ICON – the Next Generation General Circulation Model for NWP and Climate Research in Germany

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Objectives of MPI-M and DWD

DWD (NWP):

- combined regional and global prediction system for the short range to the seasonal prediction time scale
- improvement of numerical properties in comparison to current models
- enhanced use of satellite data
- synergies from cooperation with MPI-M:
  - in physical parametrisations
  - in experience in modelling of the stratosphere
  - in the use of the ocean model of the MPI-M for seasonal forecasts

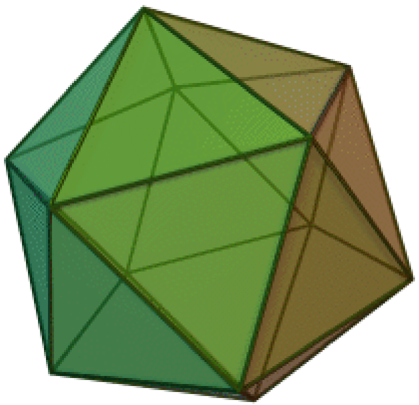
MPI-M (climate simulations):

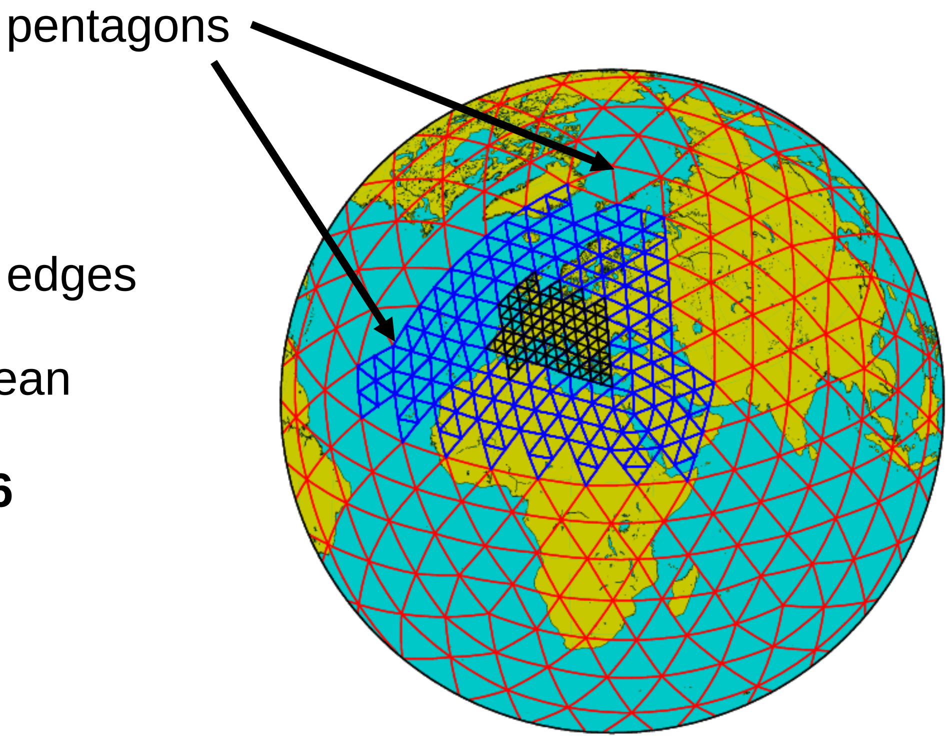
- numerical conservation properties for mass, tracer, energy, momentum
- common grid structure for ocean and atmospheric model and therefore an improved interaction of essential parts of the Earth system model
- synergies from cooperation with MPI-M:
  - in data assimilation
  - in continuous evaluation and optimization in operational use

Icosahedron

12 knots

20 equilateral triangles



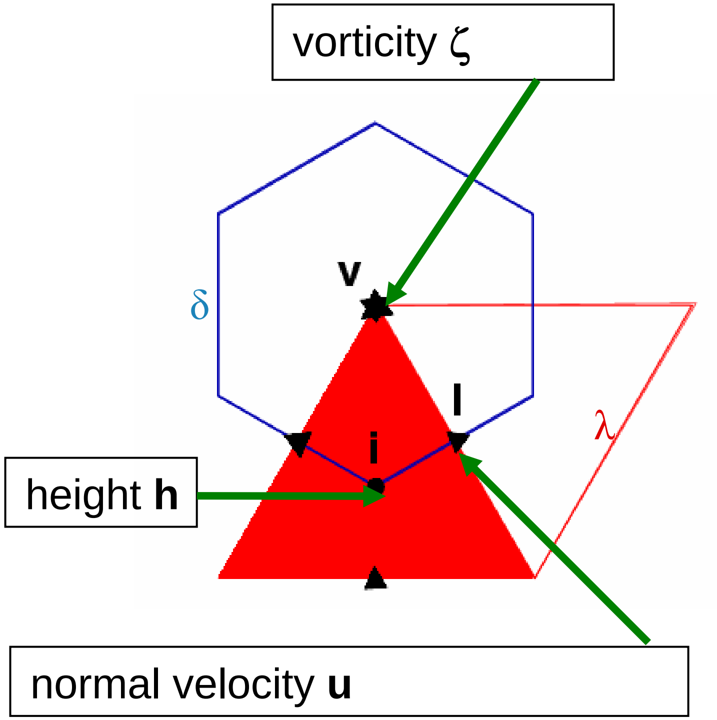


Example for local grid refinement

- Quasi-uniform base grid:
  1 icosahederon edge → 6 cell edges

- 2-step refinement in an European region by division of edges:
  1 triangle → **4 triangles** → **16 triangles**

Basic operators on the C-grid



Application of integral theorems

Divergence for triangles: Gauss theorem

$$\nabla \cdot \vec{v}\,\big|_i = \frac{1}{A_{triangle}} \sum_{l=1}^{3} u_l \lambda_l$$

The velocity points outward.

Rotation for hexagons: Stokes theorem

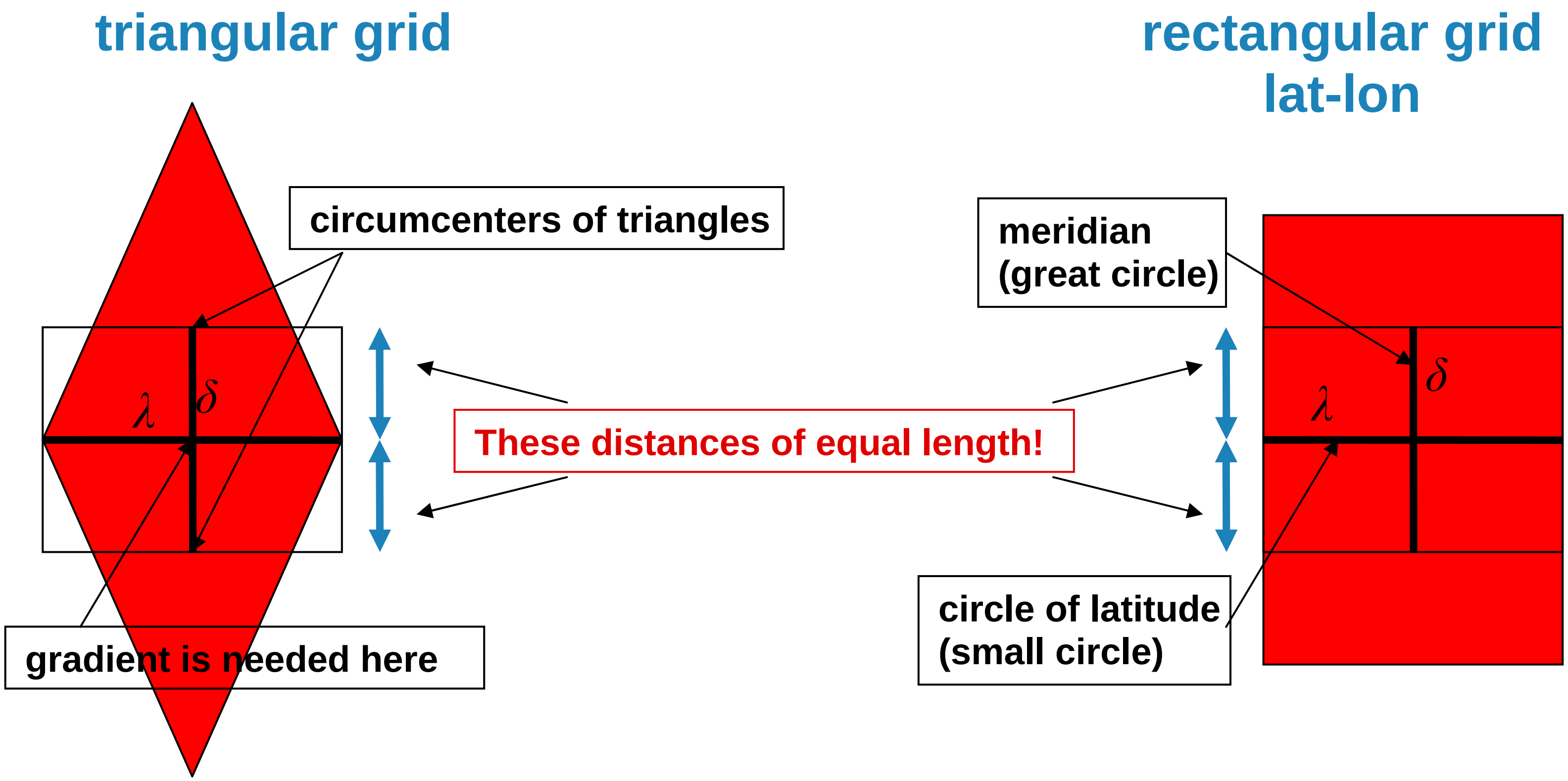
$$\vec{k} \cdot \nabla \times \vec{v}\,\big|_v = \frac{1}{A_{hexagon}} \sum_{l=1}^{6} u_l \delta_l$$

The velocity is given counterclockwise.

Gradient at an edge: given by the relation

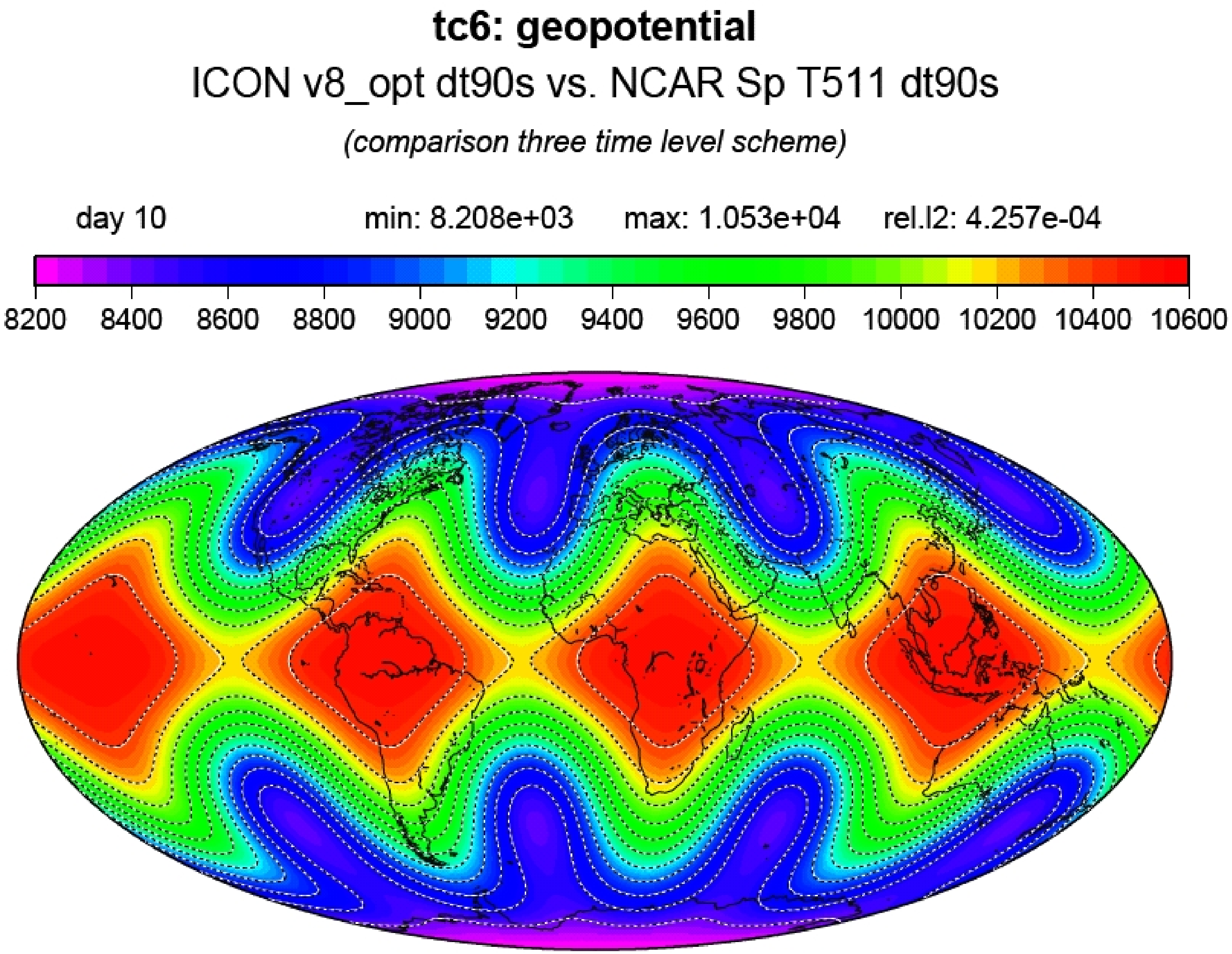
$$\nabla \cdot (\psi \rho \vec{v}) - \psi \nabla \cdot (\rho \vec{v}) = \rho \vec{v} \cdot \nabla \psi$$

Grid optimization



- Initially, all distances are great circles on the globe.
- Second, apply a Heikes/Randall grid optimization as a variational optimization of the distances δ.
- Last, add small circle optimization for the arc λ between two vertices.

Shallow water test case (1)

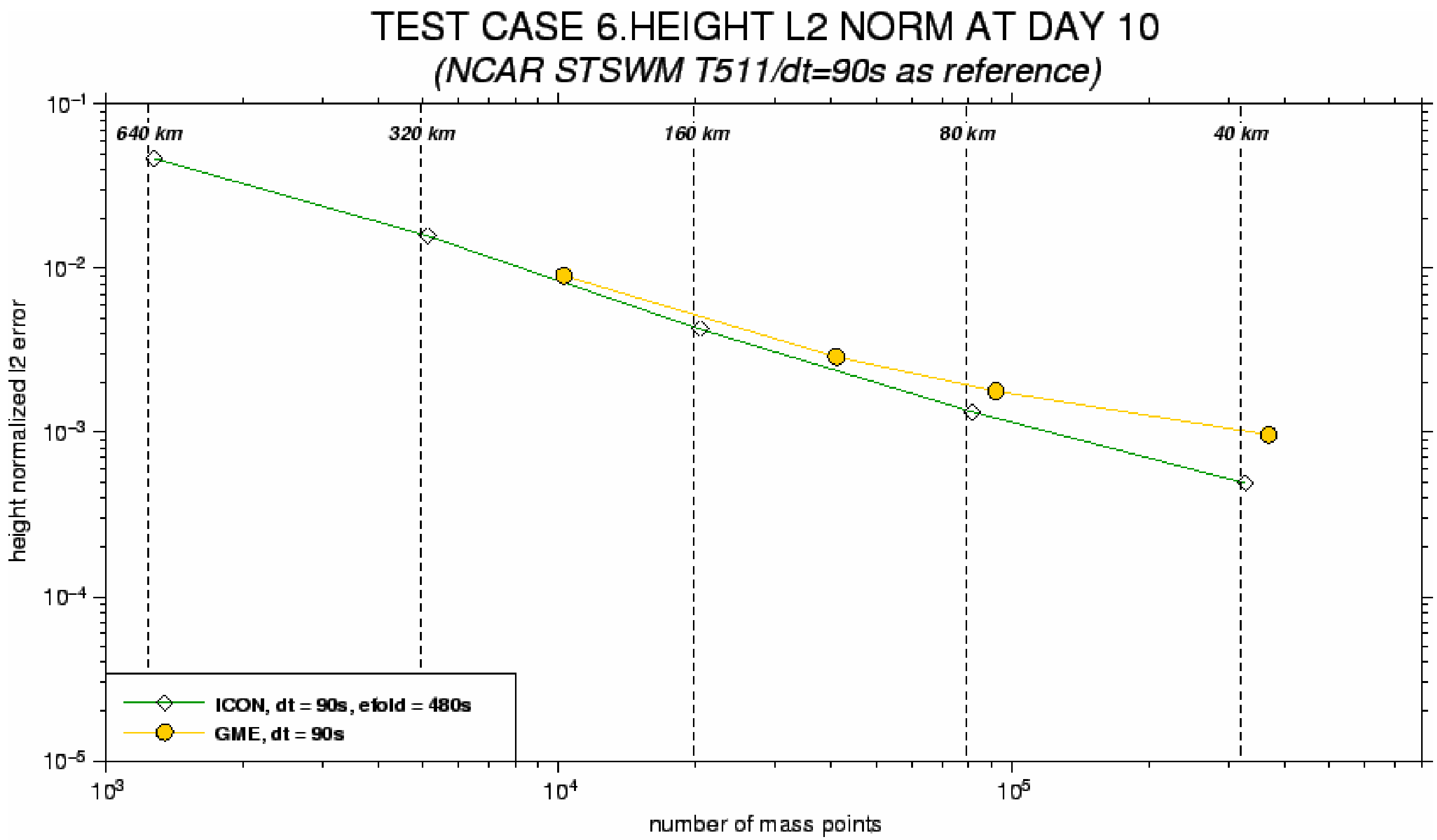


Rossby-Haurwitz-wave at day 10

Isolines: NCAR reference, Colors: ICON

rel. l_2-error: $4.257 * 10^{-4}$

Shallow water test case (2)



Comparison of GME's and ICON's three time level schemes
GME: ni32, ni64, ni96, ni192;
ICON: refinement levels 4 to 8, optimized grids;

Rossby-Haurwitz-wave

L2-norm to NCAR-reference

ICOSWP and **GME**

Concepts for conservation properties for non-hydrostatic models (1)

Conservable quantities in an ideal fluid are:

- **Energy** $$H = \iiint_V \left(\rho\varphi + \rho \frac{\vec{v}^2}{2} + \rho c_v T \right) d\tau$$
- **Mass** $$M = \iiint_V \rho d\tau$$
- **Entropy** $$\Theta = \iiint_V \rho\theta d\tau$$

(corresponds to Lagrangian conservation of potential temperature)

- **Vortex charge** $$P_a = \iiint_V \rho \frac{\vec{\omega}_a \cdot \nabla\theta}{\rho} d\tau$$

(corresponds to Lagrangian conservation of Ertel's potential vorticity)

Concepts for conservation properties for non-hydrostatic models (1)

Energy as a constituting and conserved dynamical quantity:

Hamiltonian dynamics with antisymmetric Poisson brackets {F,H}

Conservation of more dynamical quantities (Casimirs) X:

Nambu dynamics with antisymmetric Nambu brackets {F,X,H}

$$\boxed{\frac{\partial F}{\partial t} = \{F, h_a, H\} + \{F, M, H\} + \{F, \Theta, H\}}$$

$$\{F, h_a, H\} = -\iiint_V \frac{1}{\rho}\frac{\delta F}{\delta \vec{v}} \cdot \frac{\delta h_a}{\delta \vec{v}} \times \frac{\delta H}{\delta \vec{v}} d\tau$$

$$\{F, M, H\} = -\iiint_V \left(\frac{\delta F}{\delta \vec{v}} \frac{\delta M}{\delta \rho} \cdot \nabla \frac{\delta H}{\delta \rho} + \frac{\delta F}{\delta \rho} \nabla \cdot \left(\frac{\delta M}{\delta \rho} \frac{\delta H}{\delta \vec{v}} \right) \right) d\tau$$

$$\{F, \Theta, H\} = -\iiint_V \left(\frac{\delta F}{\delta \vec{v}} \frac{\delta \Theta}{\delta \rho} \cdot \nabla \frac{\delta H}{\delta (\rho\theta)} + \frac{\delta F}{\delta (\rho\theta)} \nabla \cdot \left(\frac{\delta \Theta}{\delta \rho} \frac{\delta H}{\delta \vec{v}} \right) \right) d\tau$$

$$\frac{\partial \vec{v}}{\partial t} = -\vec{\omega}_a \times \vec{v} - \nabla \left(\frac{\vec{v}^2}{2} + \Phi \right) - \theta \nabla (c_p \pi)$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{v})$$

$$\frac{\partial \rho\theta}{\partial t} = -\nabla \cdot (\rho \vec{v} \theta)$$

with *F* a functional (e.g. delta functional) of the prognosed quantity

and h_a the absolute helicity $$h_a = \frac{1}{2} \iiint_V \vec{\omega}_a \cdot \vec{v}_a d\tau$$

Nambu dynamics with dissipative source functions

$$\frac{\partial F}{\partial t} = \{F, h_a, H\} + \{F, M, H\} + \{F, \Theta_v, H\} + \left(F, \vec{R}\right) + \left(F, Q^{\theta_v}\right)$$

$$\left(F, \vec{R}\right) = \iiint_V \frac{\delta F}{\delta \vec{v}} \cdot \vec{R} d\tau \qquad \vec{R} = -\frac{1}{\rho} \nabla \cdot \overline{\rho \vec{v}'' \vec{v}''}$$

$$\left(F, Q^{\theta_v}\right) = \iiint_V \rho \frac{\delta F}{\delta \rho \theta_v} Q^{\theta_v} d\tau$$ diabatic source terms

together with the H_2O budget equations

$$\frac{\partial \rho q_i}{\partial t} = -\nabla \cdot \left(\rho \vec{v} q_i + \vec{J}_i + \overline{\rho \vec{v}'' q_i''}\right) + S_i$$

Non-dissipativer
Nambu bracket part

realistic scale interaction

mass conservation

Dissipative
source function part

control of
dissipative processes

production of entropy

Energy conservation is only achieved with both parts!
Numerical operators do no sources or sinks for entropy or energy by their own!

Numerically consistent spatial operators (1)

To keep antisymmetric structure of the Nambu brackets is a must during the discretisation process.
It explains the success of Salmon's SWE and the Arakawa Jacobian.

Simple recipe...
...approximate the integrals a sums over grid boxes!
...preserve antisymmetry of Nambu brackets
(possibly by 1/3 weights on each permutation)
...and be careful!

$$h_a = \frac{1}{2}\iiint_V \vec{\omega}_a \cdot \vec{v}_a \, d\tau$$

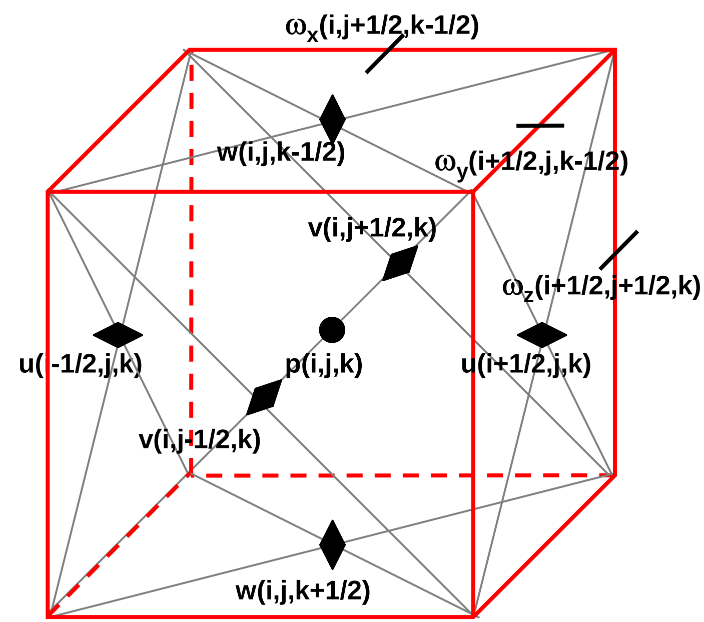
$$\Rightarrow \frac{1}{2}\sum_{i,j,k} u_{a(i+1/2,j,k)} \overline{\vec{\omega}_{a,x(i,j+1/2,k)}}^{x,y,z} V_{u(i+1/2,j,k)+\ldots}$$

$$H = \iiint_V \rho\left(\frac{1}{2}\vec{v}^2 + \Phi + c_v T\right) d\tau$$

$$\Rightarrow \sum_{i,j,k} \frac{1}{2}\overline{\rho_{(i,j,k)}}^{x} u^2_{(i+1/2,j,k)} V_{u(i+1/2,j,k)} + \ldots$$

$$\{F, h_a, H\} = -\iiint_V \frac{1}{\rho}\frac{\delta F}{\delta \vec{v}} \cdot \left(\frac{\delta h_a}{\delta \vec{v}} \times \frac{\delta H}{\delta \vec{v}}\right) d\tau$$

$$\Rightarrow -\sum_{i,j,k} \frac{1}{\rho_{(i,j,k)}} \overline{\frac{\delta F}{\delta \vec{v}}}^{x} \left(\overline{\rho w_{(i,j,k+1/2)}}^{z} \overline{\overline{\omega_{y(i,j+1/2,k)}}^{x,y,z}}^{y} - \overline{\rho v_{(i,j+1/2,k)}}^{y} \overline{\overline{\omega_{z(i,j,k+1/2)}}^{x,y,z}}^{z} \right) V_{(i,j,k)} + \ldots$$



Numerically consistent spatial operators (2)

Result for the **v**x**ω** term (the helicity bracket):

$$\frac{\partial u}{\partial t} = \overline{\overline{v}^{y}\,\overline{\overline{\omega_z}^{xyz}}^{z}}^{x} - \overline{\overline{w}^{z}\,\overline{\overline{\omega_y}^{xyz}}^{y}}^{x}$$

$$\frac{\partial v}{\partial t} = \overline{\overline{w}^{z}\,\overline{\overline{\omega_x}^{xyz}}^{x}}^{y} - \overline{\overline{u}^{x}\,\overline{\overline{\omega_z}^{xyz}}^{z}}^{y}$$

$$\frac{\partial w}{\partial t} = \overline{\overline{u}^{x}\,\overline{\overline{\omega_y}^{xyz}}^{y}}^{z} - \overline{\overline{v}^{y}\,\overline{\overline{\omega_x}^{xyz}}^{x}}^{z}$$

Comparing the result with Sadourny‘s (1975) enstrophy conserving scheme, a similarity exists if double averaging is dropped. (But care: The Nambu brackets of the SW system are completely different!)

Result for the other brackets (1-dimensional example)

$$\frac{\partial u_{i+1/2}}{\partial t} = -\frac{1}{4\Delta x}\left(u_{i+3/2}^{2} - u_{i-1/2}^{2}\right) - c_p \overline{\theta}^{x}_{v,i+1/2} \frac{1}{\Delta x}\left(\pi_{i+1/2} - \pi_{i-1/2}\right)$$

$$\frac{\partial \rho_i}{\partial t} = -\frac{1}{\Delta x}\left(\overline{\rho}^{x}_{i+1/2} u_{i+1/2} - \overline{\rho}^{x}_{i-1/2} u_{i-1/2}\right)$$

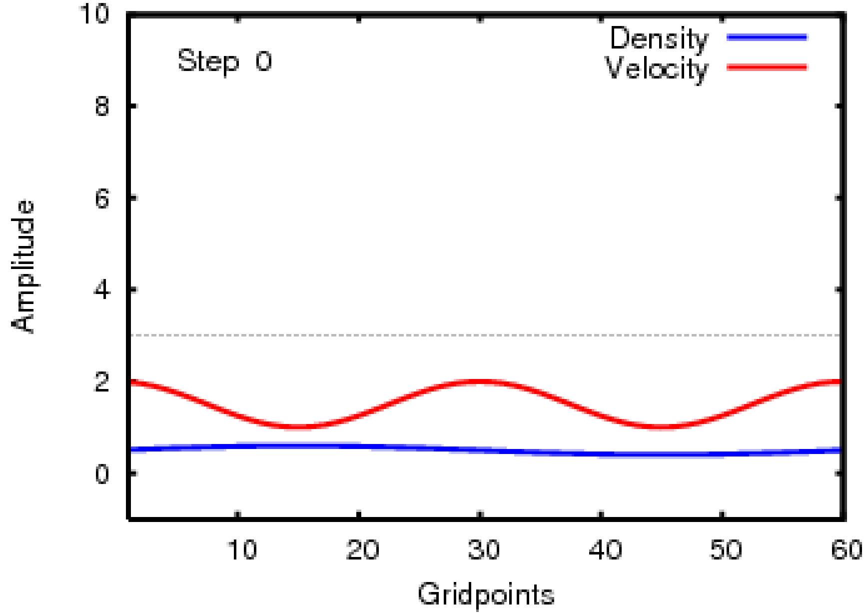
$$\frac{\partial \rho\theta_i}{\partial t} = -\frac{1}{\Delta x}\left(\overline{\rho}^{x}_{i+1/2} u_{i+1/2} \overline{\theta}^{x}_{i+1/2} - \overline{\rho}^{x}_{i-1/2} u_{i-1/2} \overline{\theta}^{x}_{i-1/2}\right)$$

The kinetic energy is obtained by first squaring and subsequent averaging. The mass flux is multiplied with θ.

General rule: guarantee for $\vec{A}\cdot\left(\vec{B}\times\vec{C}\right)$ to remain antisymmetric.

guarantee the rule $\nabla\cdot\left(\rho\vec{v}\psi\right) = \psi\nabla\cdot\left(\rho\vec{v}\right) + \rho\vec{v}\cdot\nabla\psi$

1-D example with {F,M,H} and added consistent time scheme



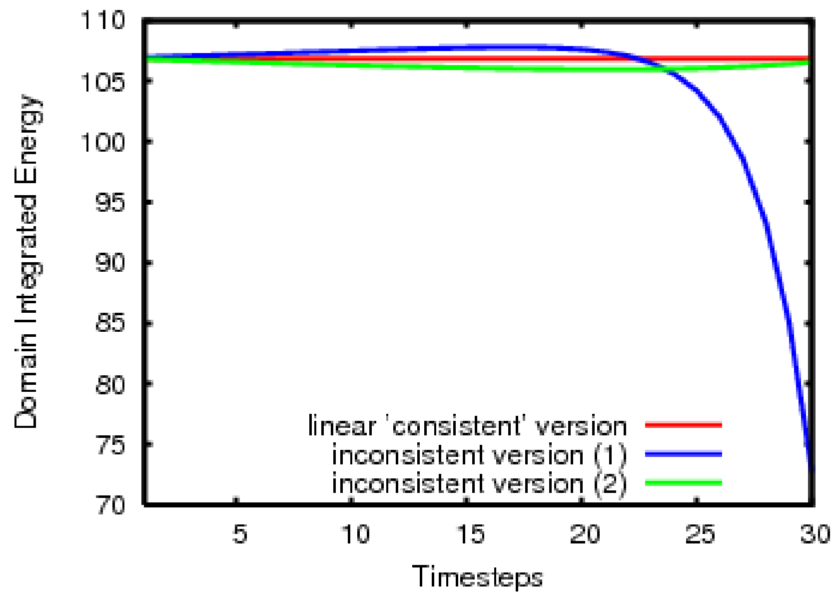
Dynamics of {F,M,H} bracket alone corresponds to Burgers equation

$$\boxed{\frac{\partial \vec{v}}{\partial t}} = -\vec{\omega}_a \times \vec{v} \boxed{- \nabla\left(\frac{\vec{v}^2}{2}\right.} + \Phi \Big) - c_p \theta \nabla \pi$$

$$\boxed{\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{v})}$$

$$\frac{\partial \rho\theta}{\partial t} = -\nabla \cdot (\rho \vec{v} \theta)$$

Dissipative processes have to be added via physical turbulence parameterization.
That is beyond the scope of discretized
Nambu brackets.



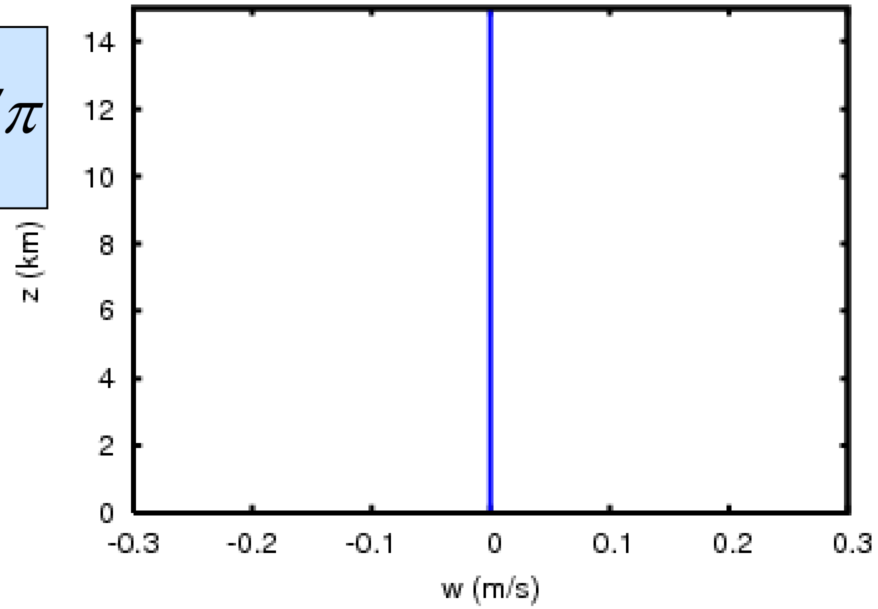
1-D example with {F,M,H} and {F,Θ,H} and consistent time scheme (1)

With the dynamics of the {F,M,H} and {F,Θ,H} brackets we can study the vertical propagation of sound waves as in Satoh*.

$$\boxed{\frac{\partial \vec{v}}{\partial t}} = -\vec{\omega}_a \times \vec{v} - \boxed{\nabla\left(\frac{\vec{v}^2}{2} + \Phi\right) - c_p \theta \nabla \pi}$$

$$\boxed{\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{v})}$$

$$\boxed{\frac{\partial \rho\theta}{\partial t} = -\nabla \cdot (\rho \vec{v} \theta)}$$



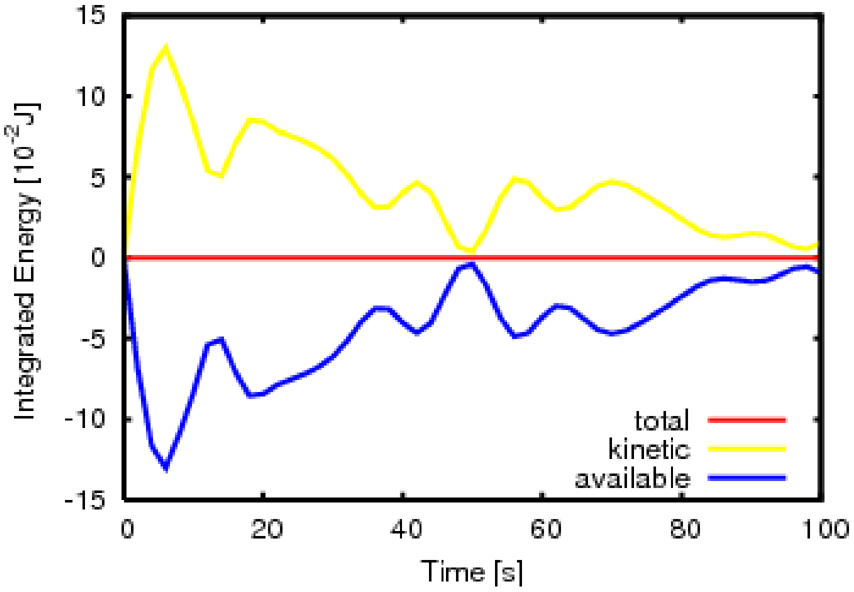
*Satoh,M., 2002: Conservative scheme for the compressible nonhydrostatic models with the horiontally explicit and vertically implicit ime integration scheme. MWR 130, 1227-1245

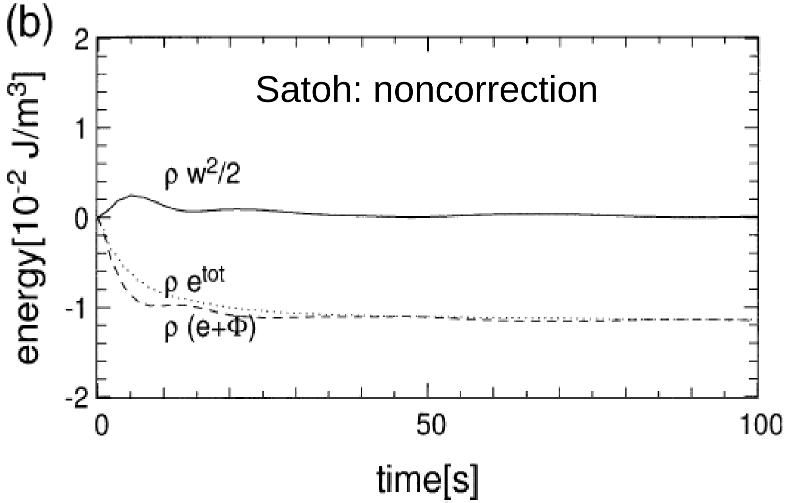
Experimental setup:
T=250 K, hydrostatic rest,
p'_ini=100Pa between 2.5 and 5 km,
dt=2sec, t_max=100sec

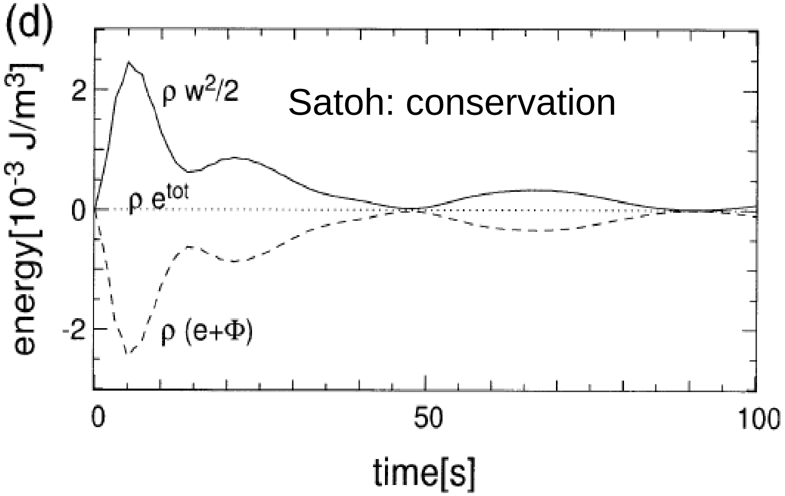
1-D example with {F,M,H} and {F,Θ,H} and conistent time scheme (2)

Energy conservation is obtained without correction or the use of energy as a prognostic quantity.

We observe transformation between availabe energy and kinetic energy as desired.

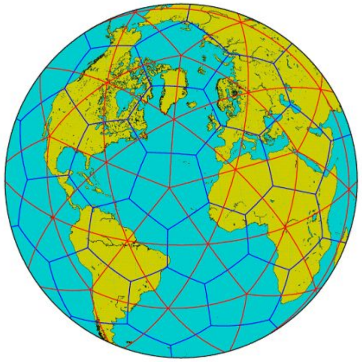






Summary

- ICON will be developed as
  - a new climate simulation and NWP model
  - with a new numerical concept
  - on a new grid



- ICON fundament is available with
  - flexible grid generator
  - existing shallow water model
  - hydrostatic model under development
  - operators build from antisymmetric Nambu brackets
  - consistently derived model equations with turbulence, diabatic source terms, and H_2O