Under-Resolved Simulation of Mesoscale Atmospheric Convection

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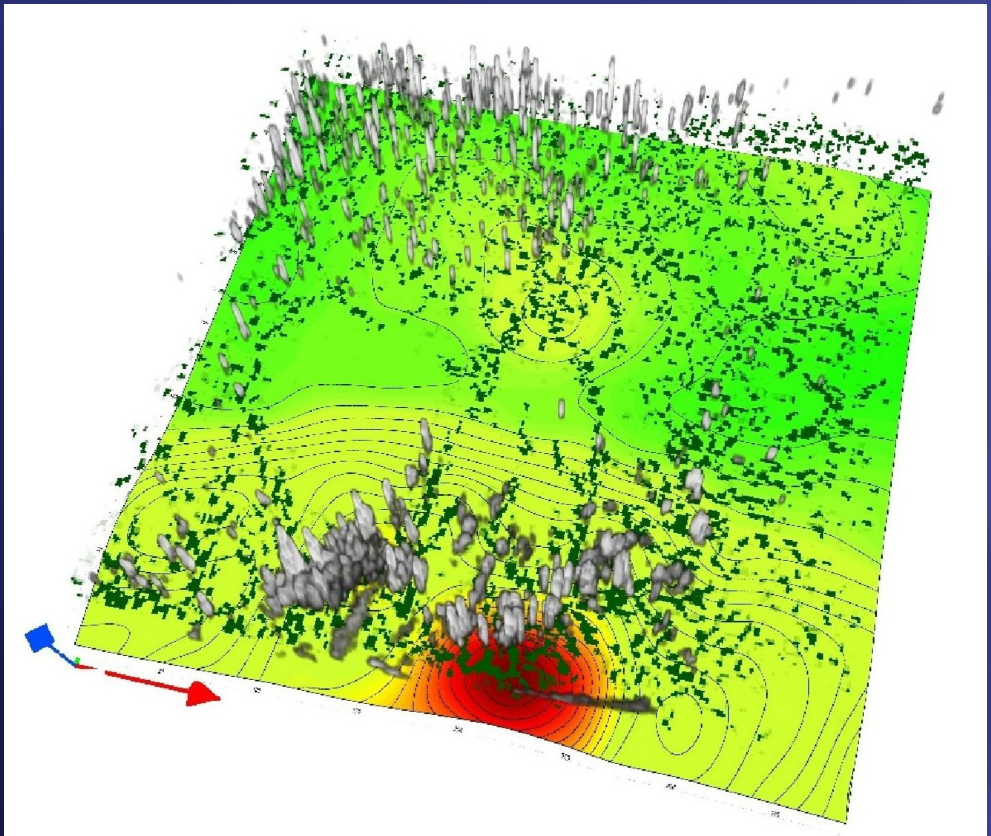
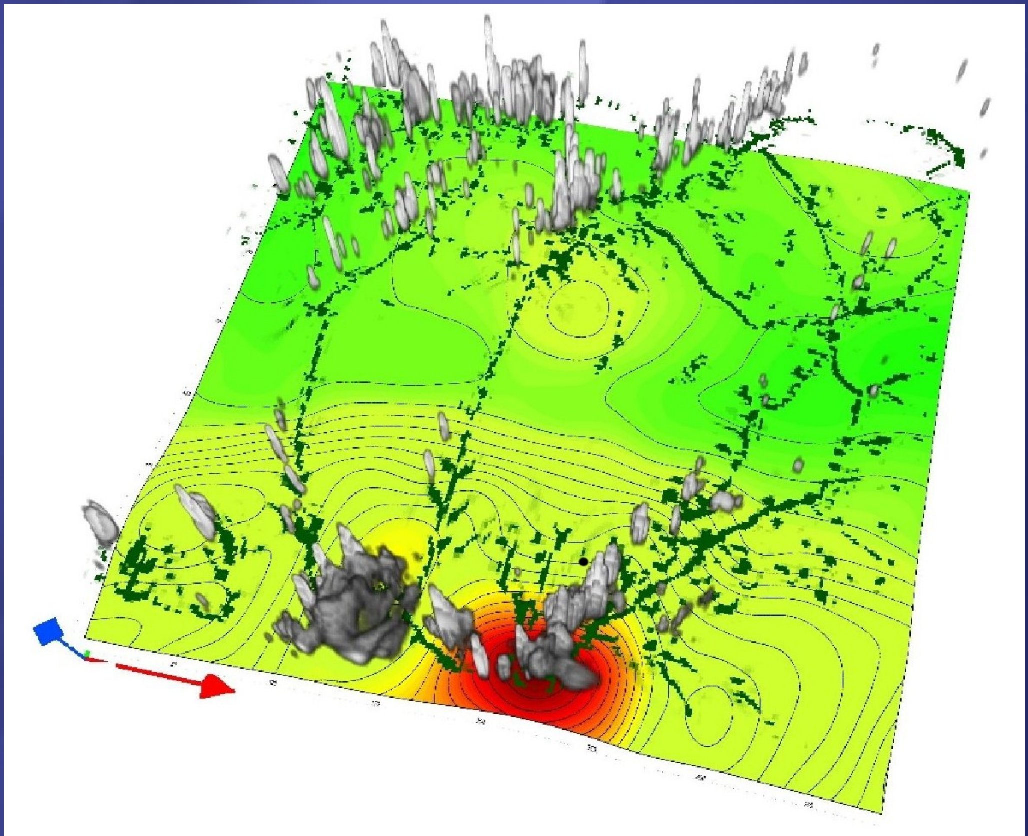
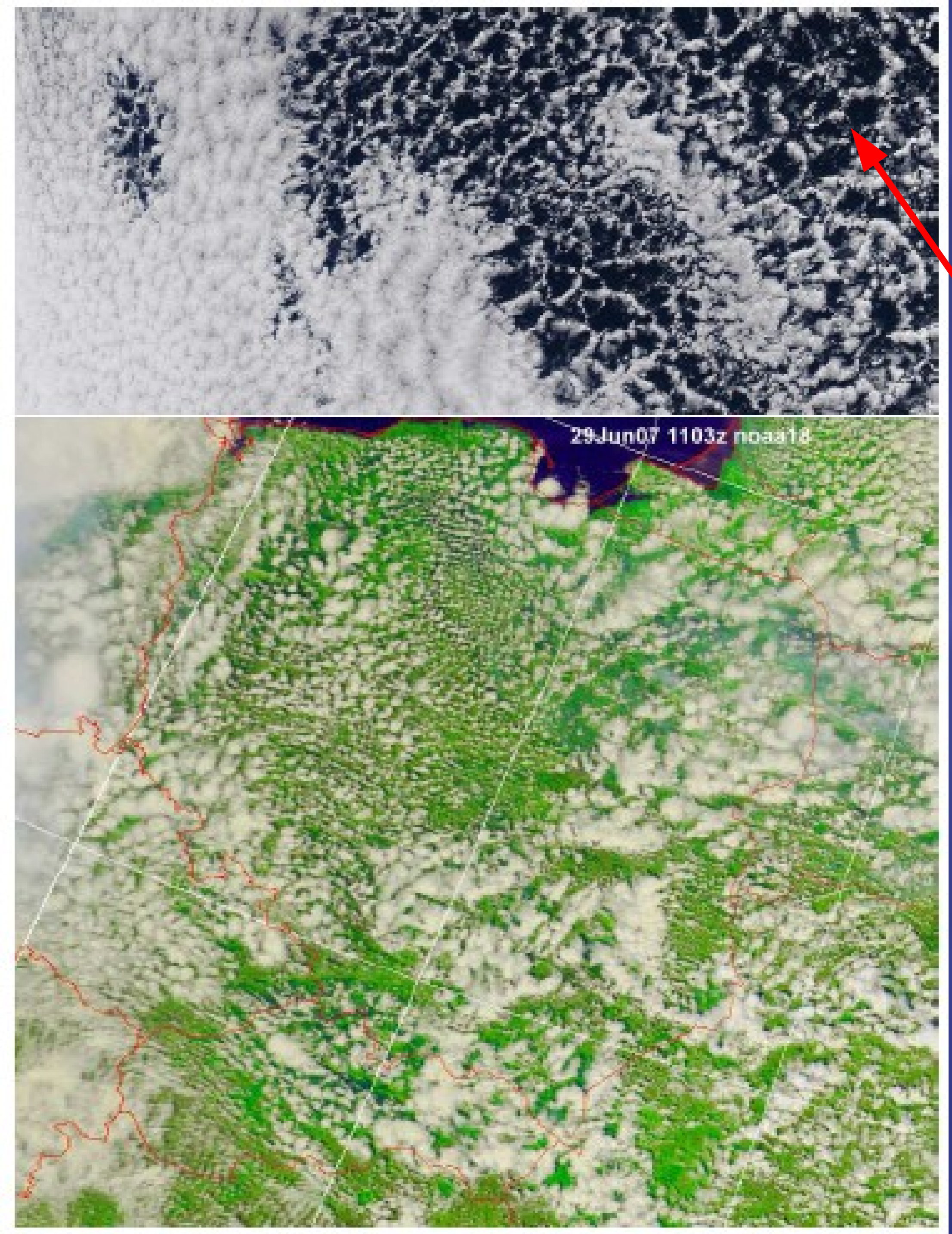
Motivation





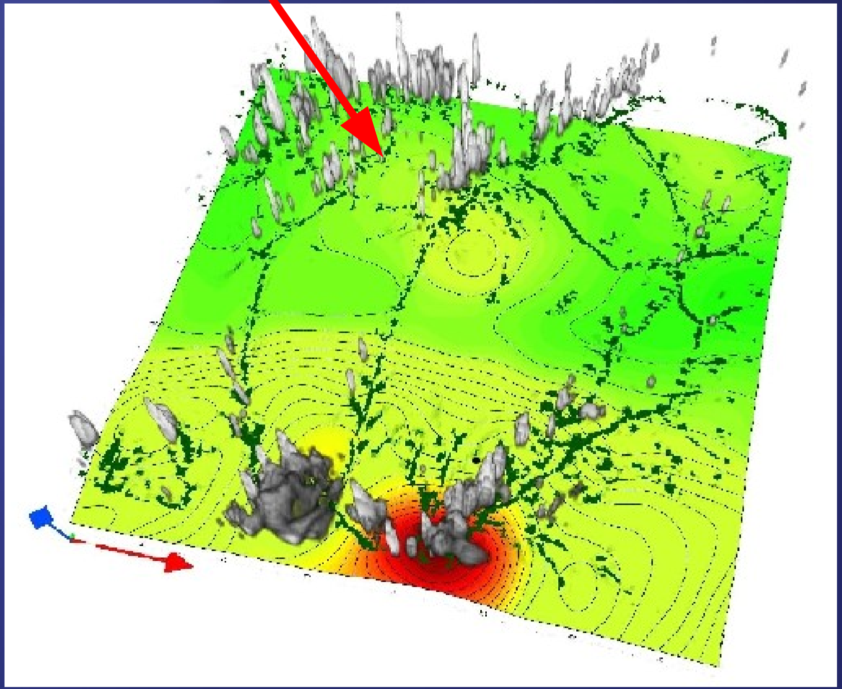
Structure of simulated convection over heated realistic terrain.

*Vertical velocities after 6h of simulated time are shown within the PBL depth. Grey iso-surfaces represent clouds, and dark green patterns mark updrafts at boundary layer top. Isolines and other colors show the topography. The only **difference** between the two simulations is the **effective viscosity of numerical advection**.*



Cellular convection with characteristic size O(10 km)

Is it related ?



Contents

- Rayleigh-Benard convection in the atmosphere and in the virtual reality
- Convection resolving in modern NWP
- Numerical artifacts in NWP-like underresolved simulations

Rayleigh number :

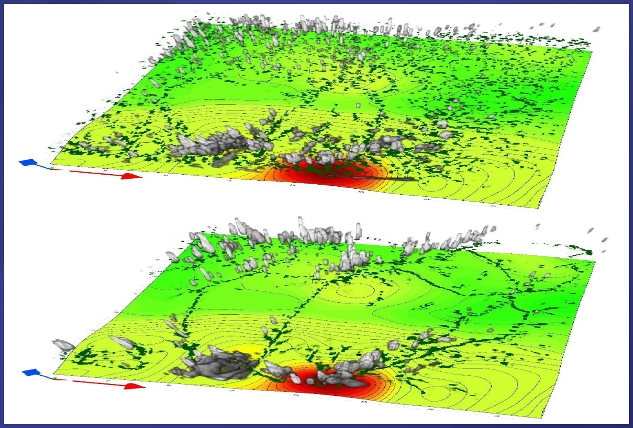
$$Ra = \frac{g\Delta\overline{\theta}h^3}{\overline{\theta}\nu\nu_\theta}$$

g – gravity acceleration
h – fluid layer thickness
ν – kinematic viscosity
ν_θ – thermal diffusivity
$\Delta\theta / \theta$ – pot. temperature, relative change over h

Ra measures relative magnitude of buoyancy and viscous forces

rigid/stress-free lower/upper boundary

$Ra_c = 1100.657$



$>>$ critical

$\gtrsim$ critical

In the dry atmosphere:

$h = 1000$ m
$\nu = 1.7 \times 10^{-5}\ m^2/s$
$\nu_\theta = 1.9 \times 10^{-5}\ m^2/s$
$\Delta\theta / \theta = O(10^{-3})$

$\longrightarrow$ **$Ra \approx O(10^{16})$**

Thus, how to explain cellular convection ?

Modified definition (Jeffreys, 1928, Priestley 1962, Ray 1965, Sheu 1980)

$$Ra = \frac{g\Delta\bar{\theta}h^3}{\bar{\theta}K_m^2}$$

Km - effective „eddy diffusivity”

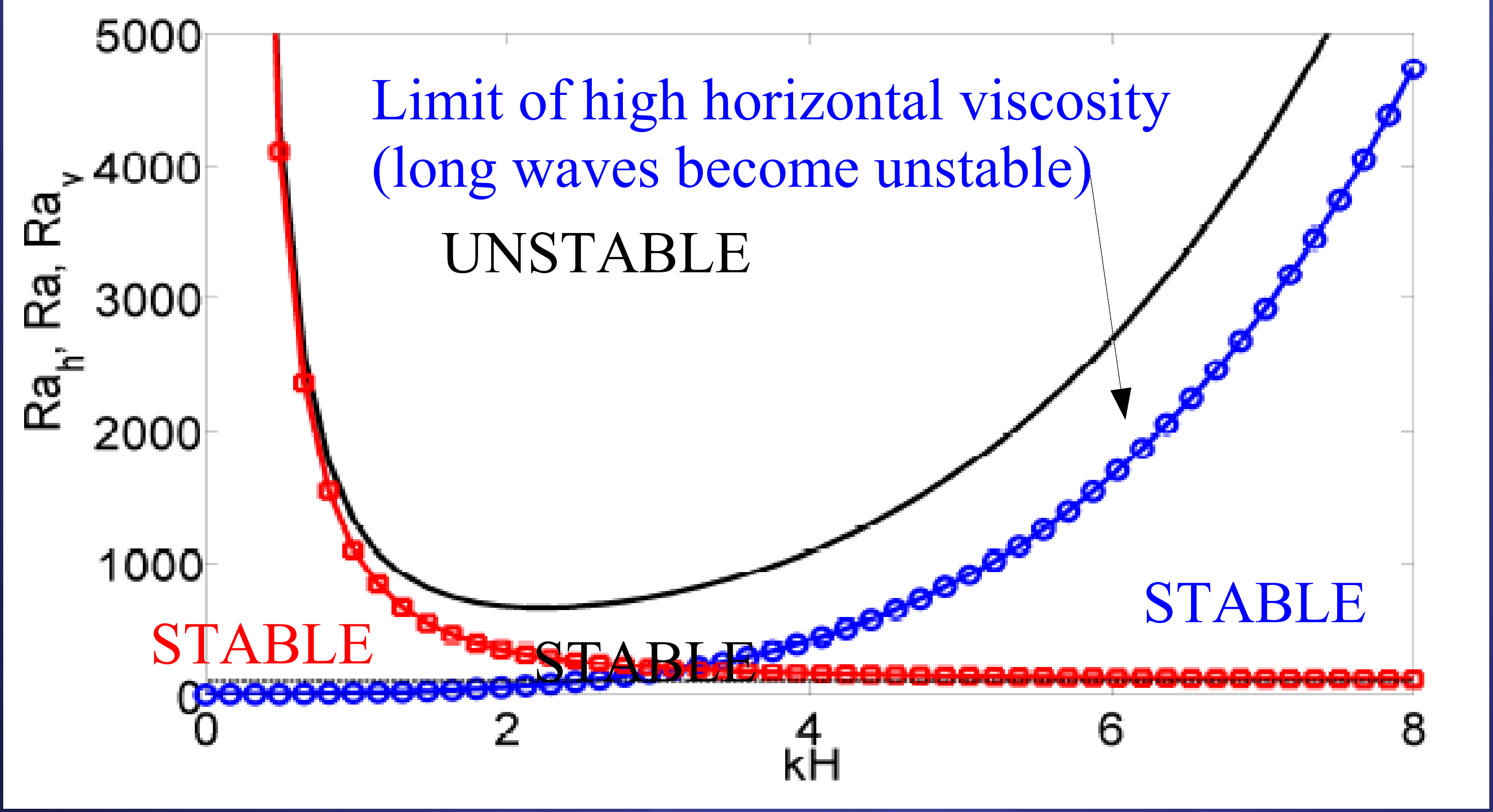
K_m can be different in the horizontal and in the vertical.

Problem lacks conclusion and calls for attention with the advent of $O(1)$ km resolution NWP.

Possible sources of K_m and *Ra* anisotropy in virtual reality

- Explicit anisotropic filtering
- Using numerical schemes with different dissipative properties in the horizontal and in the vertical
- Numerical dissipation ~**V** (flow magnitude), as oppose to ~∂**V**; e.g., first-order upwinding, or composite schemes

Linear theory – effect of viscosity anisotropy

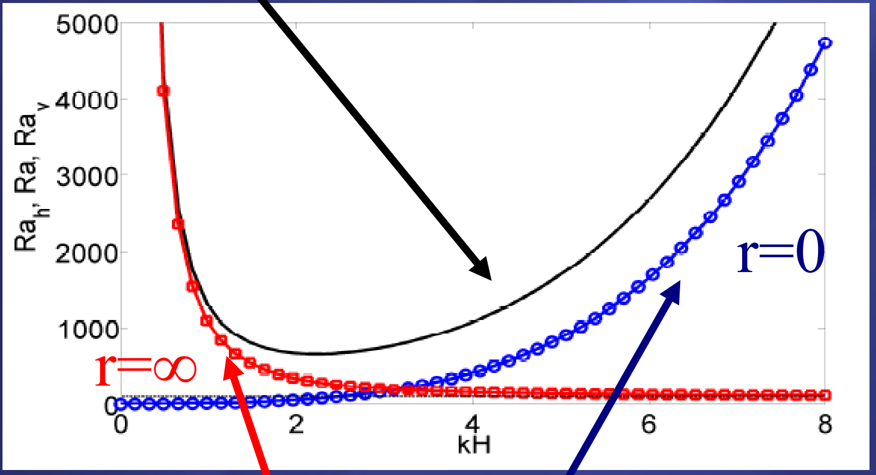


Asymptotic marginal stability relations for a finite Prandtl number and $\nu_h = \nu_v$ (black solid), $\nu_v = 0$ (blue circles) and $\nu_h = 0$ (red squares). Respective Rayleigh numbers Ra_h, Ra and Ra_v are shown in function of the horizontal wave number. Stability region is below the curves.

Marginal stability relation

$$Ra = \frac{H^4}{k^2}\left(n^2\left(\frac{\pi}{H}\right)^2 + k^2\right)^3$$

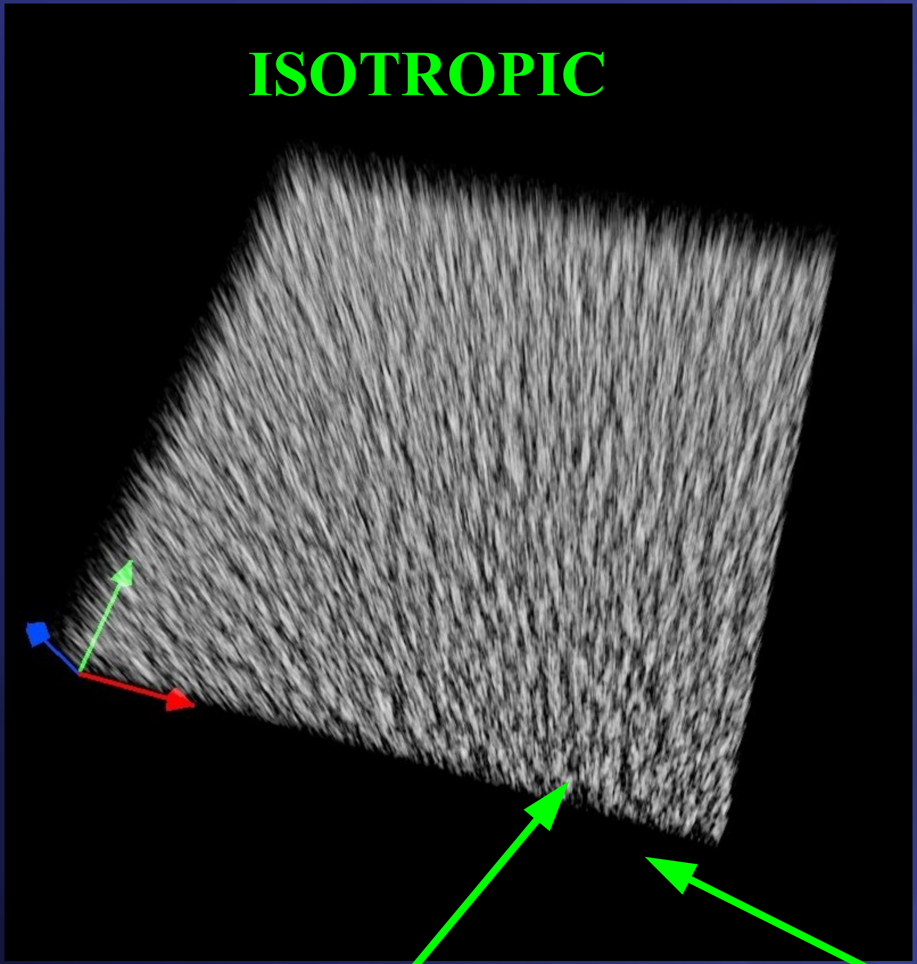
Isotropic

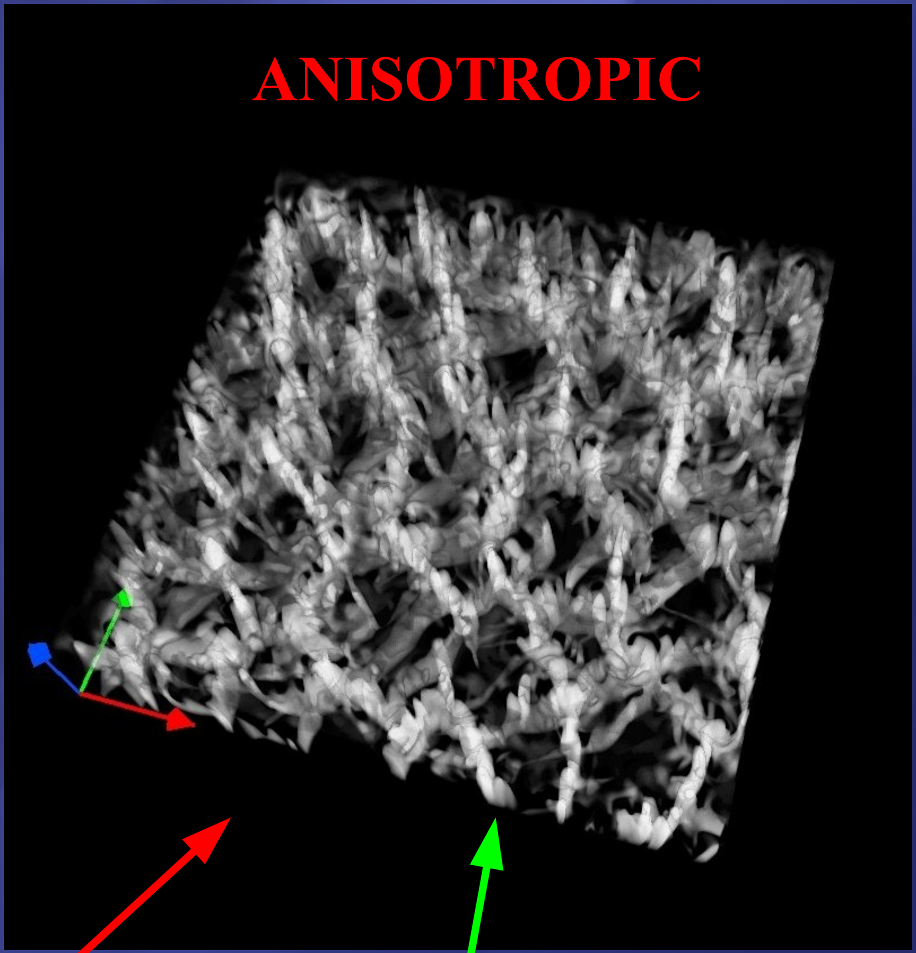


Anisotropic

$$Ra_h = \frac{H^4}{k^2}\left(n^2\left(\frac{\pi}{\frac{H}{\sqrt{r}}}\right)^2 + k^2\right)^3 \frac{\left(n^2\left(\frac{\pi}{H}\right)^2 + k^2\right)}{\left(n^2\left(\frac{\pi}{H}\right)^2 r + k^2\right)}$$

Canonical case: V=[0,0] and constant viscosities





Structure of thermal convection over heated plate. Vertical velocities after 6h of simulated time are shown within the PBL depth. Bright and dark volumes denote updrafts and downdrafts, respectively. The only difference between the two solutions is the value of viscosity in horizontal entries of the stress tensor, $\nu_h = 2.5 \; m^2 s^{-1}$ *and* $\nu_h = 70 \; m^2 s^{-1}$ *; the vertical entry* $\nu_v = 2.5 \; m^2 s^{-1}$ *both.*

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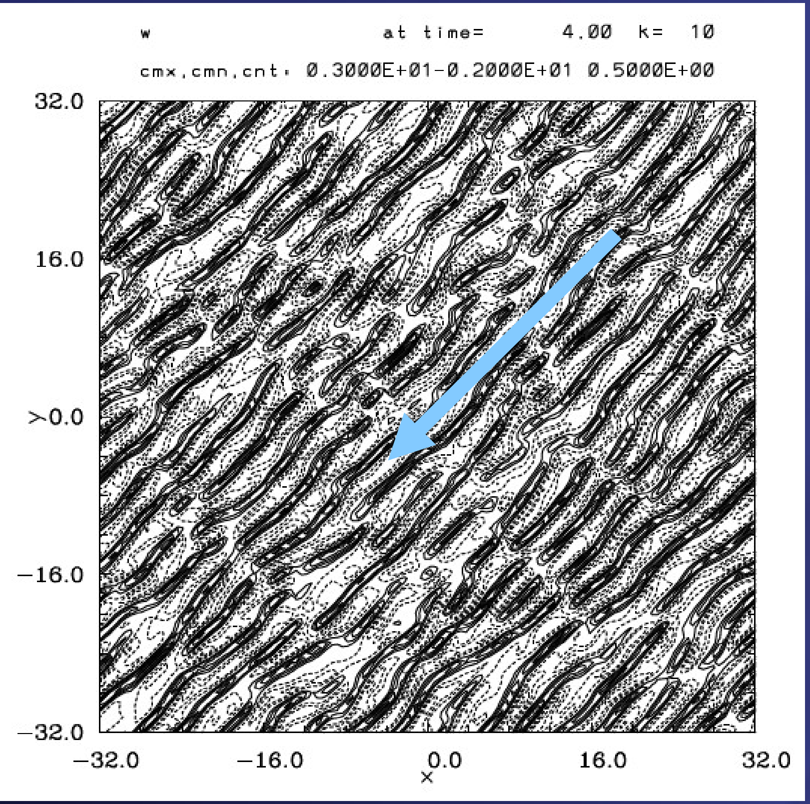
Numerical substantiation

Series of LES using the EULAG model



dz=50 m

V = [-10,-10] m/s

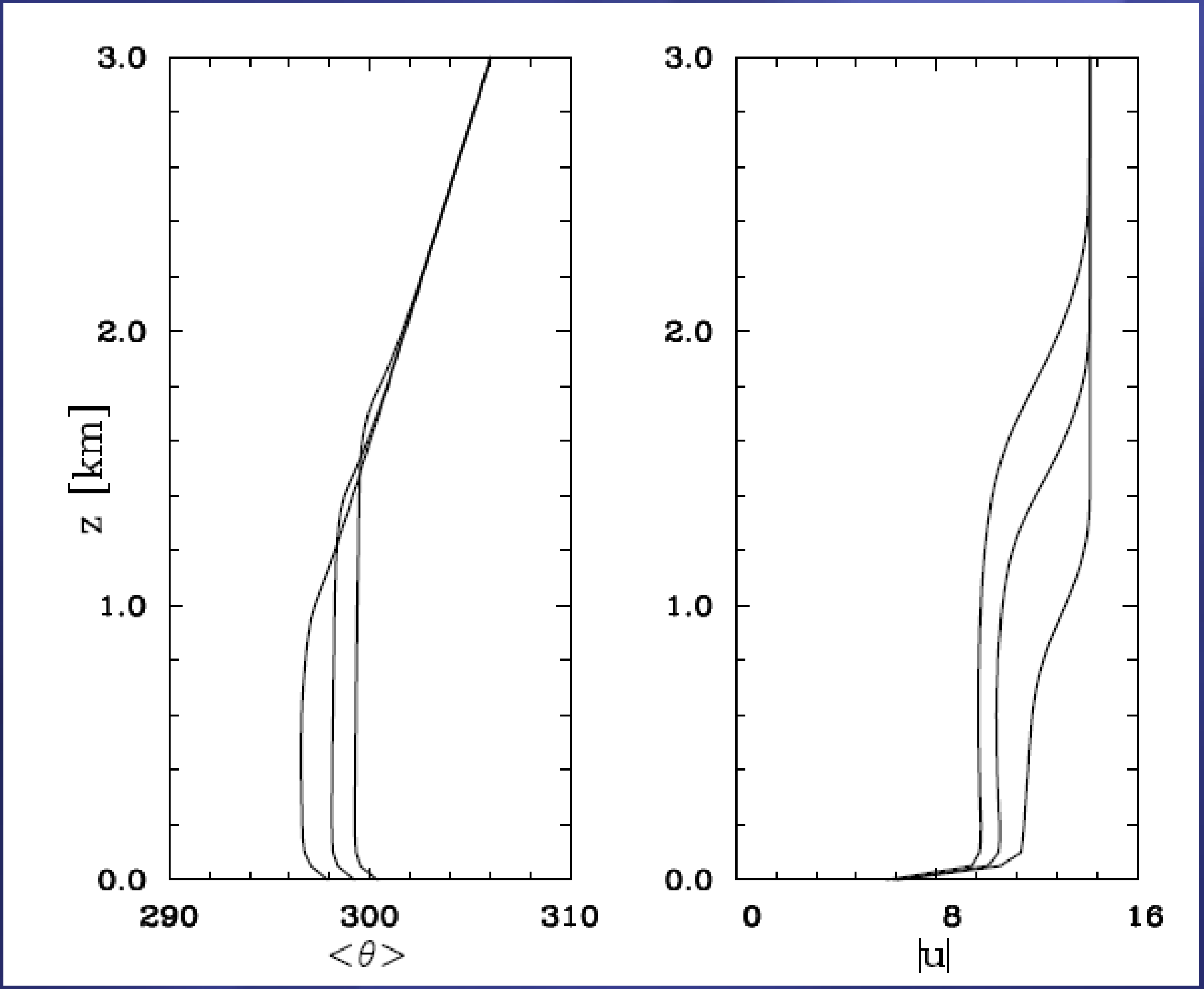
dx=dy≈500 m

Heat flux
hfx≈200 W/m^2

Flat lower boundary, doubly periodic horizontal domain, Boussinesq option

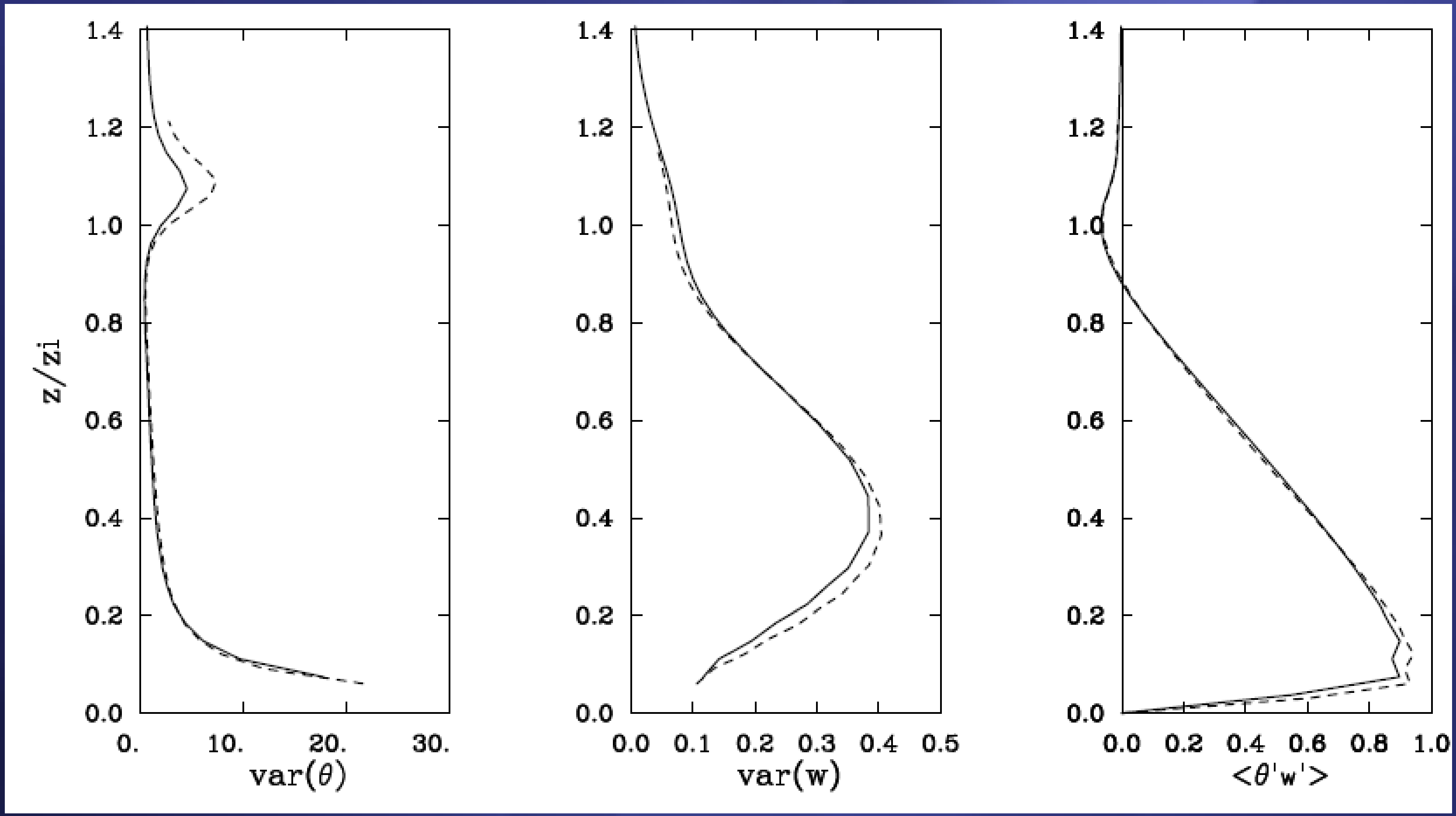
Reference setup alludes to contemporary, mesoscale cloud-resolving NWP

Potential temperature and wind speed



Profiles of potential temperature and horizontal wind speed at t=2, 4 and 6 h

Turbulent profiles



Dimensionless profiles of temperature and vertical velocity variance (left and center) and of turbulent heat flux (right), at t = 4 (solid) and 6 h (dashed).

Is the reference flow resolved ?

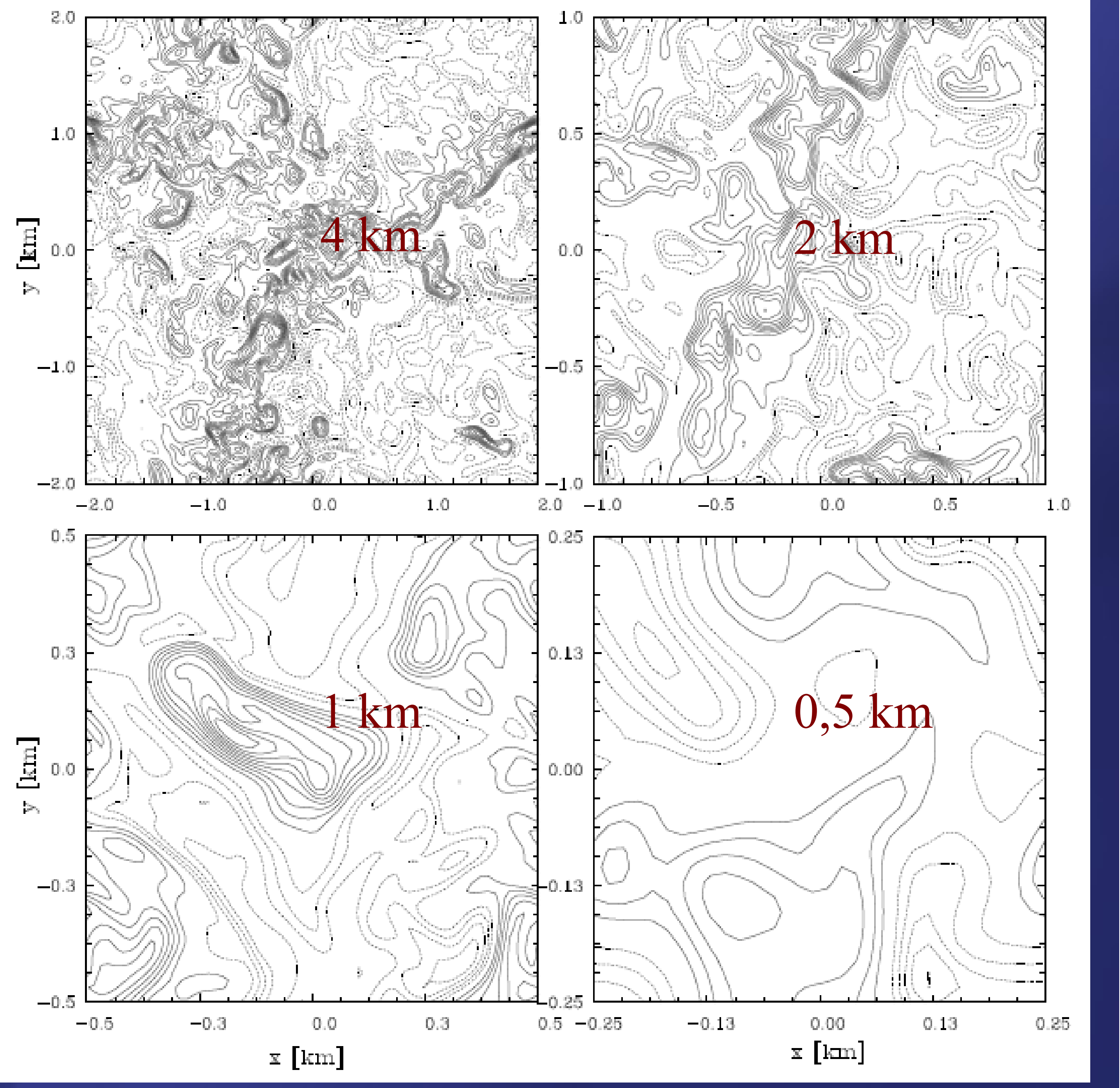
High resolution tests on the fraction of the reference domain

Domain and resolution required to faithfully represent convective structures:

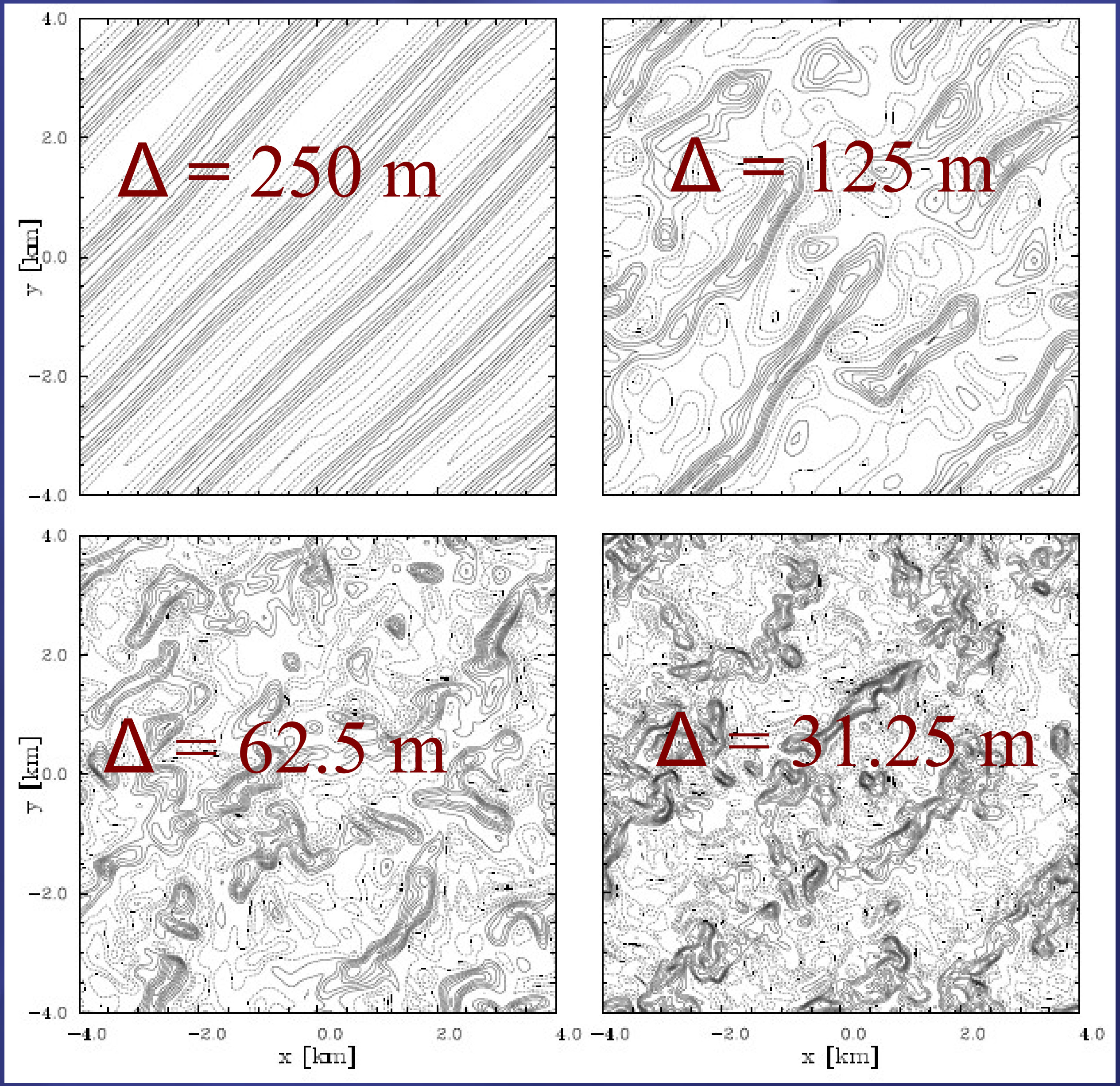
- $D = O(10)$ km x $O(10)$ km in the horizontal
- $\Delta = O(10)$ m horizontal resolution

Horizontal domain size required to capture the structural organization

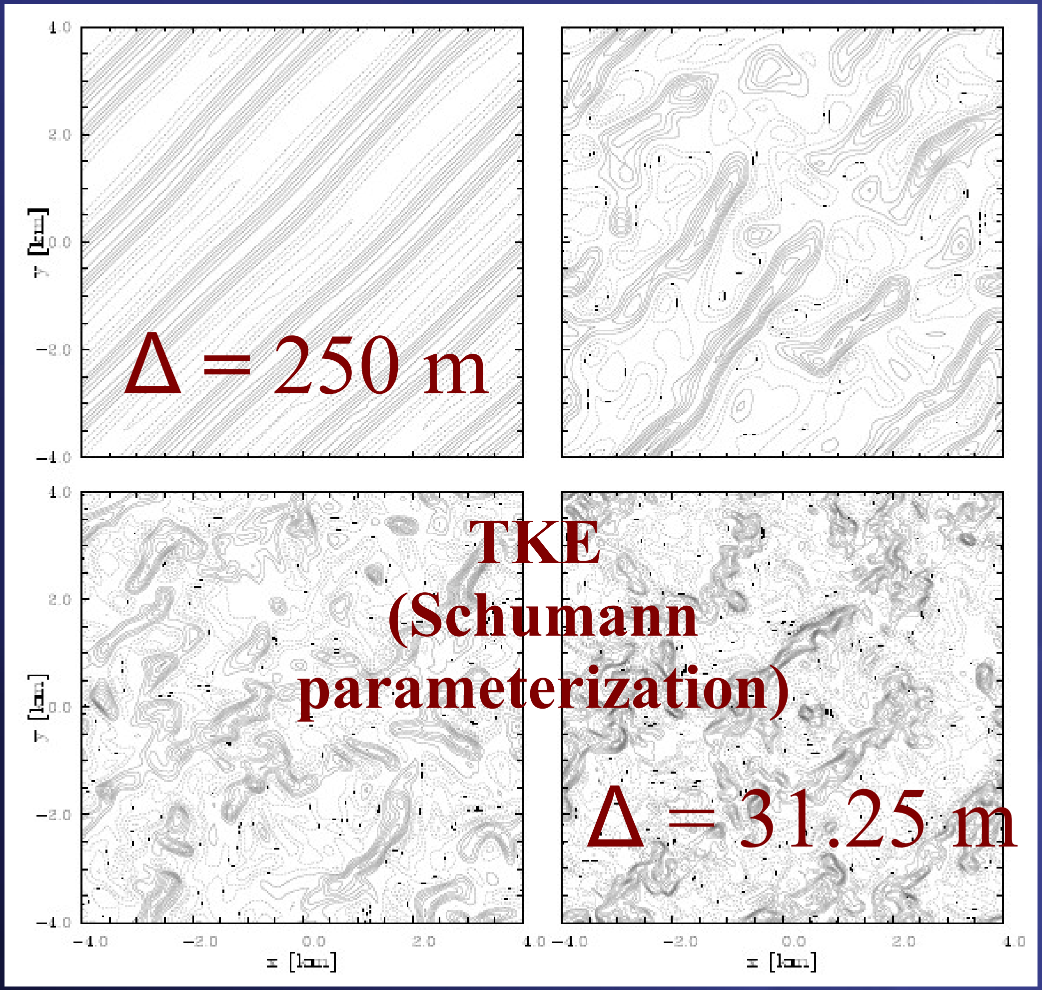
> 4 km x 4 km

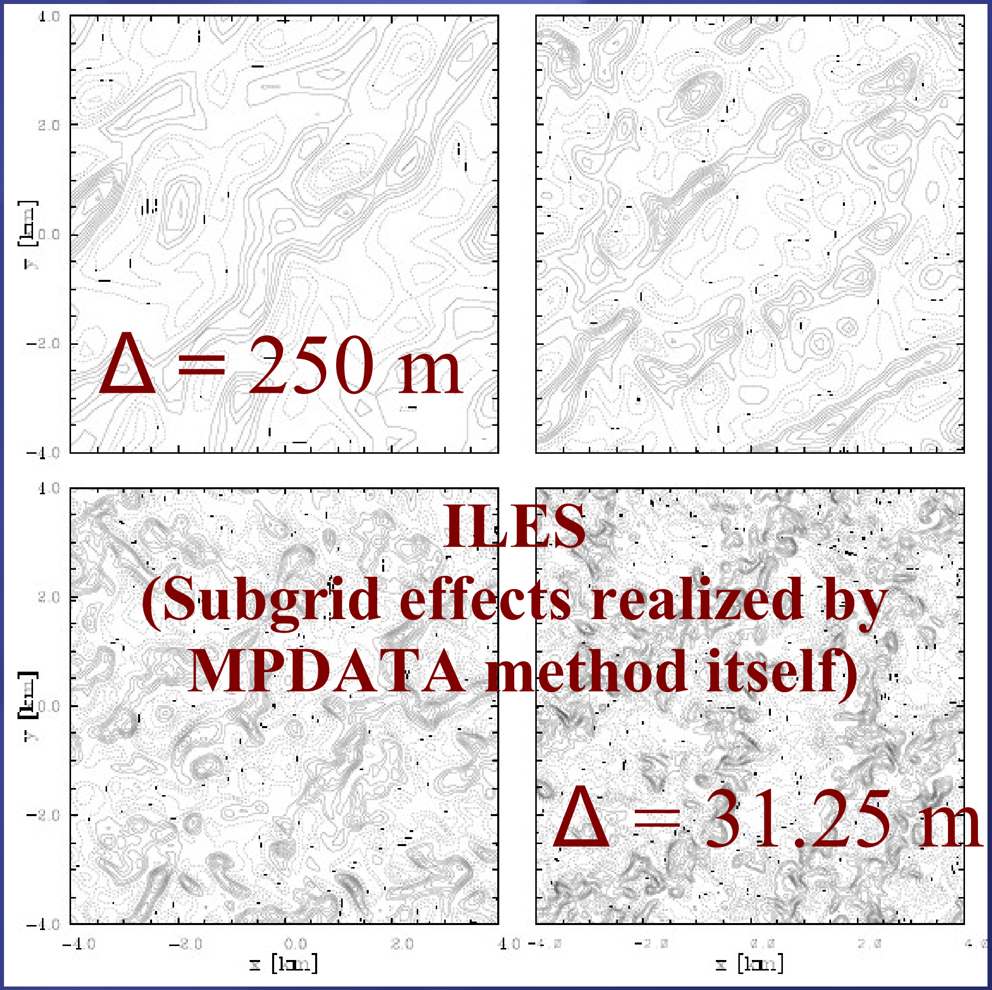


Grid spacing required to resolve the width of updrafts is of *O* (10) m



Comparision of TKE and ILES convergence tests





Implicit LES seems to minimize viscosity, which is visible in the most underresolved case, where coarse resolution is more similar to the high resolution.

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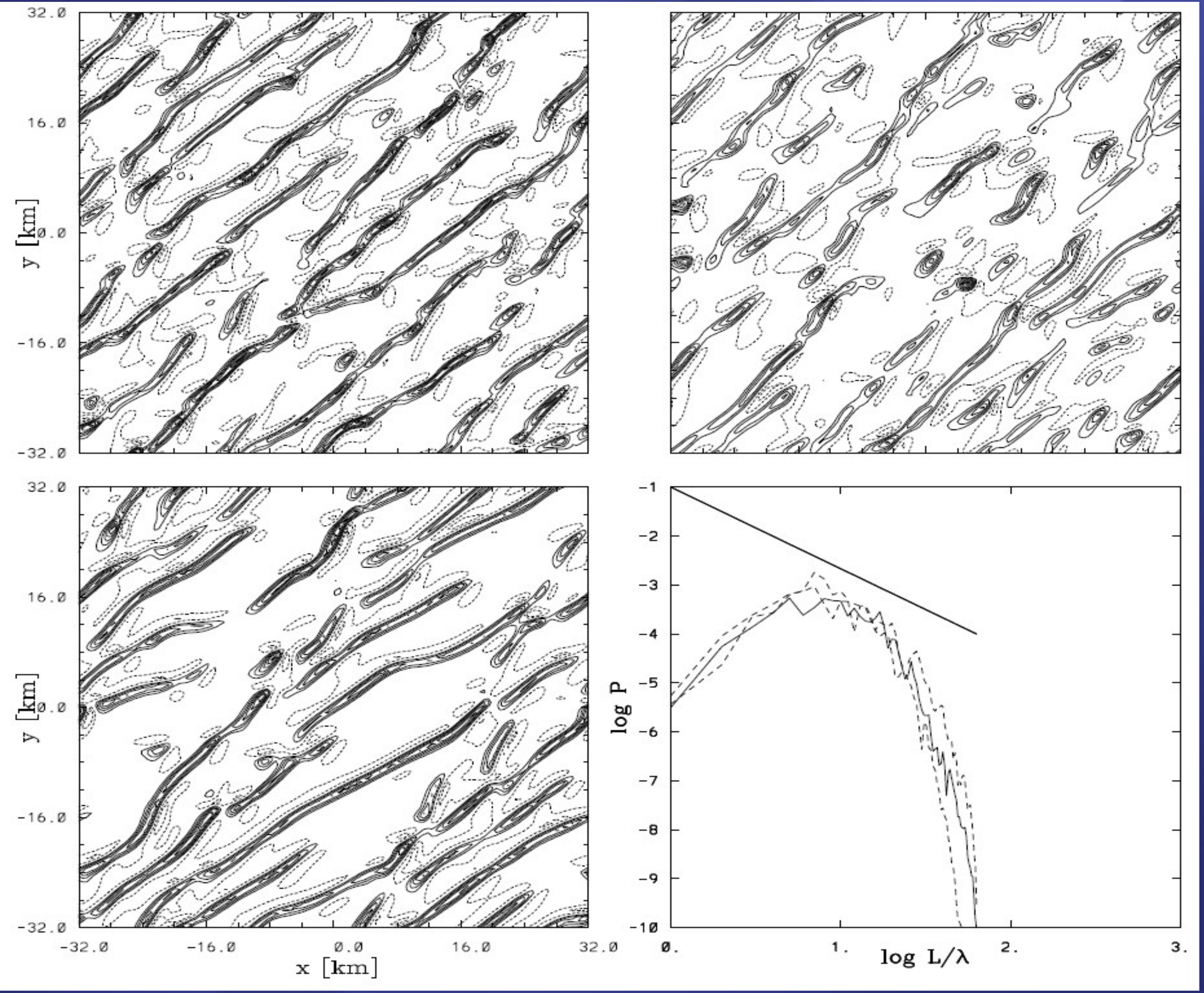
Convective picture – reference ILES simulations, but with different filters (anisotropic viscoisty)

Composite MPDATA: 1st order UPWIND every 4th dt

1-2-1filter

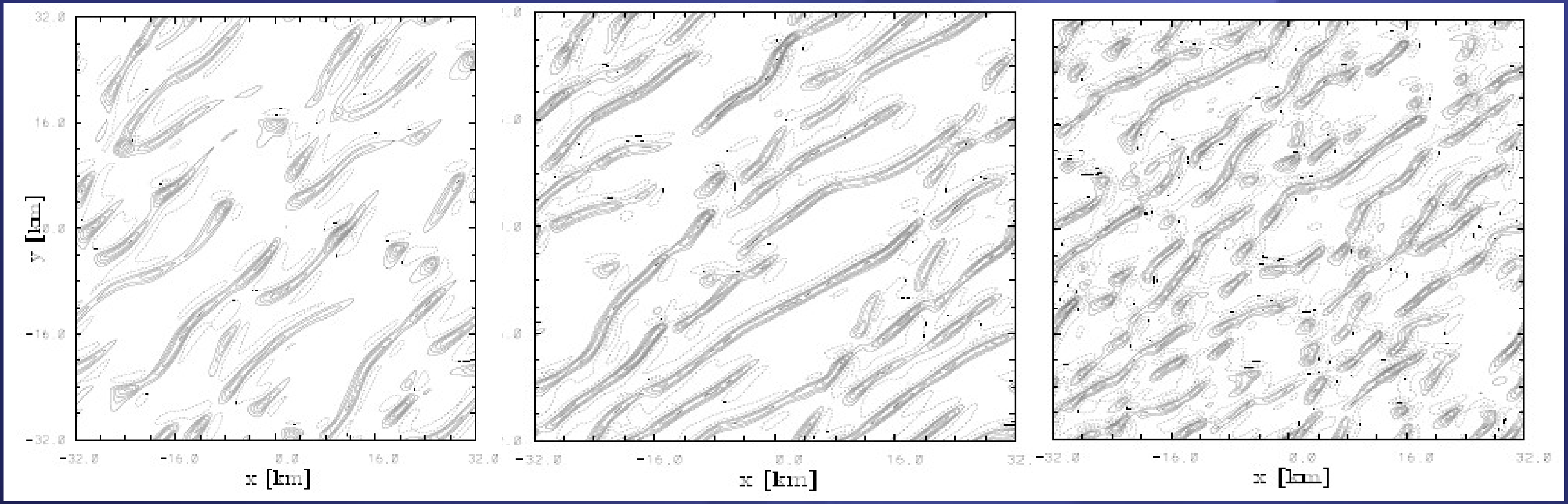


Explicit anisotropic viscosity

Diagonal spectra 2D

Different filtering gives similar results

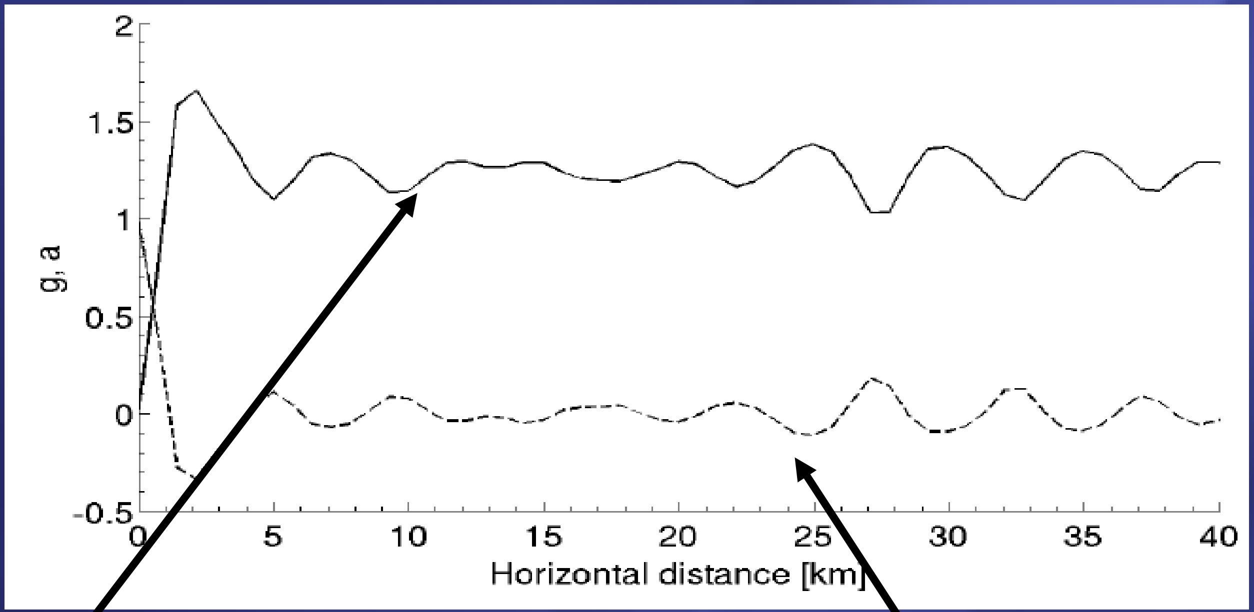
Illustration of anisotropic viscosity effects



ILES simulations with anisotropic viscosity vertical to horizontal ratio
$r = 2^k/170$, $k = -1, 0, 1$

Contours depict vertical velocities in 1/3 BL height after 4h of simulation. Updrafts are getting closer as the anisotropy ratio is getting weaker.

What do we see in the picture ? Structure quantification.



Second order structure function

$$g(\delta x, \delta y) = \frac{1}{nx \cdot ny} \Sigma_{i=1}^{nx} \Sigma_{j=1}^{ny} [w(x_i + \delta x, y_j + \delta y) - w(x_i, y_j)]^2$$

Autocorrelation function

$$a(\delta x, \delta y) = \frac{1}{\sigma^2} \Sigma_{i=1}^{nx} \Sigma_{j=1}^{ny} [w(x_i + \delta x, y_j + \delta y) - \overline{w}][w(x_i, y_j) - \overline{w}]$$

Stability vs. Growth rates of linear modes
- linear theory -

Horizontal Rayleigh number:

$$Ra_h = \frac{H^4}{k^2}\left(n^2\left(\frac{\pi}{\frac{H}{\sqrt{r}}}\right)^2 + k^2\right)^3 \frac{\left(n^2\left(\frac{\pi}{H}\right)^2 + k^2\right)}{\left(n^2\left(\frac{\pi}{H}\right)^2 r + k^2\right)}$$

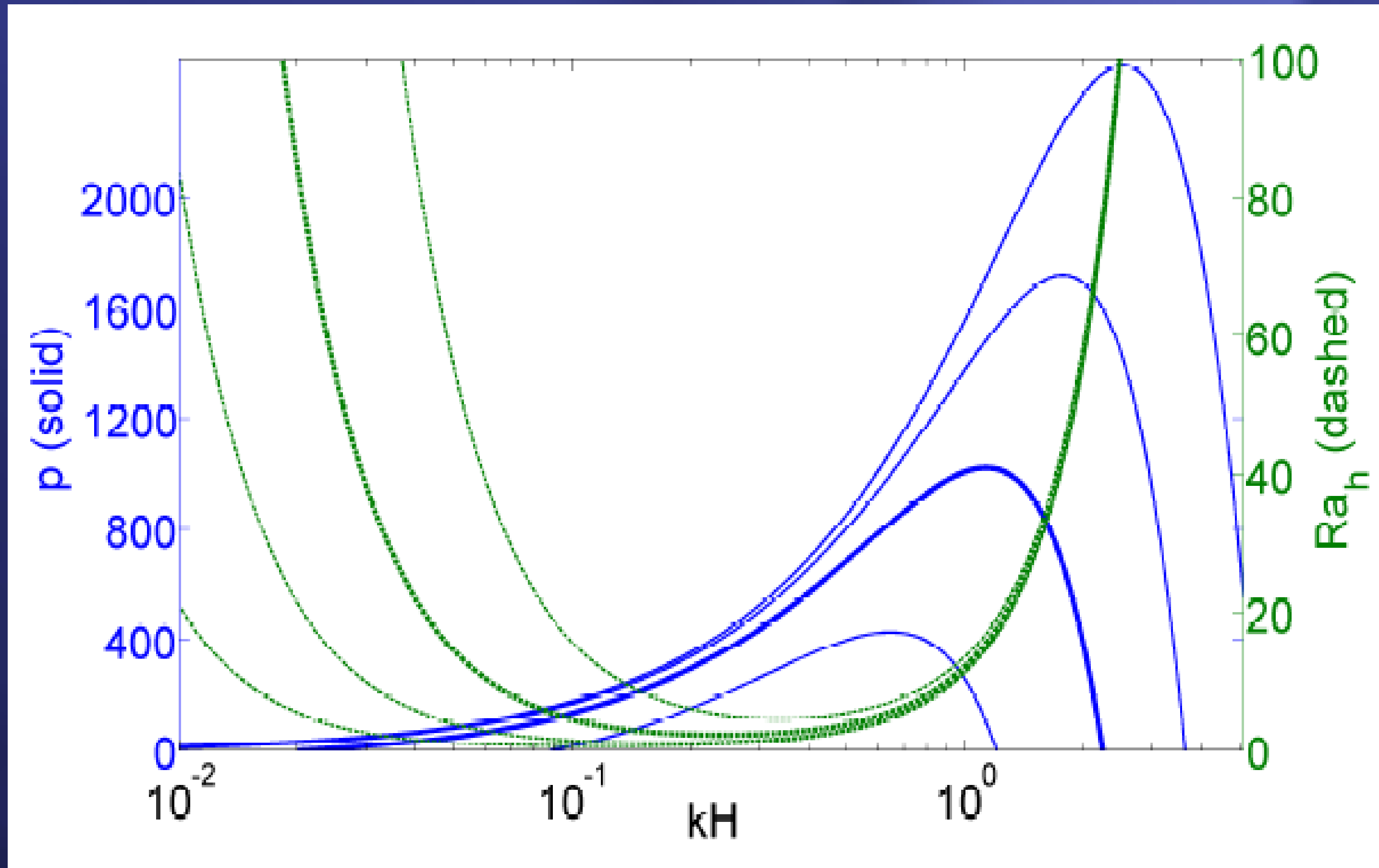
Critical wavenumber formula:

$$k_{cr}^2 = \frac{1}{4}\left(\frac{\pi}{H}\right)^2\left(\sqrt{8r+1} - 1\right)$$

Mode growth rate :

$$p_{\pm}^{n} = \frac{\nu_v}{2P}\left(\frac{\pi^2 n^2}{H^2} + \frac{k^2}{r}\right)\left(-(P+1) \pm \sqrt{(1-P)^2 + 4Ra_v P \frac{r^2}{Ra_h|_{(14)}}}\right)$$

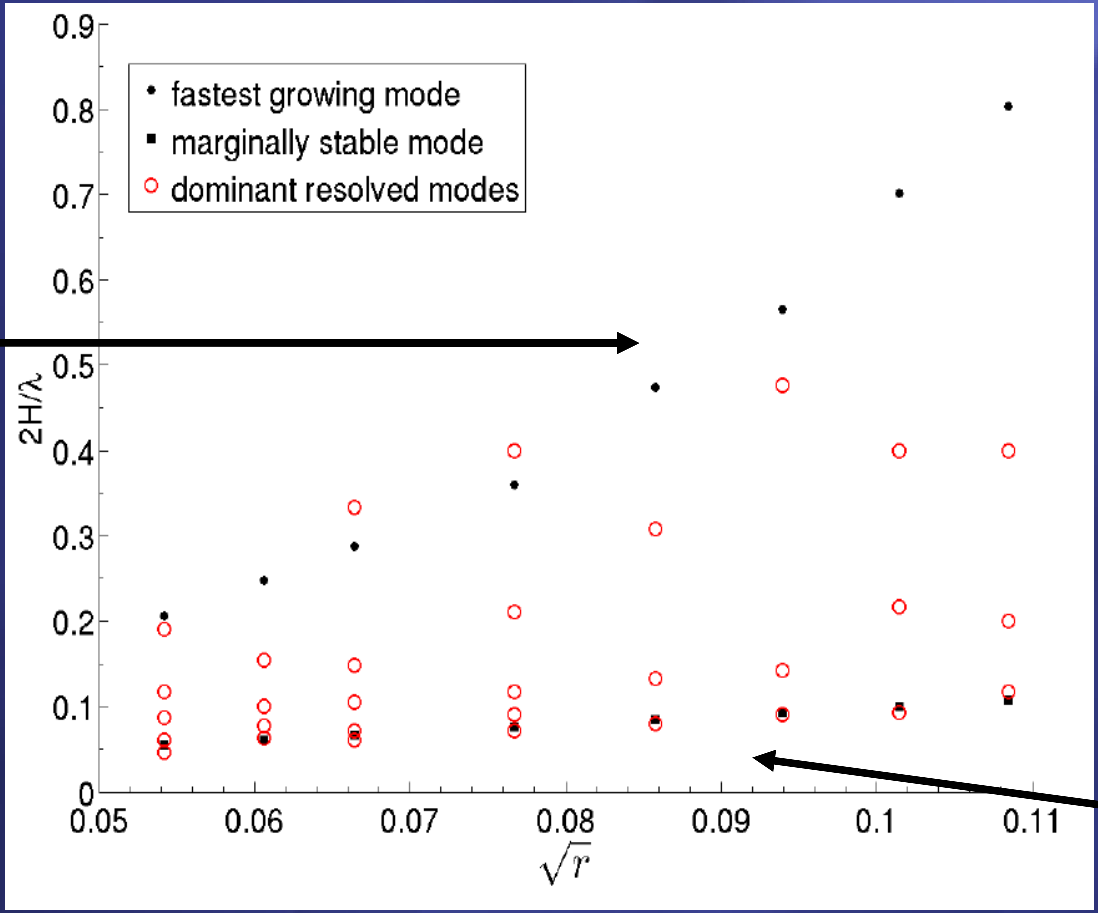
Extremum of the mode growth rate formula is quite complicated, therefore →



Growth rate curves p(kH) (blue solid) and marginal stability curves R_c(kH) (green dashed) for the critical vertical mode n = 1; thick lines correspond to the solutions $r = r_0 = 1/170$, whereas thin lines are for $r = 0.5r_0$, $2r_0$, $4r_0$ while keeping ν fixed.

The critical mode is different from the fastest growing mode

The dependence of dominant modes' wavelength on √r (at constant ν)



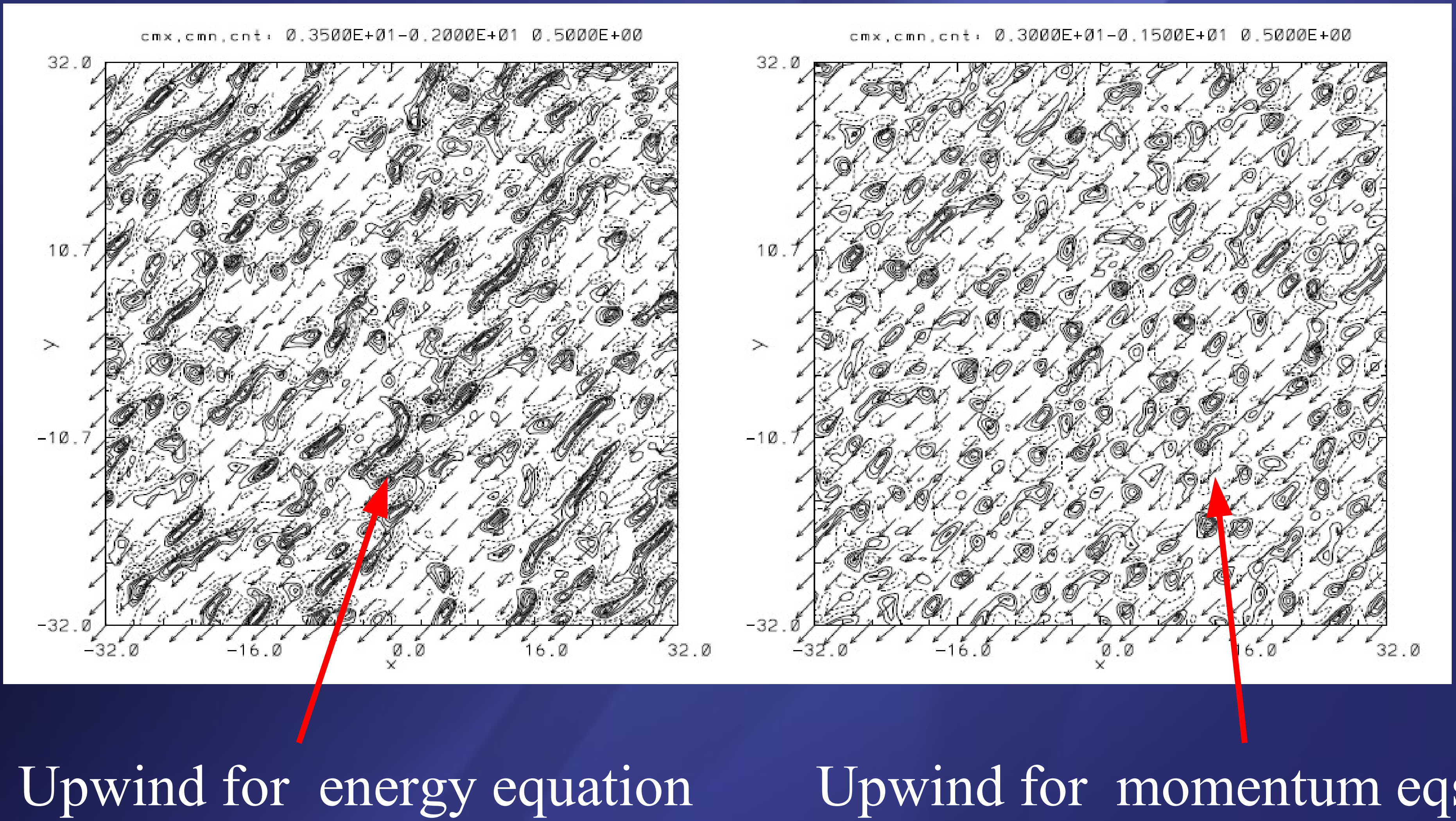
Fastest growing modes

Marginally stable modes

The lines marked with bullets and full squares are, respectively, the linear theory estimates at the maxima of growth function $p_1(k)$, for n = 1, and the marginal stability relation for the critical modes.

Observed waves depend on the viscosity anisotropy as a band of modes

Current research



Anisotropic approximation to model equations may lead to similar effects as anisotropic viscosity.

Conclusions

- Cellular convection simulated with meso- and large-scale models may be only a spurious result of the effective anisotropic viscosity
- The linear theory has a skill to quantify the anisotropic viscosity effects
- Implicit numerical viscosity and dispersion are well known. There appears to be a need for appreciating ``implicit numerical topology'' while analyzing under-resolved convective structures and cloud coverage
- Non-oscillatory methods based on MPDATA appear well suited for cloud-resolving simulations, as they:

i) do not depend on explicit subgrid-scale models;

ii) do not require filters for numerical stability; and

iii) are numerically isotropic

“Tenets” of convective-fields simulation

- Skepticism for the ``eye-pleasing’’ convective structures appearing in large-scale “cloud-resolving” simulations
- Verification of the adequacy of subgrid-scale models using convective benchmarks
- Awareness of the numerical model design; e.g., avoidance of a first-order dissipative numerics, and ad-hoc filters.
- Control of numerical viscosity: not every dissipative numerics has adequate implicit LES property.

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On numerical realizability of thermal convection

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ABSTRACT

Astounded at the regularity of convective structures observed in simulations of mesoscale flow past realistic topography, we investigate the computational aspects of a classical problem of flow over a heated plane. We find that the numerical solutions are sensitive to viscosity, either incorporated a priori or effectively realized in computational models. In particular, anisotropic viscosity can lead to regular convective structures that mimic naturally realizable Rayleigh–Bénard cells, which are unphysical for the specified external parameter range. Details of the viscosity appear to play a secondary role; that is, similar structures can occur for prescribed constant viscosities, explicit subgrid-scale turbulence models, ad-hoc numerical filters, or implicit dissipation of numerical schemes. This implies the need for a careful selection of numerical tools suitable for convection-resolving simulations of atmospheric circulations. The implicit large-eddy-simulation (ILES) approach using non-oscillatory schemes is especially attractive, as for under-resolved calculations it reproduces well the coarsened results of finely-resolved boundary layer convection.