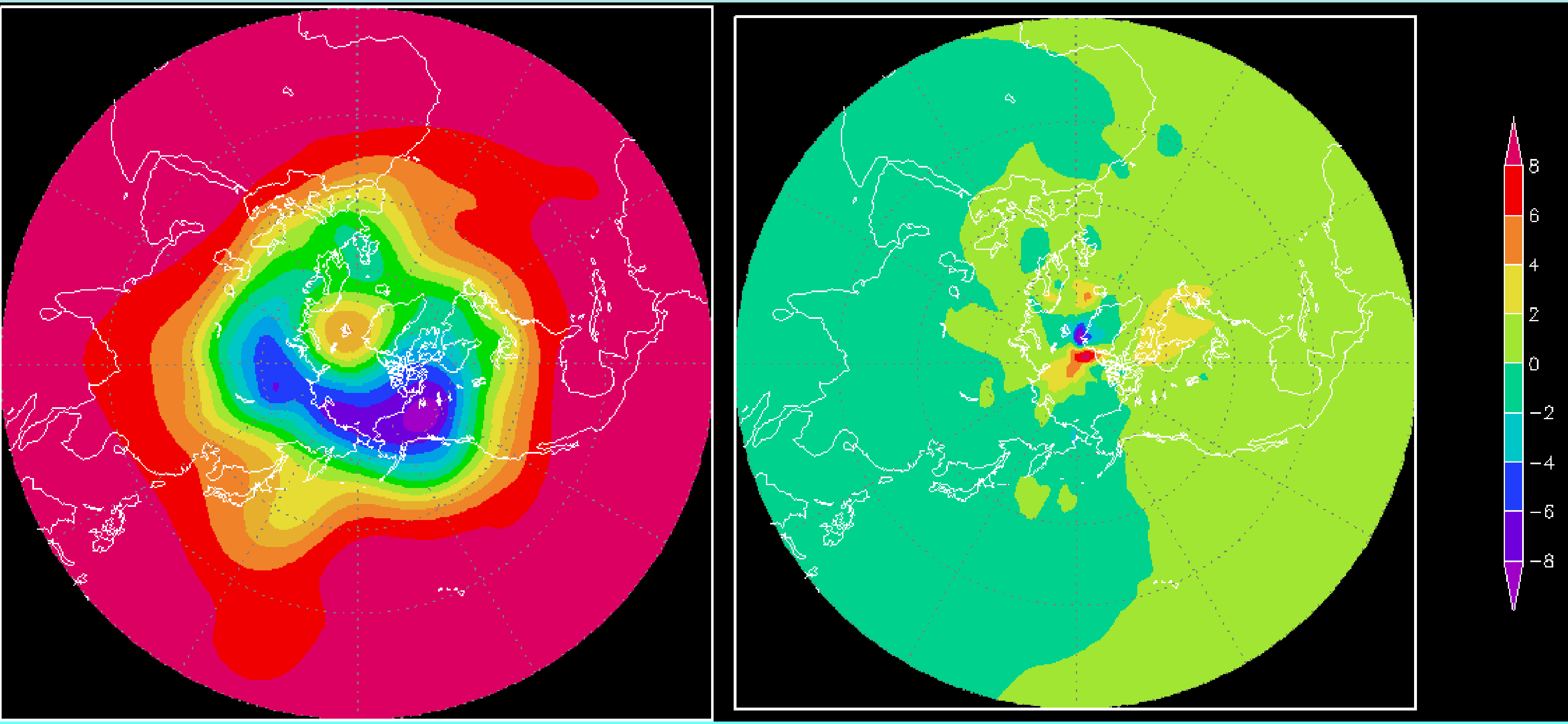


Test 7a. Normalized global RMS height error. Results for 2002 version on the full grid (left), updated version on full and reduced grids (right).





Solution (height) for Test 7a on the reduced grid (left) and difference with the full grid solution after 24hrs(right)



Constructing reduced grid for the SL-AV global model (R.Fadeev, Russian Meteorology and Hydrology,2006)

Idea: The accuracy of the SL scheme substantially depends on the interpolation procedure

Numerical approach

$f(d) = e^{-\mu d^2}, \quad \mu = -\dfrac{4\ln(\delta)}{a^2\beta^2}$ symmetric feature with

the center at $(0, \phi_0)$ and the angle size β

$\Phi(f, \phi_0, \beta, \{n_{\lambda,k}\}_{k=1}^{n_\phi})$ is the r. m. s. interpolation error

(Φ_0 is calculated on the regular grid)

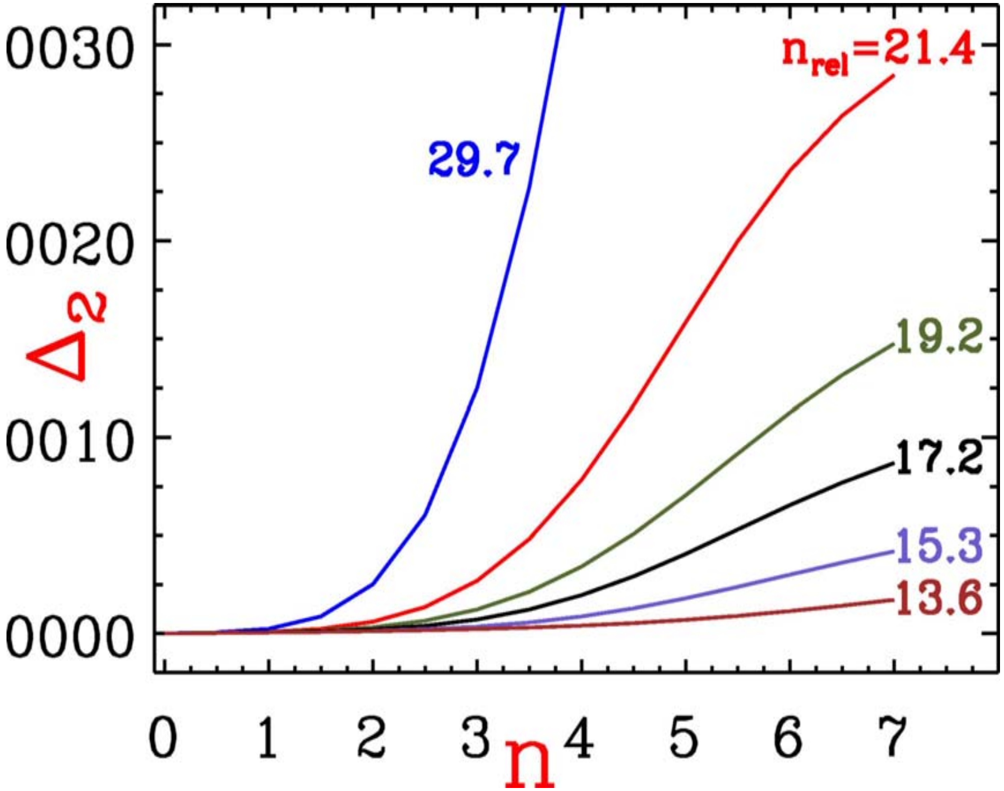
Criterion $\int_0^{\pi/2} |\Phi - \Phi_0|\, d\phi_0 \Big/ \int_0^{\pi/2} \Phi_0\, d\phi_0 \leq \epsilon_\Phi/100.$

Constructing reduced grid for the SL-AV global model

Smooth deformational flow

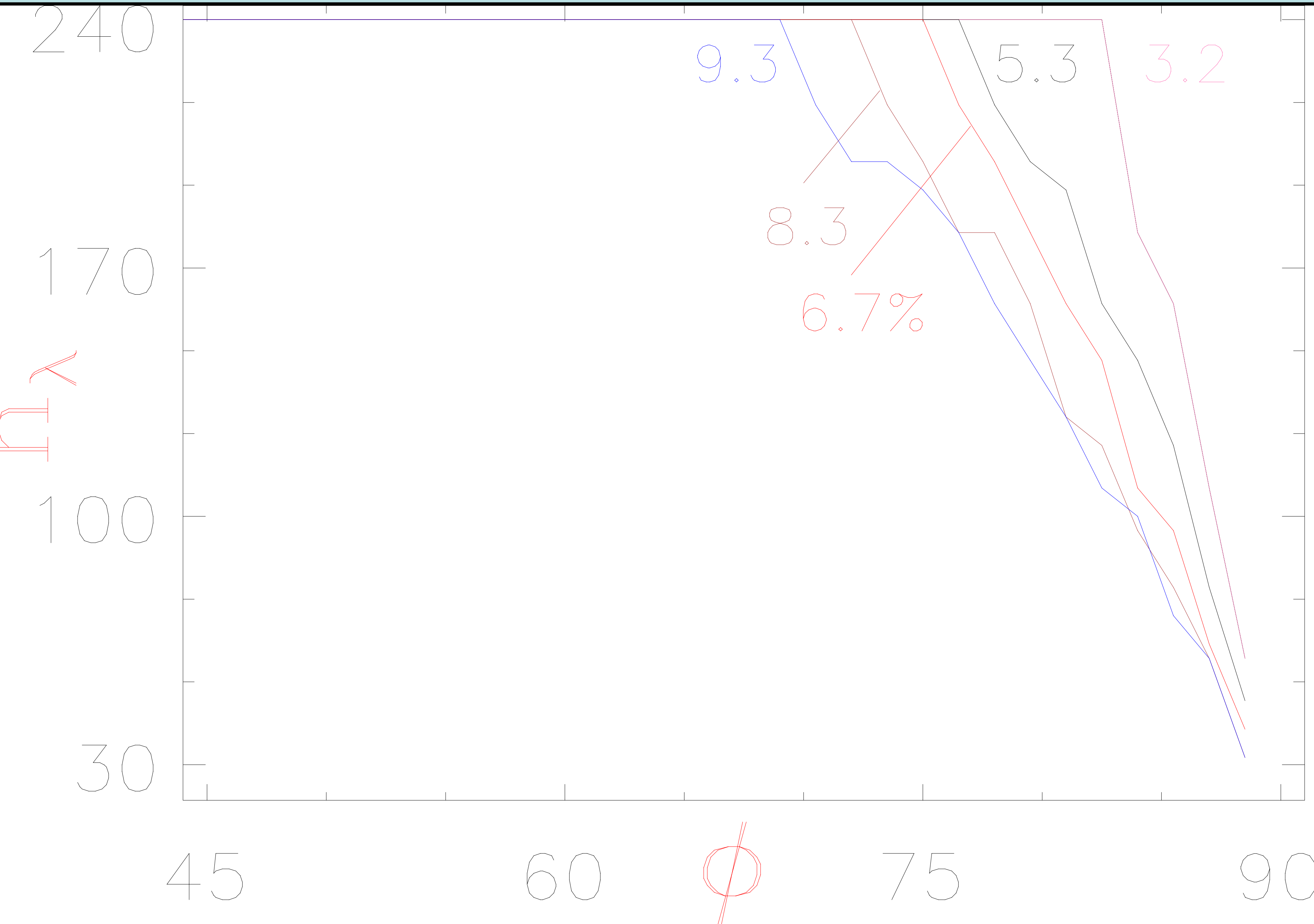
Doswell S. A. - J. Atmos. Sci., 1984, vol. 41, pp. 1242-1248.
Nair R., et. al. - Mon. Wea. Rev., 2002, vol. 130, pp. 649-667.

The normalized r. m. s. error of the numerical solution with respect to solution obtained on the regular grid

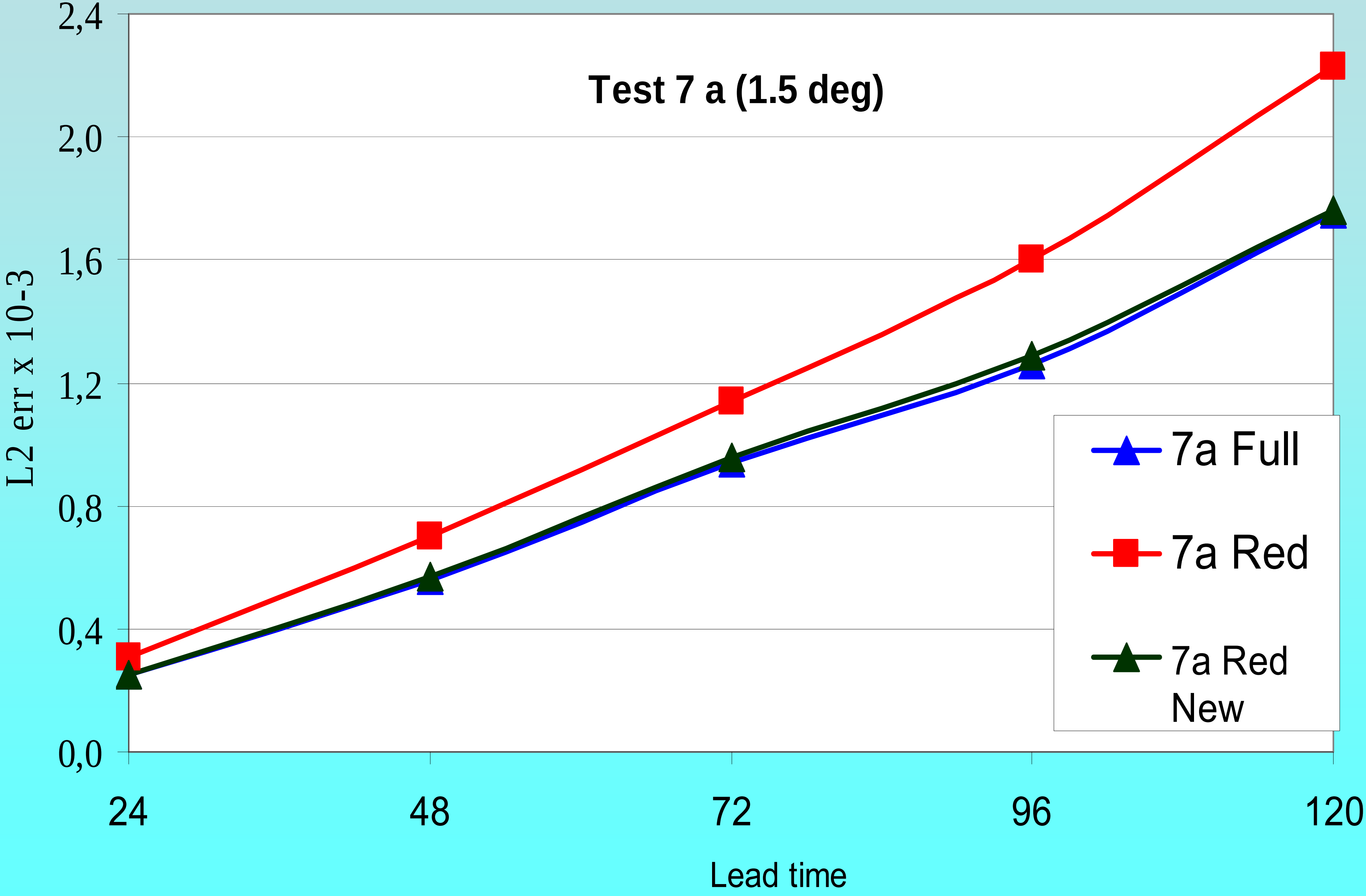


n is the number of rotations

Constructing reduced grid for the SL-AV global model



Test 7a. Normalized global RMS height error: full and two reduced grids



Development of the nonhydrostatic dynamical core

- First, quasianelastic semi-implicit semi-Lagrangian (SISL) formulation of Rööm et al (HIRLAM Rep. 65, http://hirlam.org) was implemented in the 2D (in the vertical plane) version of the SL-AV model (i.e. unstaggered grid; 4th order horizontal discretisation).
- It is planned to remove at least some of the simplifications of this formulation.

Assumptions of Room's formulation:

- quasi-anelastic approach *(simplified equation for the vertical velocity in particular),* 3-D divergence equals to zero
- non-hydrostatic component of geopotential is equal to zero at the surface.

General problem:

- This approach may lead to distorted representation of Rossby waves (T. Davies, A. Staniforth, N. Wood, J. Thuburn., Quart. J. Roy. Met. Soc., 2003)

Model equations

$$n\frac{\mathrm{dw}}{\mathrm{d}t} = g(n-1)$$

$$n\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -g\nabla z - n\mathbf{f}\times\mathbf{v}$$

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{\kappa T\omega}{p}$$

$$\frac{\mathrm{d}n}{\mathrm{d}t} + n\left(\nabla\cdot\mathbf{v} + \frac{\partial \mathrm{w}}{\partial p}\right)$$

$\rightarrow$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = -\frac{p^2}{H^2}\frac{\partial\phi}{\partial p} + \omega\frac{c_v\omega}{c_p p}$$

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\nabla(\varphi+\phi) - \mathbf{f}\times\mathbf{v}$$

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{\kappa T\omega}{p}$$

$$\nabla\cdot\mathbf{v} + \frac{\partial\omega}{\partial p} = 0$$

Density n is assumed to be 1 except for w eqn.

$$\omega = \frac{\mathrm{d}p}{\mathrm{d}t}, \qquad H = RT/g$$

$$\mathrm{w} = \frac{\mathrm{d}z}{\mathrm{d}t}, \qquad \Phi \equiv gz = \varphi + \phi$$

Pseudo-anelastic: White 1989,
Miller and Pearce 1974,
Salmon and Smith 1994

Discrete equations

$$\frac{\mathrm{d}_{\mathrm{k}+1/2}\omega}{\mathrm{d}t} = -(W\Delta\phi)_{\mathrm{k}+1/2} + (a^0_\omega)_{\mathrm{k}+1/2}$$

$$\frac{\mathrm{d}_{\mathrm{k}}\mathbf{v}}{\mathrm{d}t} = -\hat{\mathbf{G}}_{\mathrm{k}}(\varphi+\phi) - \mathbf{f}\times\mathbf{v}_k$$

$$\frac{\mathrm{d}_k T'}{\mathrm{d}t} = S_{\mathrm{k}}\overline{\omega}^{\sigma}_{\mathrm{k}}$$

$$\Delta_k\sigma\frac{\mathrm{d}_k\chi}{\mathrm{d}t} = -\frac{\Delta_k p}{p_s}D_k - \frac{\Delta_k(p_s\dot{\sigma})}{p_s} - \Delta_k\sigma\mathbf{v}_k\cdot\nabla(\ln\hat{p}_s)$$

$$\hat{\mathbf{G}}^{+}_{k}\cdot\mathbf{v} + \frac{\Delta_k\omega}{\Delta_k p} = 0$$

$$(a^0_\omega)_{\mathrm{k}+1/2} = \omega_{\mathrm{k}+1/2}\left(\frac{c_v\omega}{c_p p}\right)_{\mathrm{k}+1/2}$$

$$W_{\mathrm{k}+1/2} = \left(\frac{(gp)^2}{\overline{(RT)^2\Delta p}^{\sigma}}\right)_{\mathrm{k}+1/2}$$

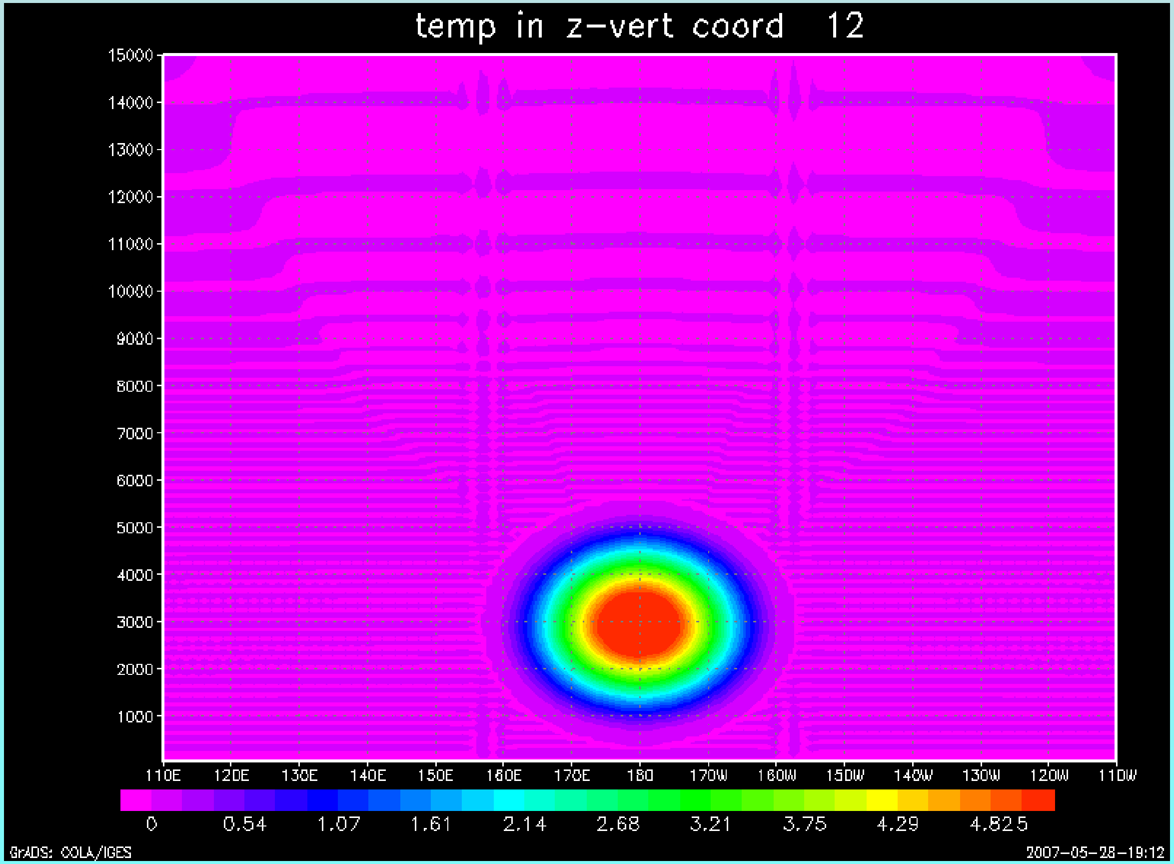
$$S_k = \left(\frac{\kappa T}{\overline{p}^{\sigma}}\right)_k - \frac{\Delta_k T^0}{\Delta_k p}$$

$$\hat{\mathbf{G}}_k\varphi = \nabla\phi_k - \frac{1}{\Delta p_k}\overline{\nabla p\Delta\varphi}^{\sigma}_k$$

$$\hat{\mathbf{G}}^{+}_{k}\cdot\mathbf{v} = \nabla\cdot\mathbf{v} - \frac{1}{\Delta p_k}\overline{\nabla p\cdot\Delta\mathbf{v}}^{\sigma}_k$$

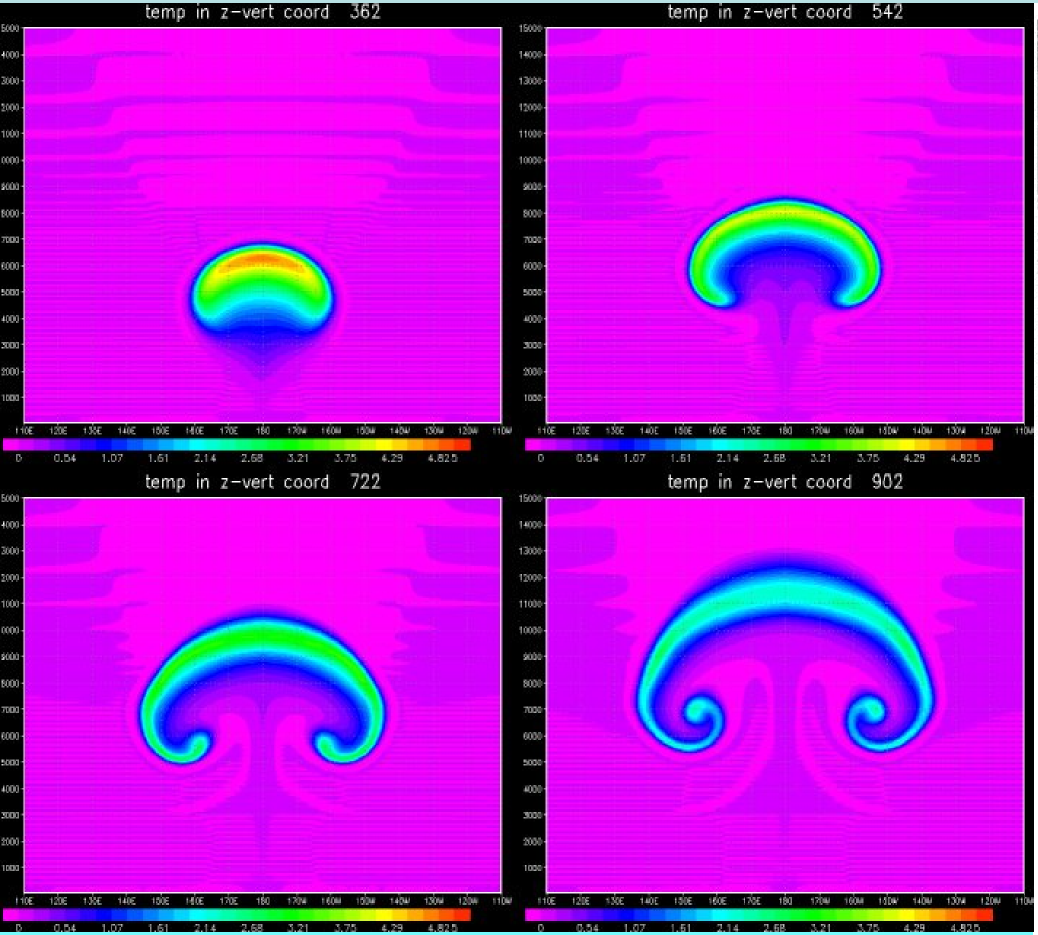
- Rigid lid at the top of the atmosphere. Free-slip at lower boundary
- Currently, Davies relaxation on lateral boundaries in 2D version

Warm-bubble test



Neutral atmosphere with the potential temperature of 300K.
An initial disturbance of the potential temperature has the bell shape centered at (0,2750m) and the radius of 2500m
The maximum value of deviation from the basic state was about 6.6K
dx = 100m; dz = 100m from surface to 8000m.
Time step = 1s.

Warm-bubble test



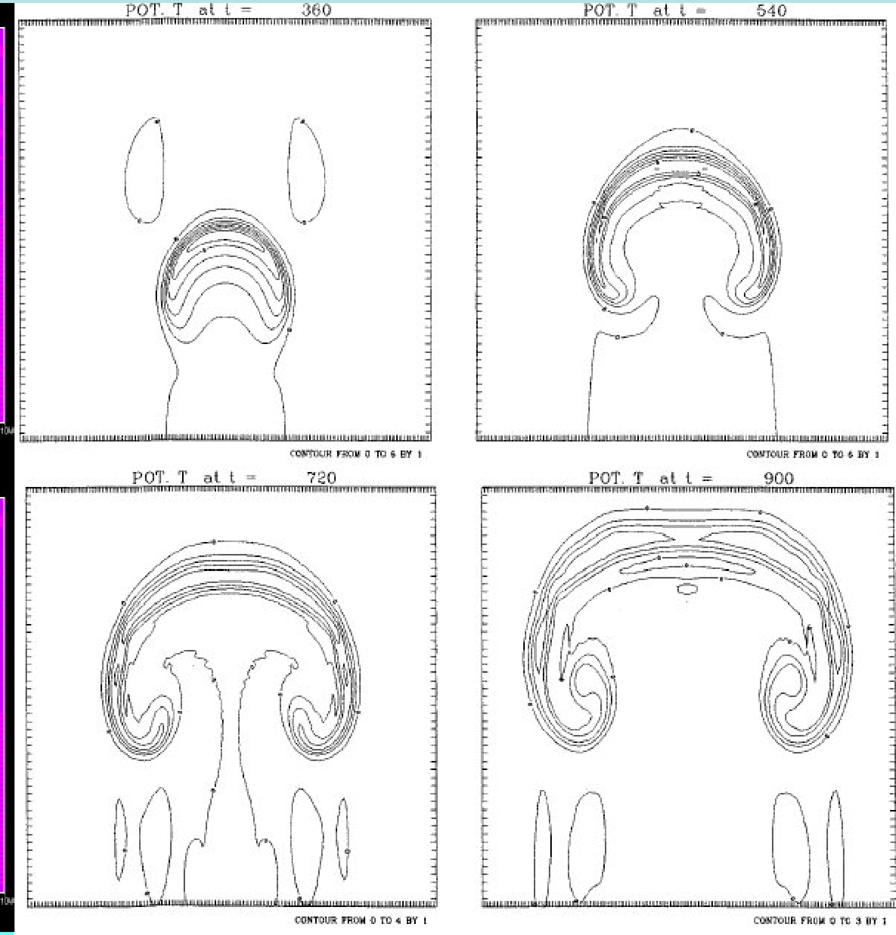


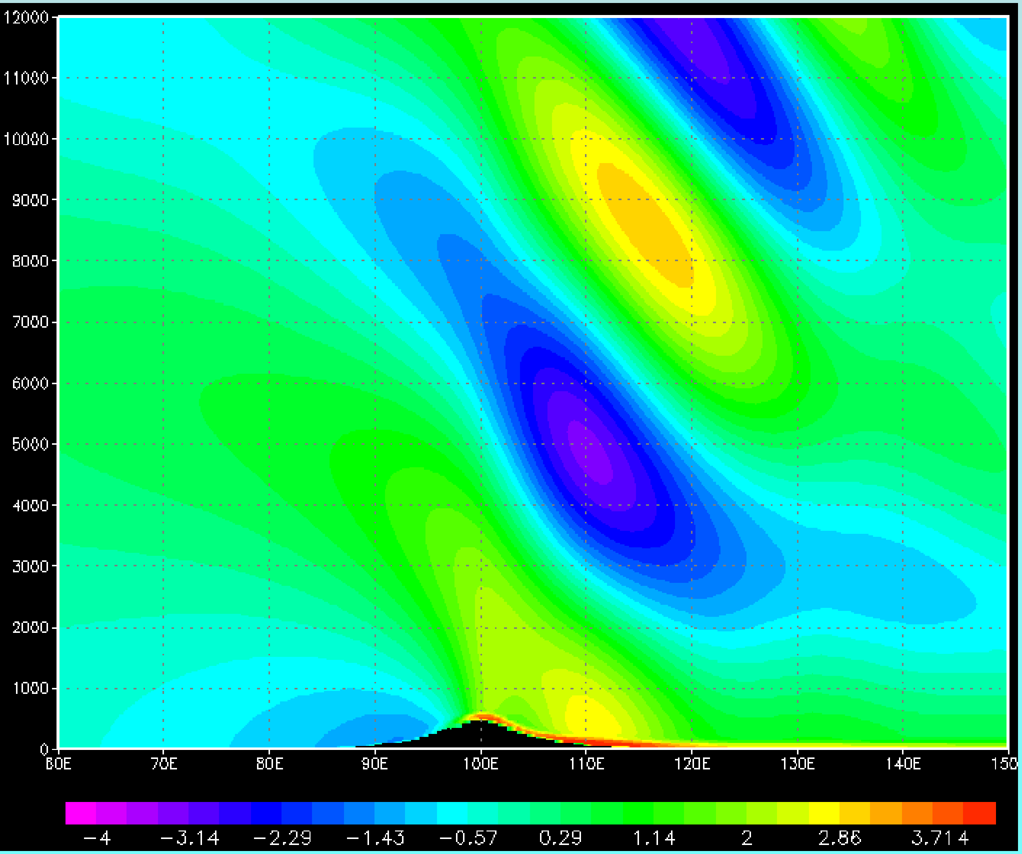
FIG. 3. The potential temperature deviation after 360, 540, 720, and 900 s (from upper left to lower right panel, respectively) in the warm bubble test. The area shown extends 16 km along the x axis, and from 0 to 13 200 m along the z axis. The contour interval is 1 K.

Setup is the same as in Janjic et al 2000,
but the time step is 1 s vs 0,3 s

WRF NMM from
Janjic et al. *An alternative approach to nonhydrostatic modeling* MWR 2000

Nonlinear mountain-wave test

Deviation of the horizontal velocity from the background value



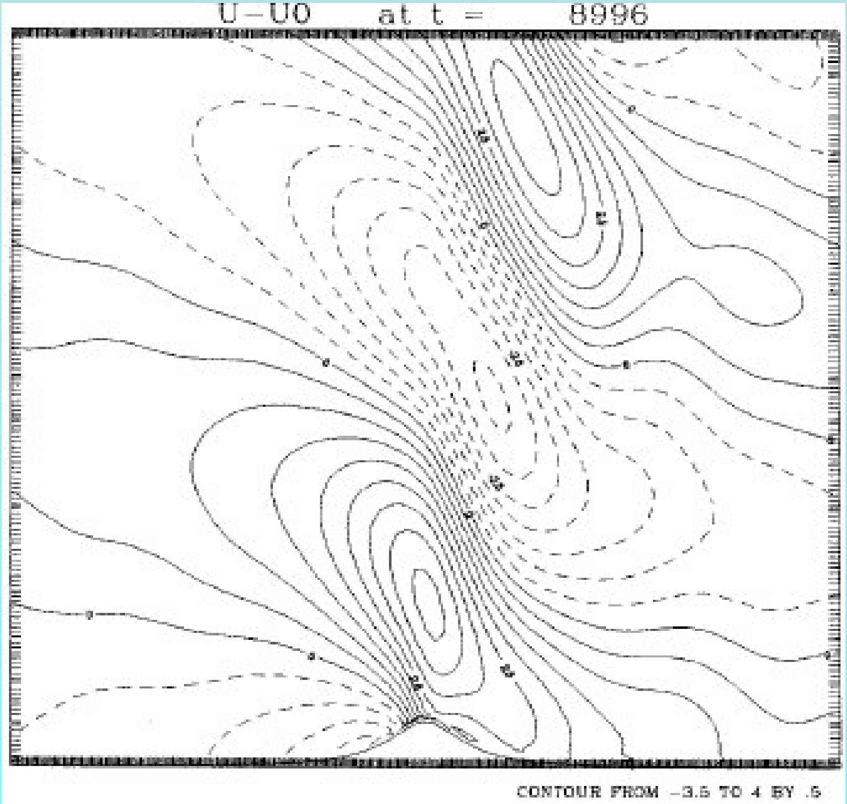


FIG. 4. The deviation of the horizontal wind from its basic-state uniform value (10 m s⁻¹) after 9000 s. The area shown extends 18 400 m on each side of the center of the mountain, and from 0 to 8000 m in the vertical. The contour interval is 0.5 m s⁻¹ and the dashed contours indicate negative values.

Bell shaped mountain height: 500 m, half-width 2000m. Background horizontal velocity: 10 m/s

Time step 20s, dx= 400m,
Brunt-Vaisala stability parameter N ~ 0.01 s-1
101 vertical layer, provides non-regular grid step in z coordinate (125m grid step in Janjic)

Right panel from:
Janjic et al. *An alternative approach to nonhydrostatic modeling* MWR 2000

Conclusions

- SL-AV vorticity-divergence formulation on the unstaggered grid is capable to produce nonhydrostatic solutions.
- Reduced grid implementation will enable high horizontal resolution in future.

Future work

- Modifications of model equations; study of their impact.
- Possible modification of model vertical coordinate.
- Implementation of 3D version (in particular, semi-implicit solver)

Thank you for attention!