First experiences with the non-hydrostatic ICON model

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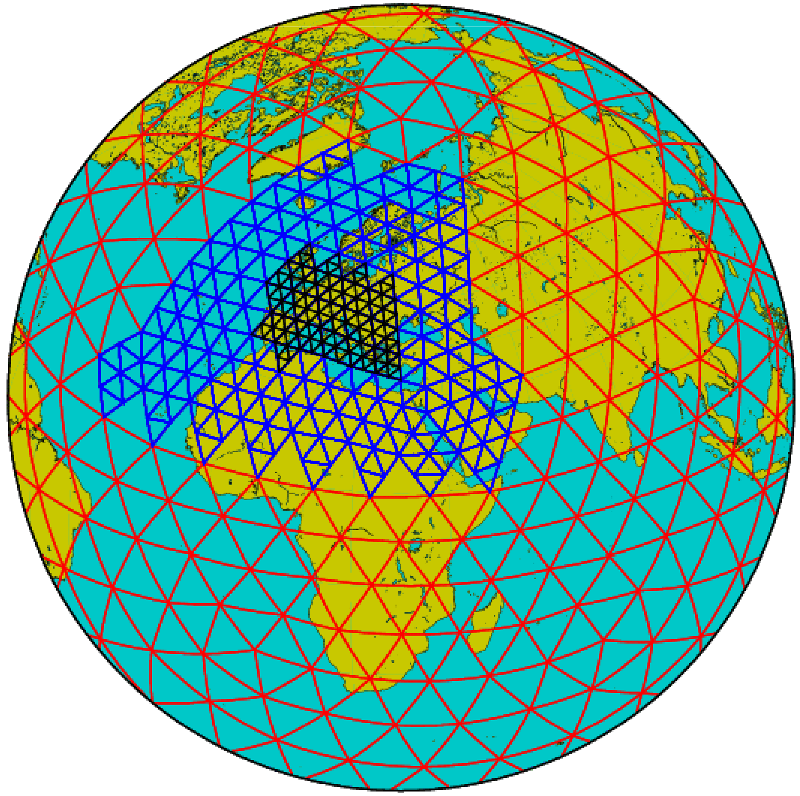
ICON

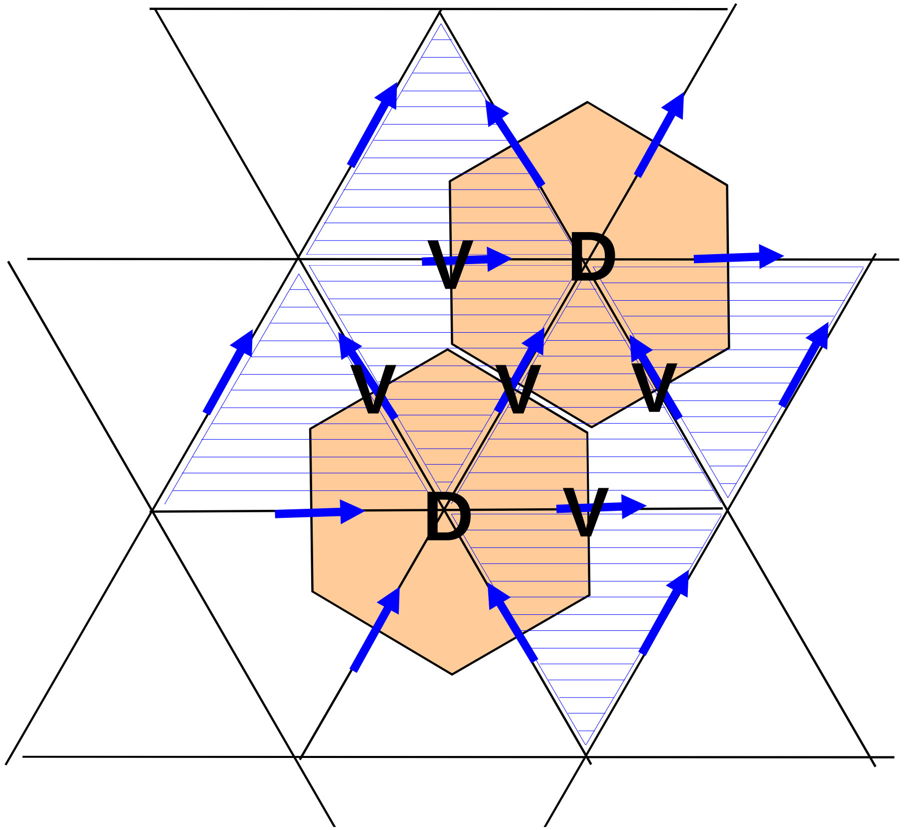
Joint effort of the Max Planck Institute for Meteorology and the German Weather Service to develop

- Models for climate research and weather forecast
- Consistent discretizations for air mass and tracers
- Compatible numerical methods for a coupled atmosphere-ocean GCM system
- Static local grid refinement



Outline

- Grid options
- Special topic: nonlinear instability
- NH model equations
- Special topic: time discretisation
- First example runs

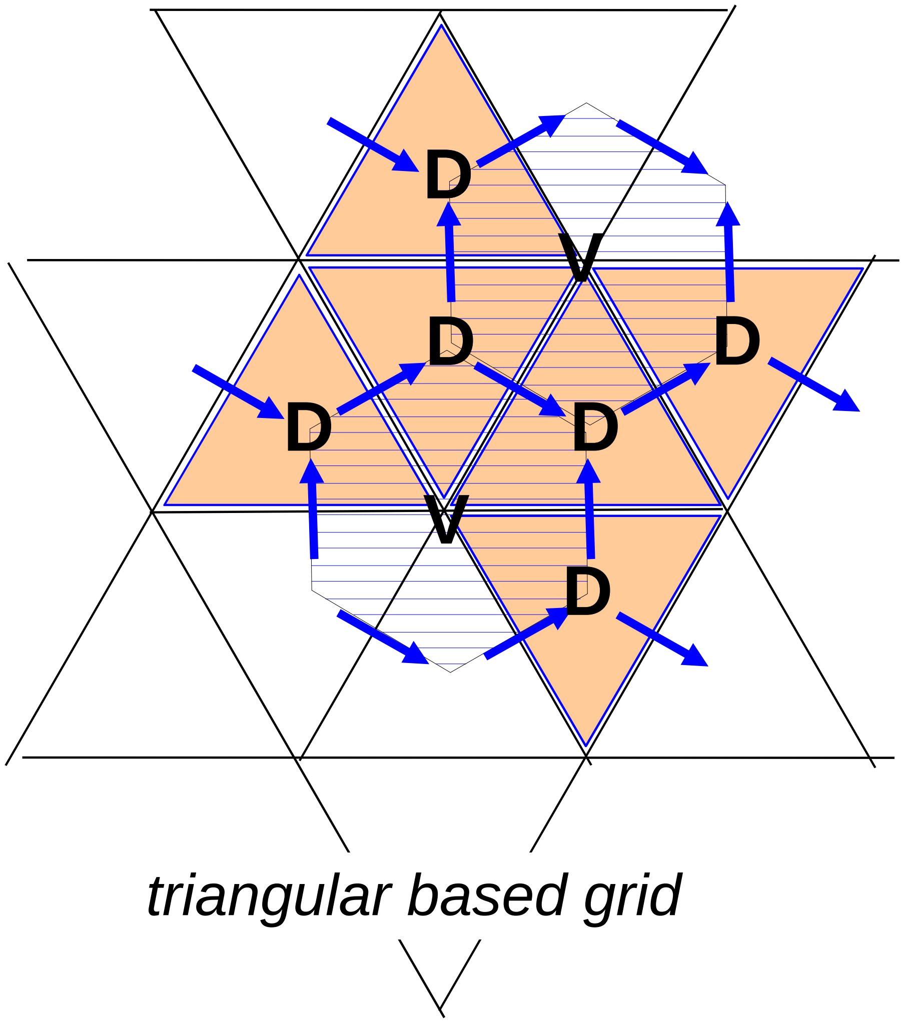


hexagonal based grid

vector invariant form of
momentum advection

$$-\mathbf{k}\eta_z \times \varrho\mathbf{v} - \nabla K$$

Arakawa C grid
Divergence/kinetic energy at centers
Vorticity at corners
Normal velocities at edges



triangular based grid

Quart. J. R. Met. Soc. (1983), **109**, pp. 417–428

causes serious model crashs

An internal symmetric computational instability

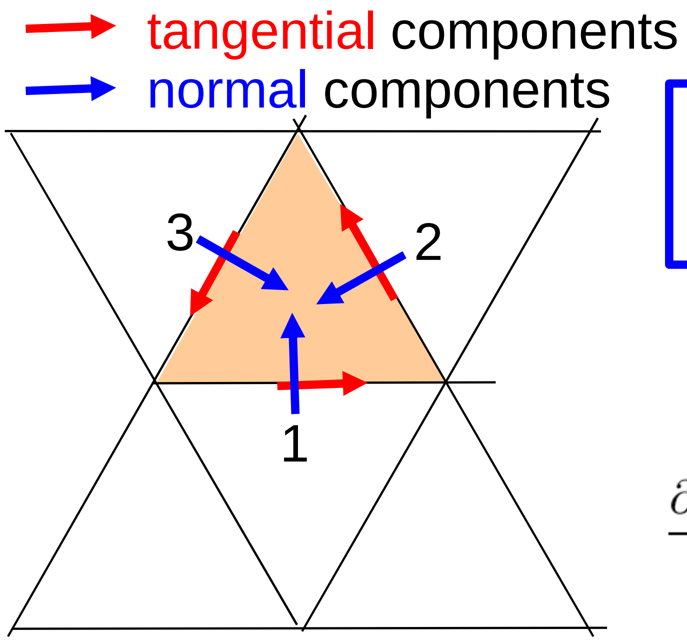
A. HOLLINGSWORTH[1], P. KÅLLBERG[1], V. RENNER[2] and D. M. BURRIDGE[1]
[1] *European Centre for Medium Range Weather Forecasts, Reading.* [2] *Deutscher Wetterdienst, Offenbach am Main, Federal Republic of Germany.*

$$\boxed{-\mathbf{v}\cdot\nabla\mathbf{v} = -\mathbf{k}\eta_z \times \varrho\mathbf{v} - \nabla K}$$

Is that true at least for the linearized discretized equations? - NOT ALWAYS!

Degrees of freedom to accomplish that goal:

triangular based grid	hexagonal based grid
adjust kinetic energy K	adjust potential vorticity η_z
(this is already understood)	(this is still a suspicion)



Triangular based grid

advective form $$\frac{\partial u_1^N}{\partial t} = -\frac{2}{3}\left(u_1^T \frac{\partial u_1^N}{\partial x_1^T} + u_2^T \frac{\partial u_1^N}{\partial x_2^T} + u_3^T \frac{\partial u_1^N}{\partial x_3^T}\right)$$

vorticity $$\zeta = \frac{2}{3}\left(\frac{\partial u_1^N}{\partial x_1^T} + \frac{\partial u_2^N}{\partial x_2^T} + \frac{\partial u_3^N}{\partial x_3^T}\right)$$

$$\frac{\partial u_1^N}{\partial t} = -u_1^T \zeta + \frac{2}{3} u_1^T \frac{\partial u_2^N}{\partial x_2^T} + \frac{2}{3} u_1^T \frac{\partial u_3^N}{\partial x_3^T} - \frac{2}{3} u_2^T \frac{\partial u_1^N}{\partial x_2^T} - \frac{2}{3} u_3^T \frac{\partial u_1^N}{\partial x_3^T}$$

kinetic energy is ambivalent $$K = \frac{1}{3}\left(u_1^{N2} + u_2^{N2} + u_3^{N2}\right) = \frac{1}{3}\left(u_1^{T2} + u_2^{T2} + u_3^{T2}\right)$$

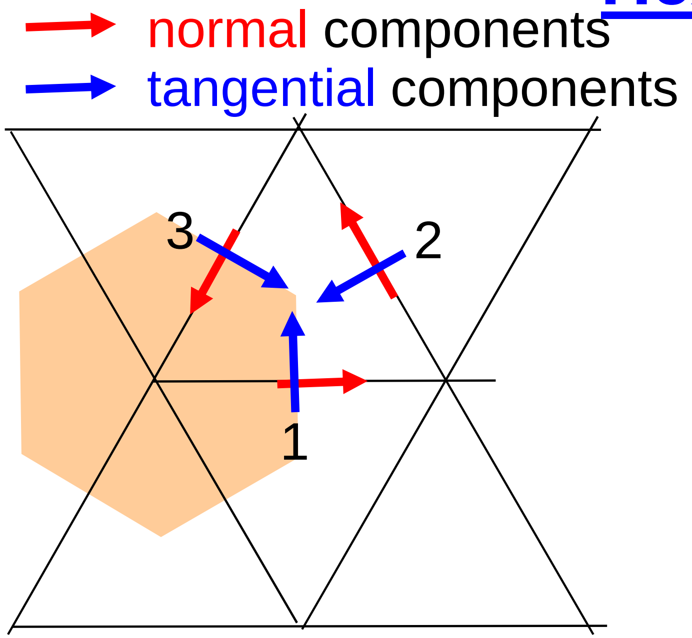
vector invariant form $$\frac{\partial u_1^N}{\partial t} = -u_1^T \zeta - \frac{1}{\sqrt{3}}\left(\frac{\partial K}{\partial x_2^T} - \frac{\partial K}{\partial x_3^T}\right)$$

Discretisation and linearization around $u_1^T = v_c$ and $u_1^N = 0$

shows that equivalence is only possible if : $$K = \frac{1}{6}\left(u_1^{N2} + u_2^{N2} + u_3^{N2} + u_1^{T2} + u_2^{T2} + u_3^{T2}\right)$$

and the tangential reconstruction is done with a 4-point stencil.

Hexagonal based grid (under investigation)



advective form $$\frac{\partial u_1^N}{\partial t} = -\frac{2}{3}\left(u_1^N \frac{\partial u_1^N}{\partial x_1^N} + u_2^N \frac{\partial u_1^N}{\partial x_2^N} + u_3^N \frac{\partial u_1^N}{\partial x_3^N}\right)$$

kinetic energy $$K = \frac{1}{3}\left(u_1^{N2} + u_2^{N2} + u_3^{N2}\right)$$

vorticity is ambivalent

$$\zeta_1 = \frac{2}{\sqrt{3}}\left(\frac{\partial u_3^N}{\partial x_2^N} - \frac{\partial u_2^N}{\partial x_3^N}\right)$$

$$\zeta_2 = \frac{2}{\sqrt{3}}\left(\frac{\partial u_1^N}{\partial x_3^N} - \frac{\partial u_3^N}{\partial x_1^N}\right) \qquad \zeta_3 = \frac{2}{\sqrt{3}}\left(\frac{\partial u_2^N}{\partial x_1^N} - \frac{\partial u_1^N}{\partial x_2^N}\right)$$

vector invariant form $$\frac{\partial u_1^N}{\partial t} = -\frac{\partial K}{\partial x_1^N} + \frac{1}{\sqrt{3}}\zeta_3 u_2^N - \frac{1}{\sqrt{3}}\zeta_2 u_3^N$$

Aside: The reconstruction of tangential velocities needs a special discretisation.

Discretisation and linearization around $u_1^T = v_c$ and $u_1^N = 0$ indicates that only neighboring vorticities at 2 and 3 points may enter the u_1^N equation.

Numerical experiments are not yet performed!

Non-hydrostatic compressible atmospheric model

Special emphasis devoted to Hamiltonian consistency during the discretisation

- divergence and gradient are dual operators
- scalar triple product A.(B xC) is antisymmetric

- mass, energy, entropy (potential temperature) conservation

$$
\begin{aligned}
\frac{\partial \mathbf{v}}{\partial t} &= -\frac{\vec{\omega}}{\varrho} \times \varrho \mathbf{v} - \nabla K - c_{pd}\theta_v \nabla \Pi \\
\frac{\partial \varrho}{\partial t} &= -\nabla \cdot (\varrho \mathbf{v}) \\
\frac{\partial \Pi}{\partial t} &= -\frac{R_d \Pi}{c_{vd}\varrho\theta_v} \nabla \cdot (\theta_v(\varrho \mathbf{v})) \\
(\varrho\theta_v)^{n+1} &= (\varrho\theta_v)^n \left(\frac{c_{vd}}{R_d} \left(\frac{\Pi^{n+1}}{\Pi^n} - 1 \right) + 1 \right)
\end{aligned}
$$

- terrain-following coordinates
- Lorenz vertical grid staggering
- no background state
- explicit time stepping in the horizontal
- implicit time stepping in the vertical
- third order upstream advection of θ_v (both: horizontal and vertical)

Time stepping scheme

May energetic consistency be achieved throughout the time stepping scheme?

continuous equations

energy conservation

$$hv \cdot \frac{\partial v}{\partial t} = -hv \cdot \nabla_x (K + gh)$$

$$hv \cdot \frac{\partial v}{\partial t} - (K + gh)\nabla_x \cdot (hv) = -\nabla_x \cdot (hv(K + gh))$$

$$hv \cdot \frac{\partial v}{\partial t} + K\frac{\partial h}{\partial t} + gh\frac{\partial h}{\partial t} = -\nabla_x \cdot (hv(K + gh))$$

$$\frac{\partial (hK + g/2\, h^2)}{\partial t} = -\nabla_x \cdot (hv(K + gh))$$

discretized equations

implicit midpoint scheme

$$\frac{v^{n+1} - v^n}{\Delta t} = -\nabla_x \frac{v^{n+1}v^n}{2} - \nabla_x g\frac{h^{n+1} + h^n}{2}$$

$$\frac{h^{n+1} - h^n}{\Delta t} = -\nabla_x \cdot \frac{v^{n+1}h^{n+1} + v^n h^n}{2}$$

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energy conservation

$$g\frac{(h^{n+1} + h^n)}{2}\frac{h^{n+1} - h^n}{\Delta t} = \frac{g/2(h^{n+1}h^{n+1} - h^n h^n)}{\Delta t}$$

$$\frac{h^n v^n + h^{n+1}v^{n+1}}{2}\frac{v^{n+1} - v^n}{\Delta t} + \frac{v^n v^{n+1}}{2}\frac{h^{n+1} - h^n}{\Delta t} = \frac{h^{n+1}v^{n+1}v^{n+1}/2 - h^n v^n v^n/2}{\Delta t}$$

Time scheme – explicit stepping?

implicit not centered scheme

energy conserving

$$\frac{v^{n+1}-v^n}{\Delta t} = -\nabla_x \frac{v^{n+1}v^{n+1}}{2} - \nabla_x g \frac{h^{n+1}+h^n}{2}$$
$$\frac{h^{n+1}-h^n}{\Delta t} = -\nabla_x \cdot \frac{(v^{n+1}+v^n)h^n}{2}.$$

$$\frac{h^{n+1/2}-h^n}{\Delta t/2} = -\nabla_x \cdot v^{n+1/2}h^n$$
$$\frac{v^{n+1}-v^n}{\Delta t} = -\nabla_x K^{n+1} - \nabla_x g h^{n+1/2}$$
$$\frac{h^{n+1}-h^{n+1/2}}{\Delta t/2} = -\nabla_x \cdot v^{n+1/2}h^n.$$

$$\frac{v^{n+1}-v^n}{\Delta t} = -\nabla_x K^{n+1} - \nabla_x g h^{n+1/2}$$

$$\frac{h^{n+3/2}-h^{n+1/2}}{\Delta t} = -\nabla_x \cdot v^{n+1}h^{n+1/2}$$

Explicit forward-backward scheme for waves.
But kinetic energy term is implicit!

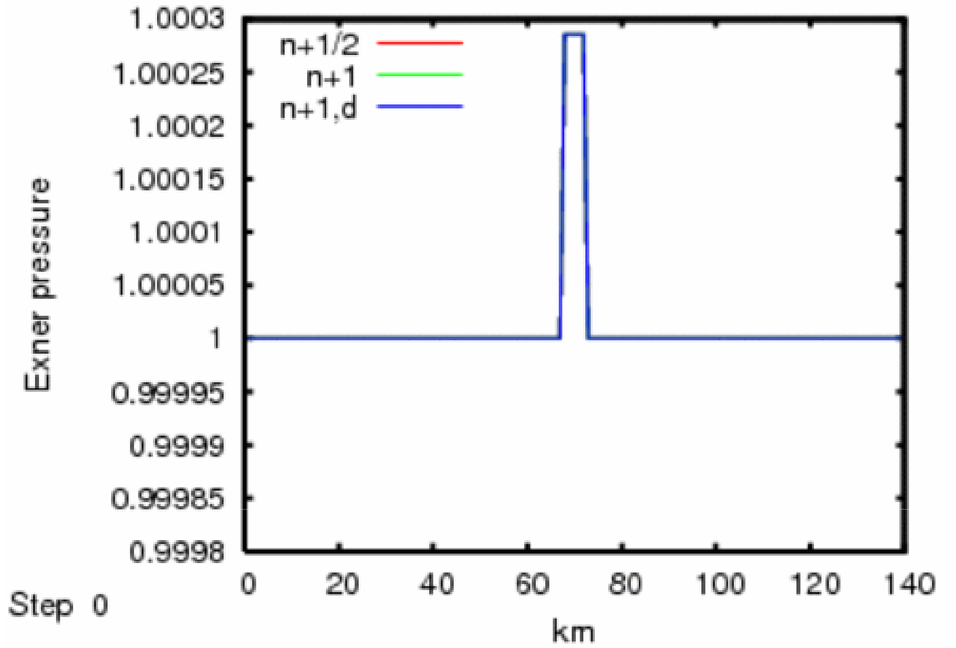
$$\frac{v^{n+1}-v^n}{\Delta t} = -\nabla_x K^{n+1} - \nabla_x g h^n$$
$$\frac{h^{n+1}-h^n}{\Delta t} = -\nabla_x \cdot v^{n+1}h^n$$

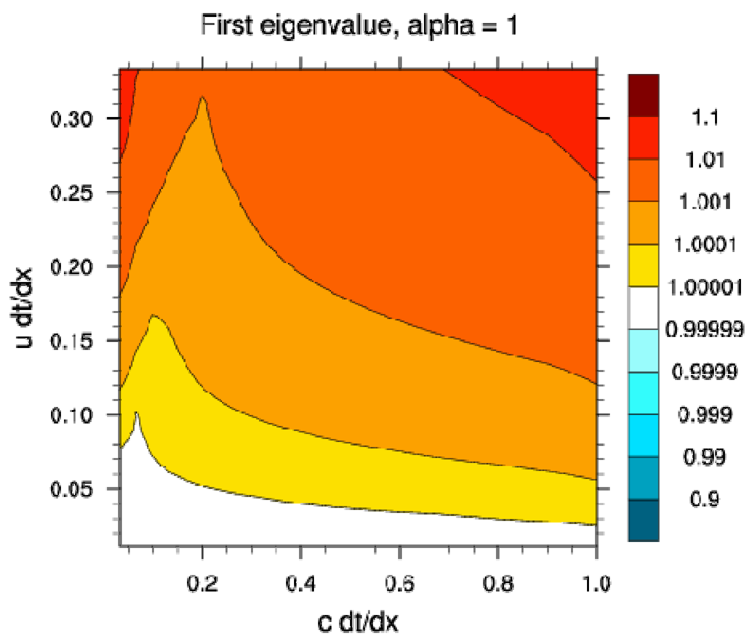
Predictor step

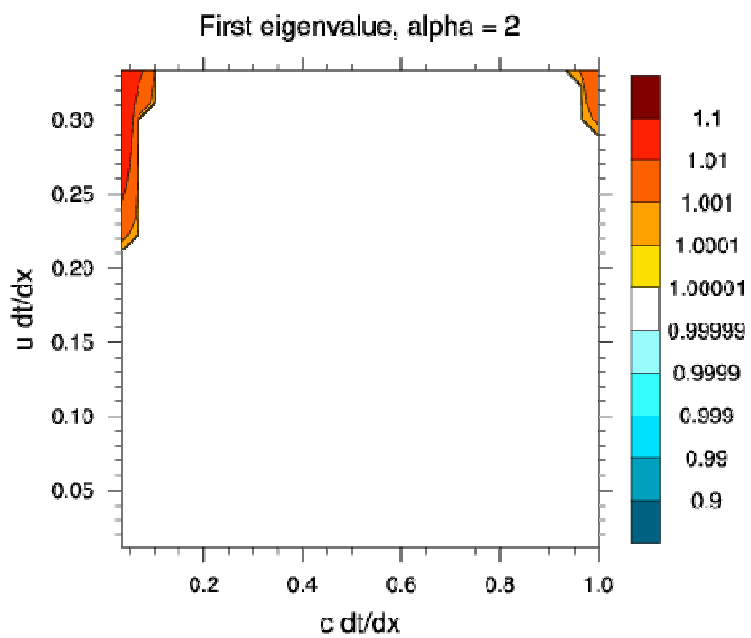
$$
\begin{aligned}
v^{est} &= v^n - \Delta\tau\alpha\nabla_x K^n - \Delta\tau\nabla_x gh^n \\
v^{n+1} &= v^n - \Delta t\nabla_x K^{est} - \Delta t\nabla_x gh^n
\end{aligned}
$$

$\Delta\tau = \Delta t/2$ •similar to RK2 procedure

$\Delta\tau = \Delta t$ •suggested prediction step

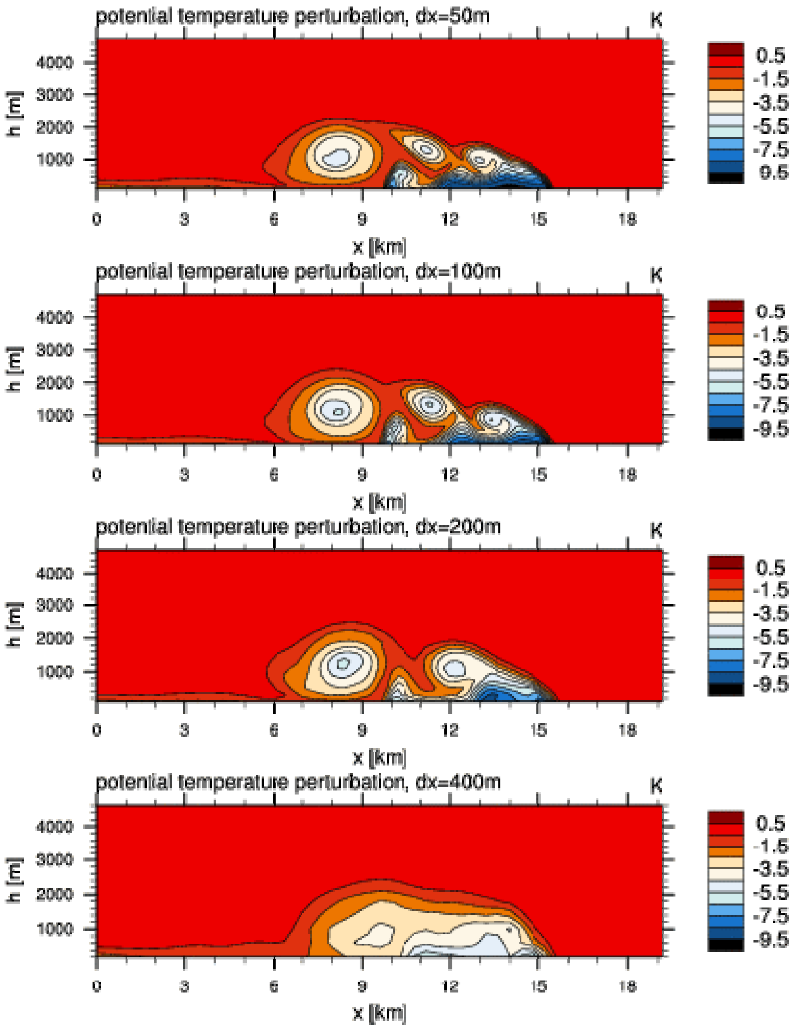






Straka test case - density current

ICON results



WRF-ARW results from Bill Skamarock's webpage