Third Order Schemes with Conservation

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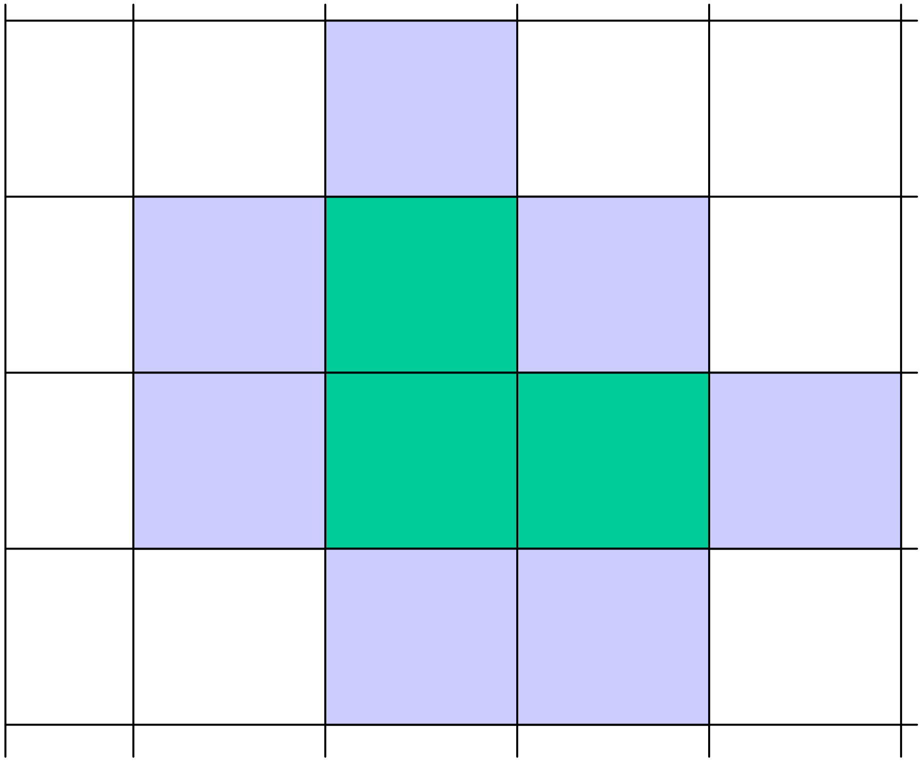
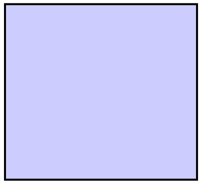
Design Features of a Future Modular Model

- Non-hydrostatic
- Global Cloud Resolving
- **Icosahedral**
- **Adaptive/variable Resolution**
- Efficient and Suitable for Multiprocessing (Semi-Implicit Integration and Serendipity grids)
- **Accurate (Order > 2)**
- **Conservation of Mass, Energy and Enstrophy/dual grids**
- **Approximation Conditions: Z-Co-ordinate /** Physics Interface
- New Features of the Less Challenging Kind: Positivity, Elliptical Shape of Earth, etc.

2-Way Nesting and Conservation

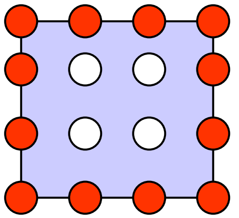


Refined Serendipity Cells



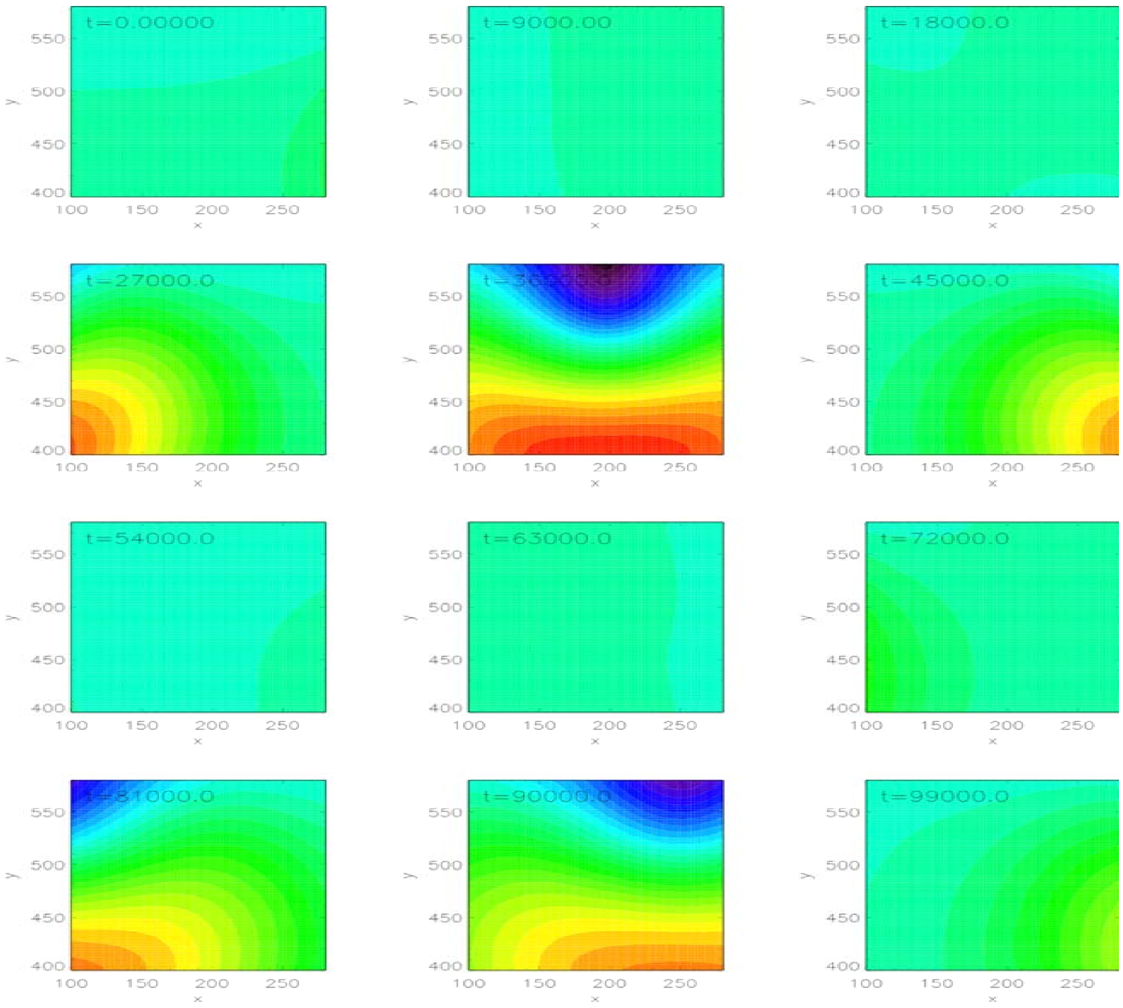
Serendipity Cells to Pose Boundary Values:

Disjunct interpolation:
Only points from blue cells are used to interpoate within the blue cell.



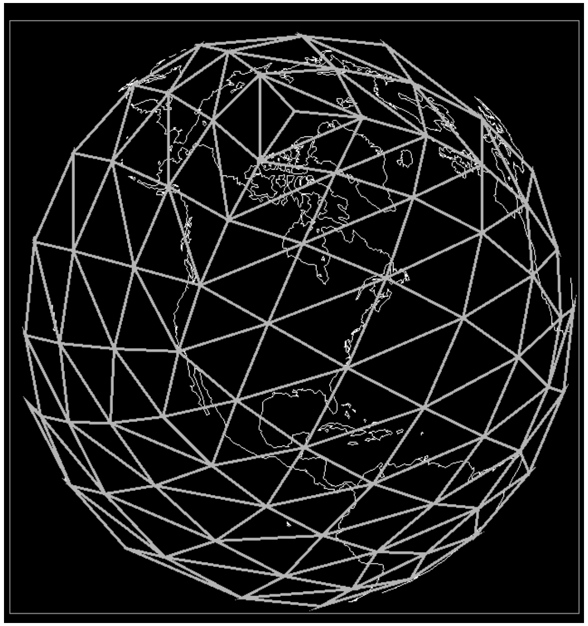
Serendipity Cells Allow to use the Skamarock 2-Way Nesting without Overlap

Rossby Wave Computed by 2-Way Nesting and Serendipity Interpolation

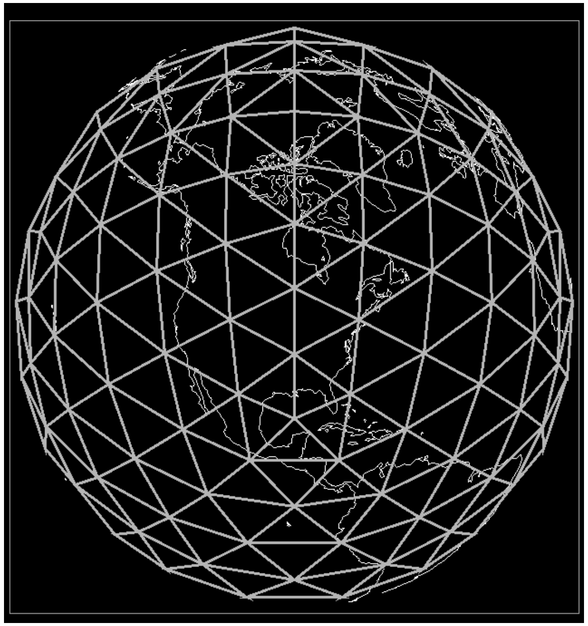


Rhomboidal Divisions of the Sphere

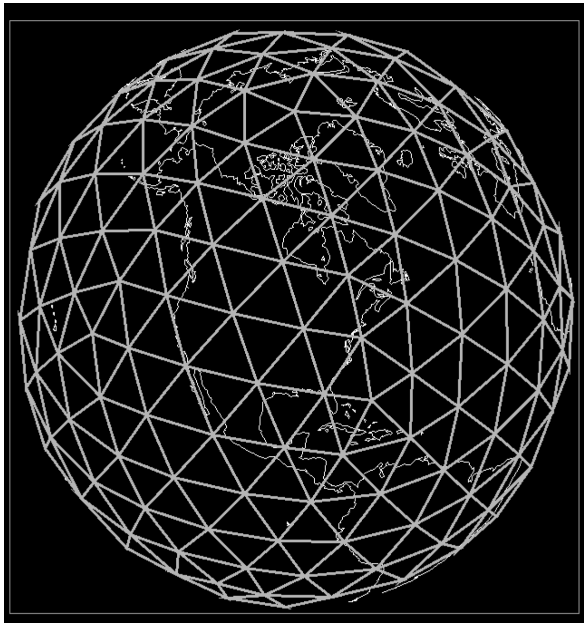
NP=3
Cube



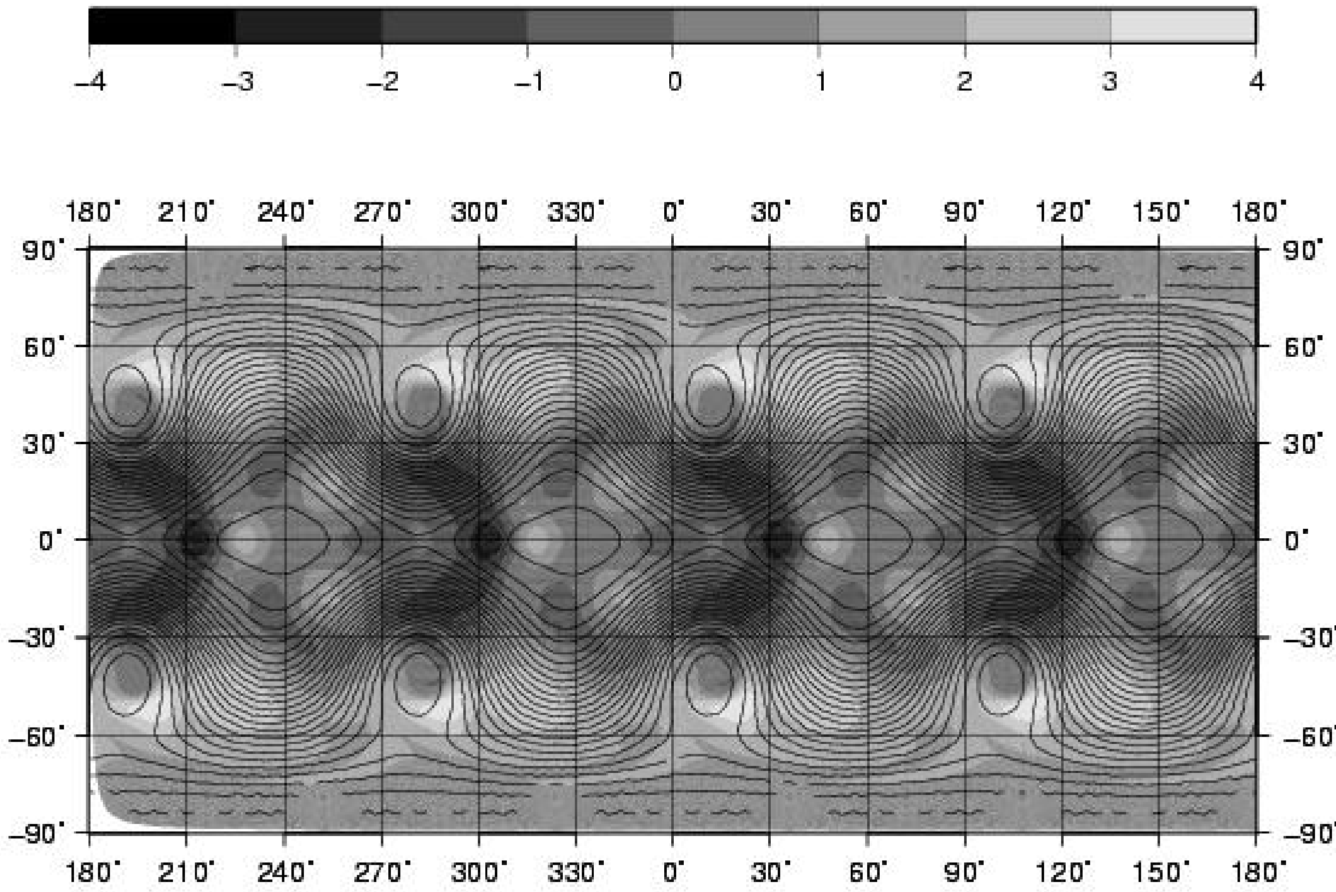
NP=4
4-body



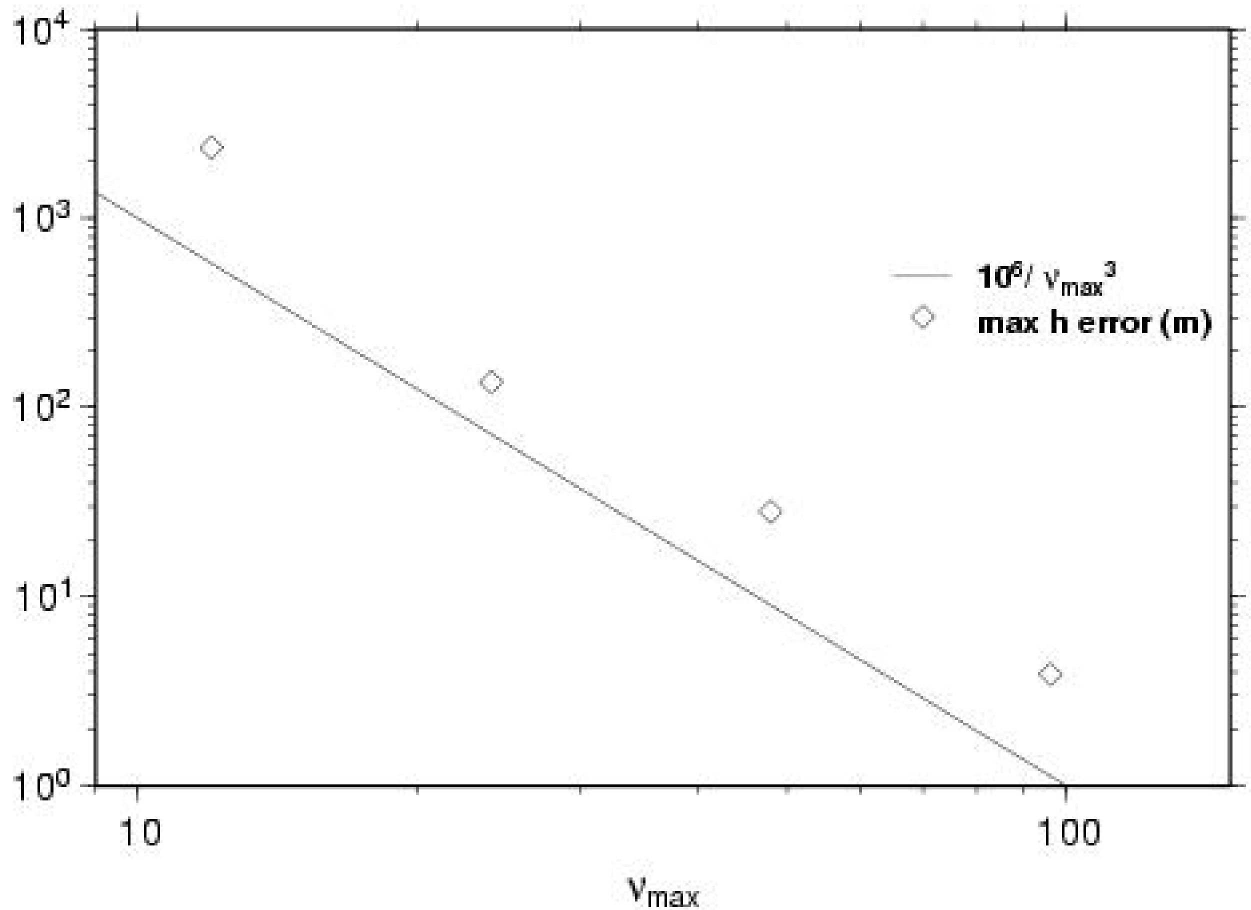
NP=5
Isocahedron



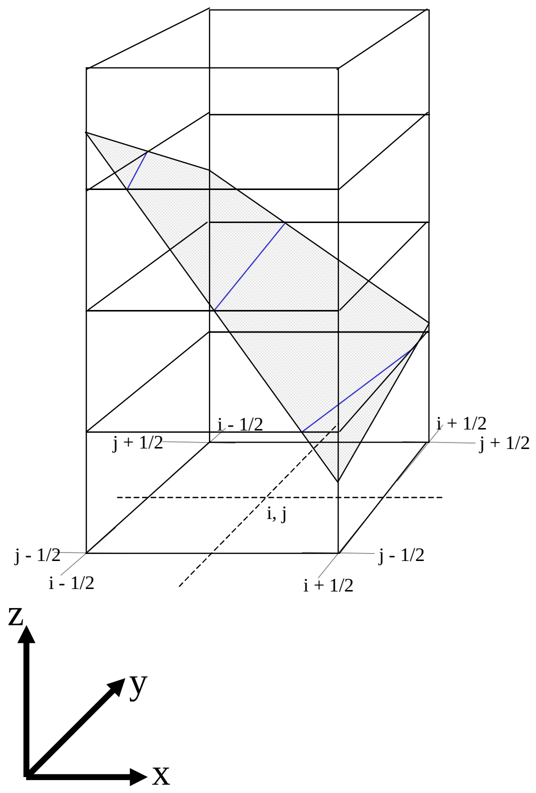
5 Days, Case 6 Williamsson

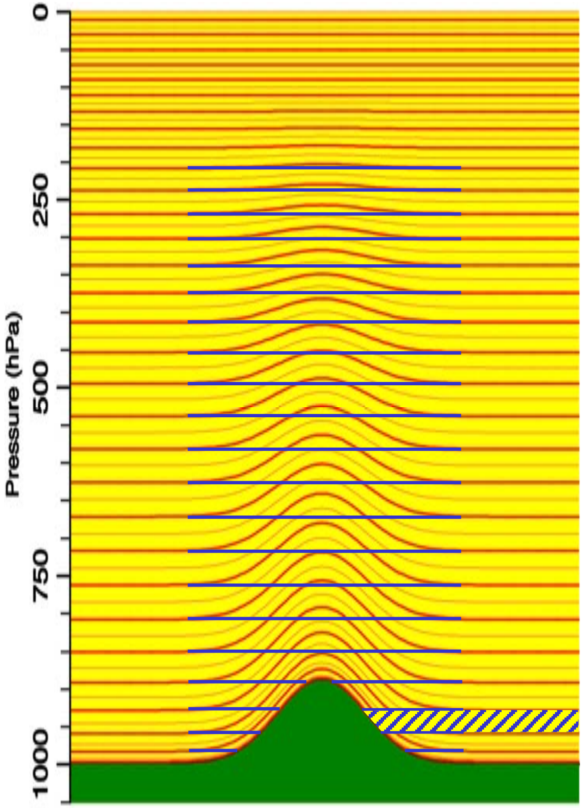


Test Case 6, Convergence, 5 Days



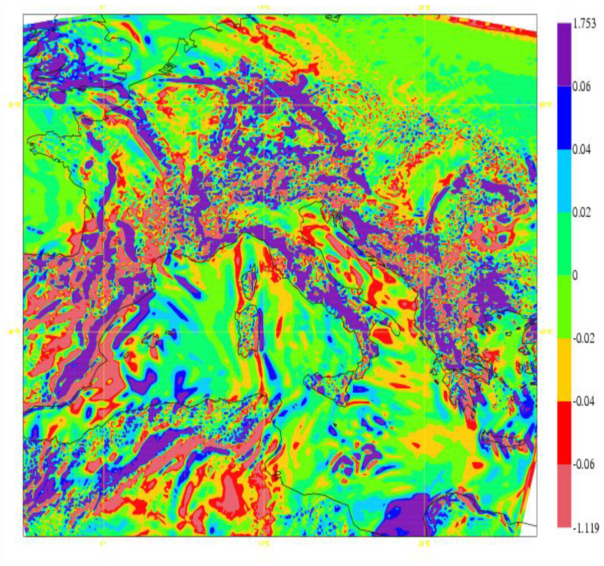
Cut Cells in 3-D and Layer Structure



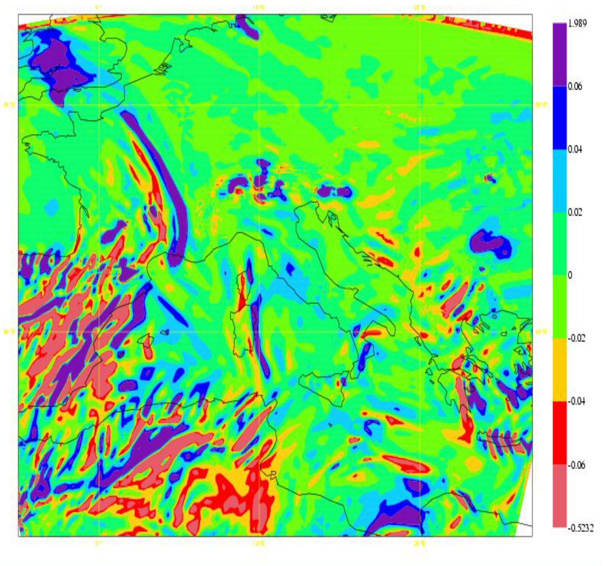


12 h Forecasts of w (5000m)

LM

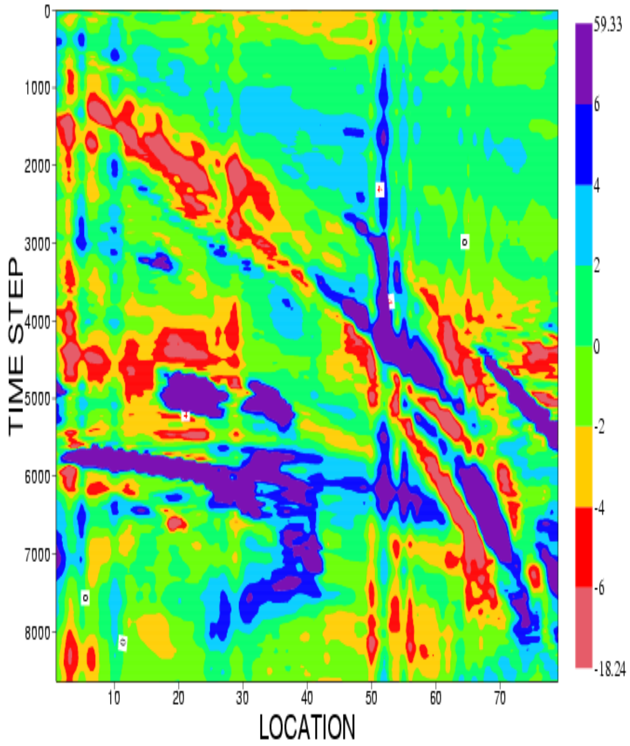


LM-z

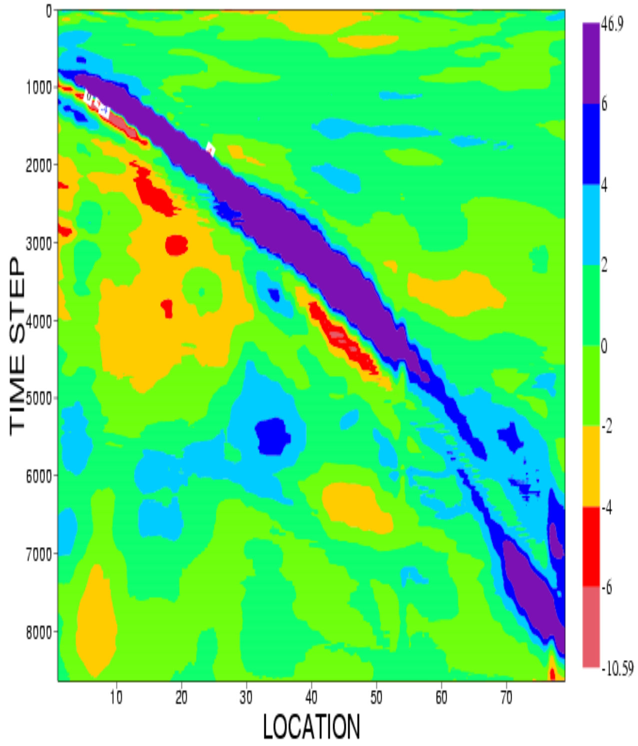


Space Time Cross Section for Height 5000m on the Line Nantes - Luxemburg

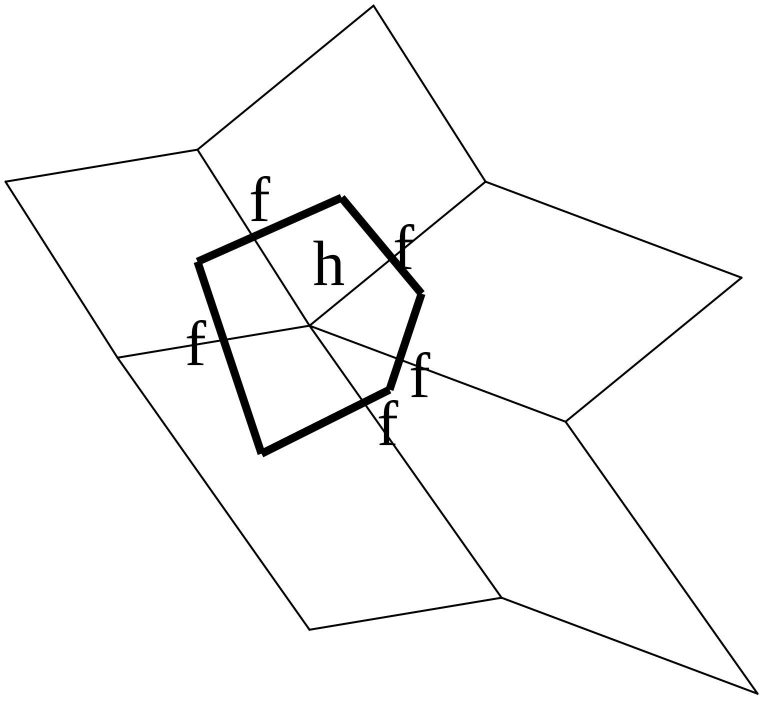
LM



LM-z



Conservation and the Dual Grid



- The dual cell of a corner of 5 rhomboidal cells is a pentagon; the dual grid of a rectangular grid is a rectangular grid
- The finite volume method leads to conservation
- For regular rectangular grids centred differences are obtained
- These methods tend to be of first or second order
- Staggered grids result into good wave characteristics

High Order Conserving Schemes on the Icosahedron

- With great circles methods and whole models on rectangular transfer easily to the icosahedron if the dual grid is not invoked.
- As the dual grid is completely different for rectangular and icosahedral grids, methods invoking the dual grid will not transfer easily to the icosahedron.
- Conservation using dual grid cells is in practice limited to second approximation order.
- Variational principles allow to conserve mass and energy using the original grid cells and easily allow for high order.
- Spectral methods and spectral elements loose their conservation properties due to approximations for efficiency

How to Do High Order Conserving Schemes

- Using variational (Galerkin) schemes on the original grid allows for high order and allows a scheme on rectangles to be transferred easily to the icosahedron

- The usual approximations with existing Galerkin schemes (Spectral and spectral elements) are unnecessary

- FE schemes are conserving. For high order the solution of the mass matrix it is not more expensive than first order elements.

- Computational efficiency (up to current operational level and much beyond) is to be invoked using other approximations, which do not prohibit conservation.

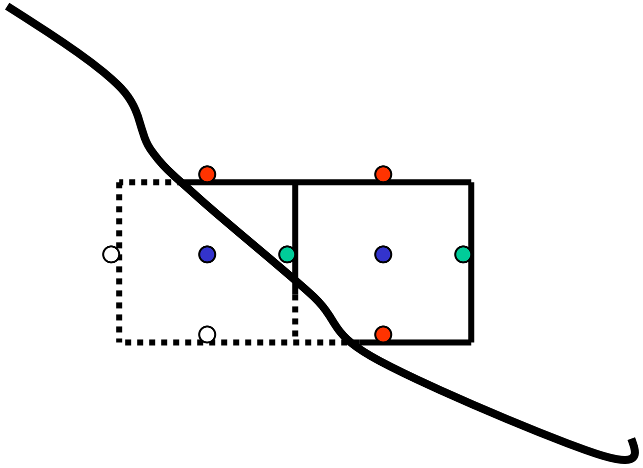
- In order to achieve efficiency the conservation form of the equs is approximated by making use of a high order finite difference solution of the equs in non-conservation form.

- High order fluxes are easily obtained using serendipity cells.

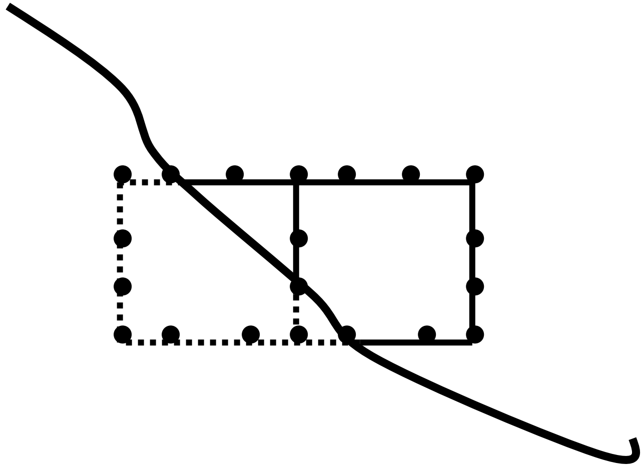
Conservation Using Serendipity Cells

- The conservation should be maintained by time truncation
- The volumes to compute fluxes can be small with z-co-ordinates
- With homogeneous difference schemes and small cells the fluxes determine the difference scheme uniquely
- High order approximations for the conserved quantities are necessary

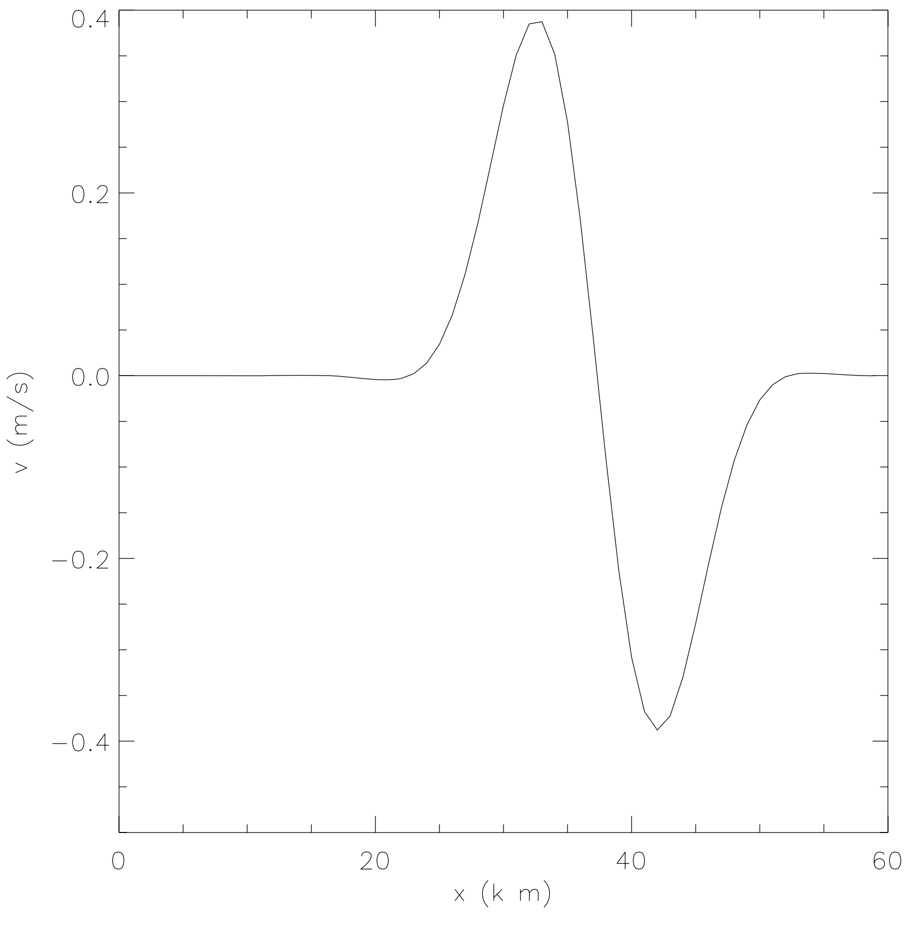
Small Cell, C-Grid



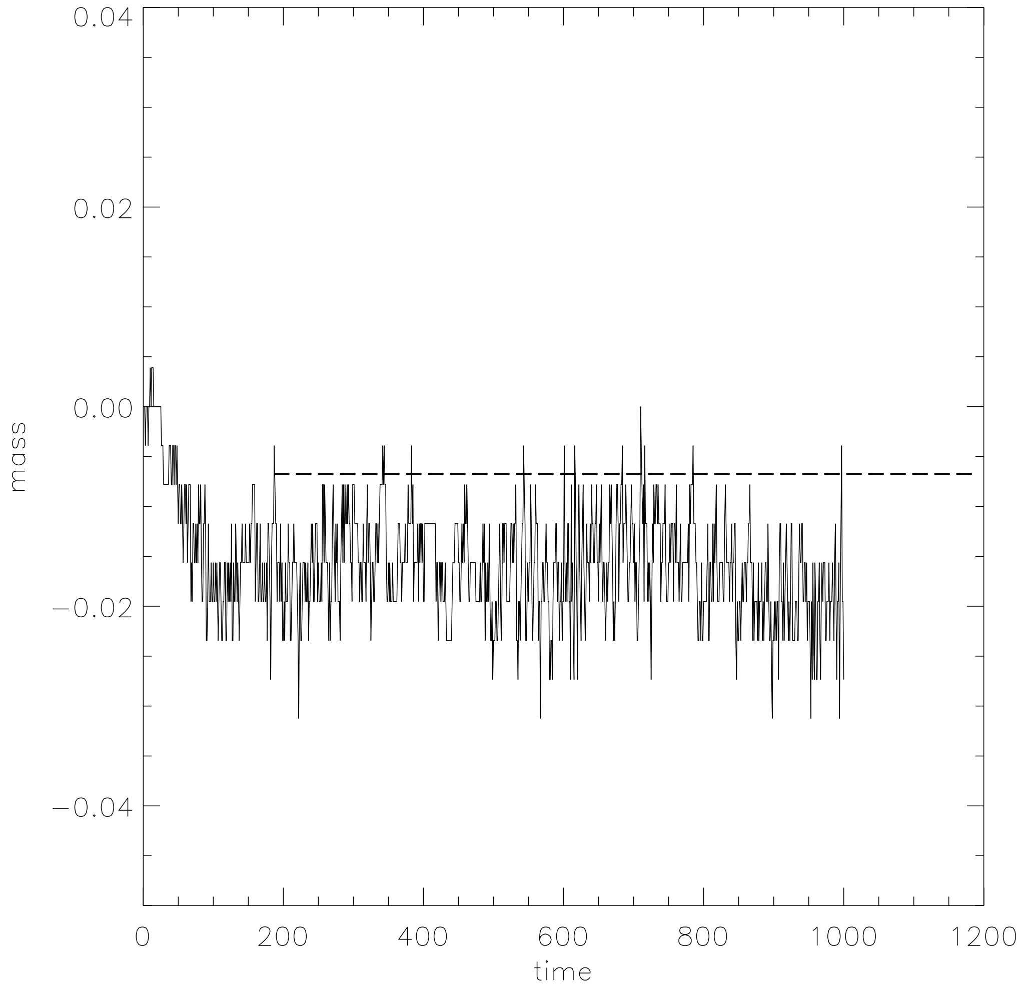
Large Cell, A-Grid



1- D Rossby Wave, V- Field



Total Mass for 1-D Model



Conclusions

- A modular way to obtain a conserving model (Z-co-ordinate, variable resolution, high order, conservation, etc. was proposed)

- The scientific development of the modules a can be done separately.

- The modules: high order, Icosahedron, z-co-ordinate are developed.

- First results of modules variable resolution and conservation were obtained.