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Some conservation issues for the dynamical cores of NWP and climate models

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Thuburn (2006)

Some reasons why a model should conserve.

1. Complex equations - so we should capture properties we know for certain.
2. Accuracy of certain aspects of the solutions are closely related to conservation.
3. Some conservation properties imply stability in certain norms.
4. Enforcement of conservation properties restricts the dimension of the solution manifold.
5. Exact conservation allows the diagnosis of a closed budget.
6. May help in debugging.

Thuburn (2006)

Continuous equations possess an infinite number of conserved quantities.

- Robust Invariants: conserved in adiabatic frictionless limit.
- Non-Robust Invariants: not conserved in the limit even though they are conserved by adiabatic frictionless flow.

Thuburn (2006)

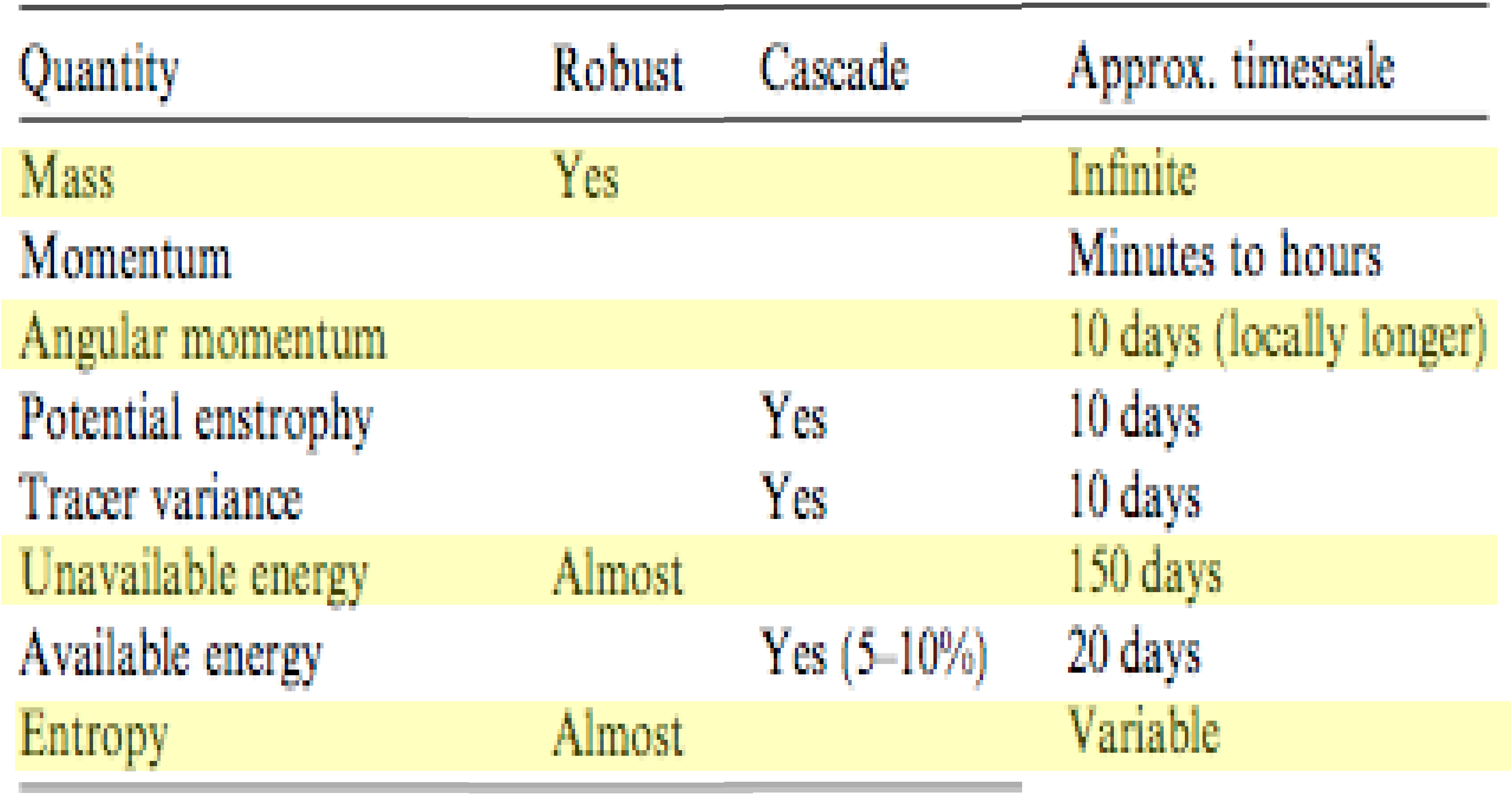
Energy

Downscale cascade for L < 100’s km argues against strict conservation (energy flux ~10^{-5} m^{-2} s^{-3}; 0.1 Wm^{-2}).

Climate model dissipation rates are 1-2 Wm^{-2}

Thuburn (2006)

Table 1



Quantity	Robust	Cascade	Approx. timescale
Mass	Yes		Infinite
Momentum			Minutes to hours
Angular momentum			10 days (locally longer)
Potential enstrophy		Yes	10 days
Tracer variance		Yes	10 days
Unavailable energy	Almost		150 days
Available energy		Yes (5–10%)	20 days
Entropy	Almost		Variable

Constructing conservative models two approaches

1. Construct model using the conserved quantity and its conservation equation.

2. Construct the discrete system such that the discrete equations can be shown to conserve the desired quantity (usually by forming the discrete conservation equation, even though it is not directly integrated).

(1) Is typically more straightforward than (2), and more-easily allows the use of higher-order methods.