# *On the selection of prognostic moments in parameterisation schemes for drop sedimentation*

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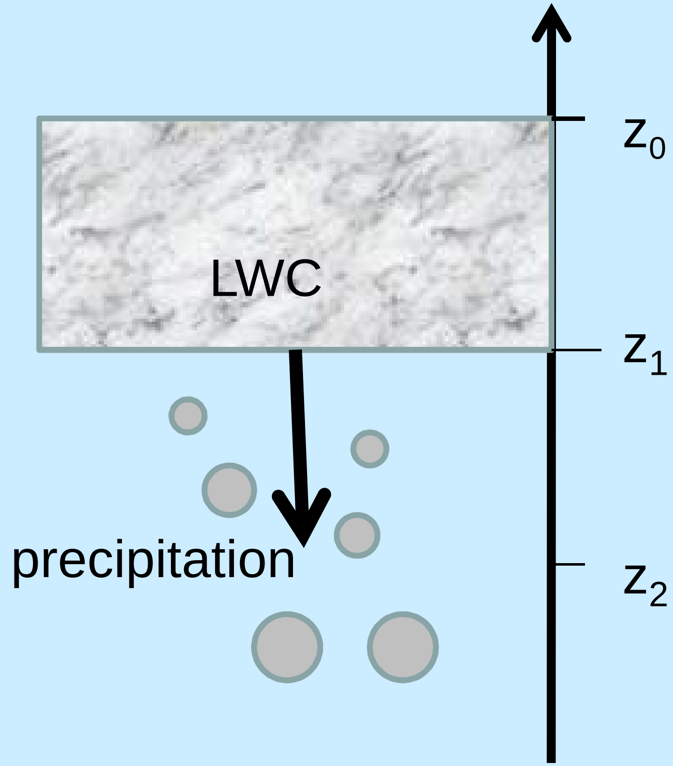
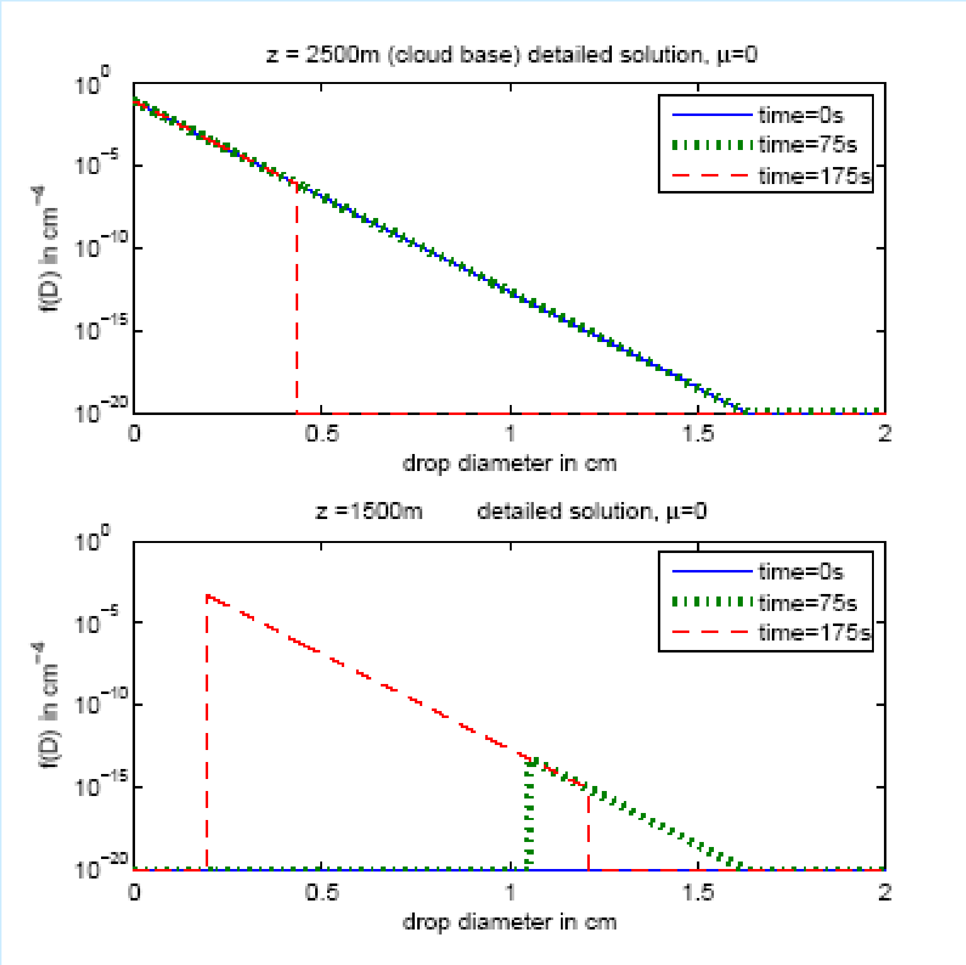
# Physical Model




# Sedimentation

cloud base

below cloud




*gravitational sorting*

What happens in a parameterization model?

## Parameterization model

Forecast few Moments $M_k(t)$ instead of size distribution $f(D,t)$

Moment of order $k$ $\quad M_k(\vec{r},t) = \int_0^\infty D^k f(D,\vec{r},t)\, dD$

$M_0 = N$ number density, $\quad M_3 \propto L$ mass density

Assume *closure conditions* (e.g. for rain drops):

(1) self preserving size distribution:

$f(D) = n_0 D^\mu \exp(-\lambda D)$ with parameters $n_0$, $\mu$, $\lambda$

(2) **'1-Moment Model':** Use **1** prognostic moment $M_k$, any $k$ possible.

Let $n_0, = \mu = const. \longrightarrow M_k = M_k(\lambda)$

Any moment $M_l = fct(M_k)$!

Sedimentation flux $F_k = \int_0^\infty v_T(D) D^k f(D)\, dD = \alpha M_{k+\beta} = fct(M_k)$

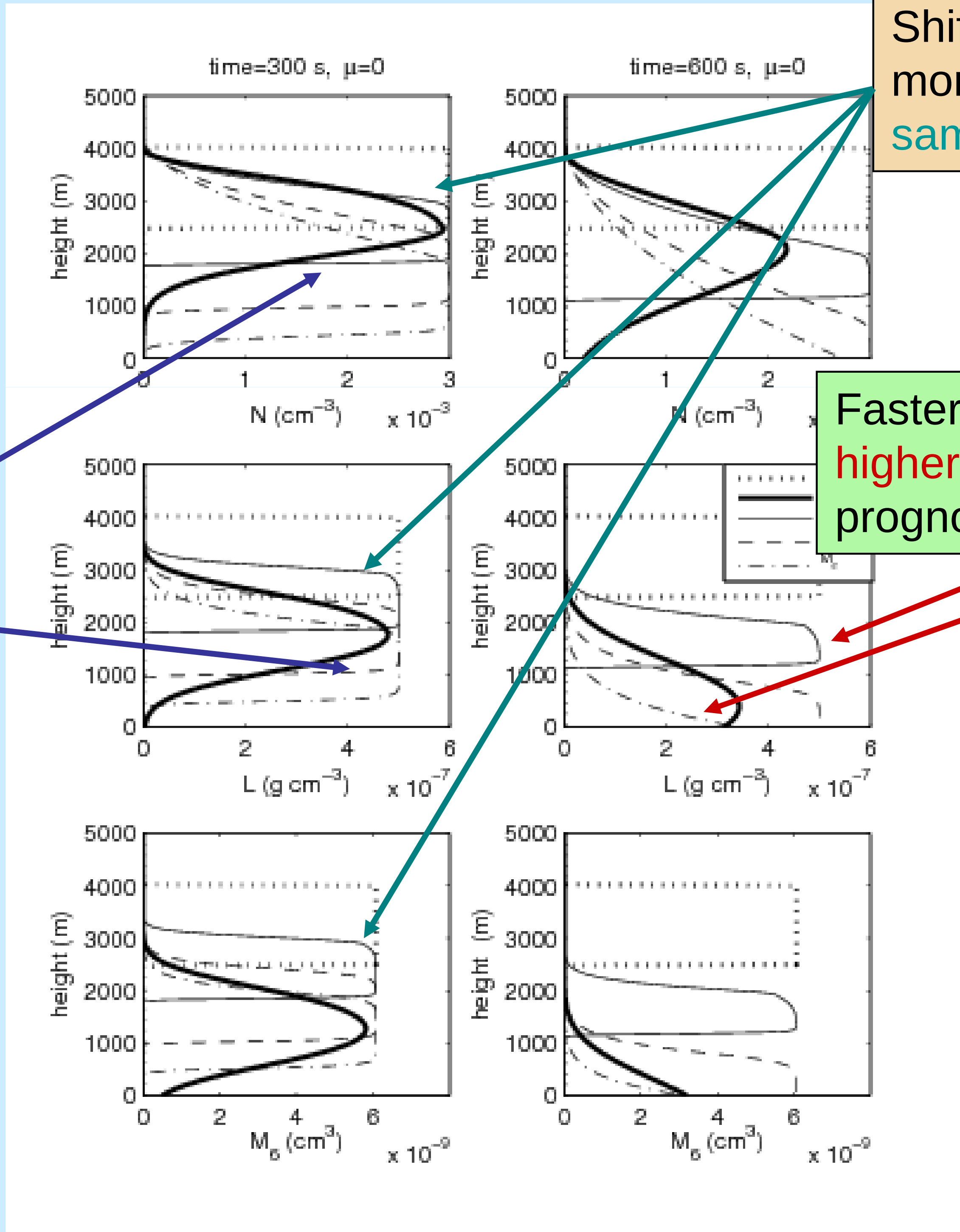
with sedim. velocity $v_T = \alpha D^\beta$

1-moment model

—— $M_0$ progn.

------ $M_3$ progn.

-·-·- $M_6$ prog.




Shift of signal of all moments $M_l$ with same speed

Faster shift of signal for higher order k of prognostic moment $M_k$

Shock wave

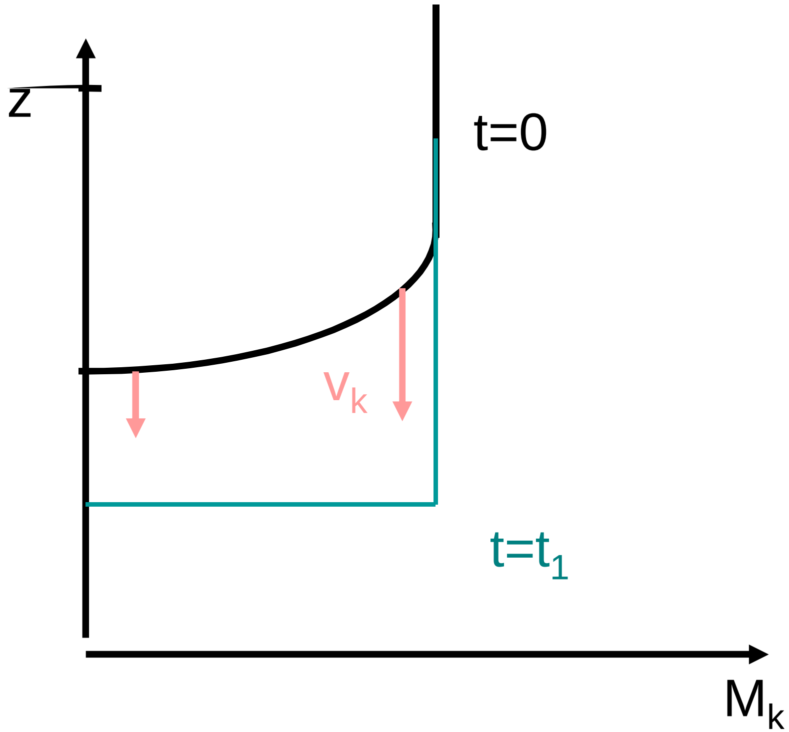
## Shock Wave in the parameterization model

$$\frac{\partial M_k}{\partial t} - \frac{\partial F_k(M_k)}{\partial z} = 0$$

$$\frac{\partial M_k}{\partial t} - \tilde{v}_k(M_k)\frac{\partial M_k}{\partial z} = 0 \qquad \text{quasi-linear advection equation}$$

$\tilde{v}_k(M_k)$ apparent advection velocity; advects **all** drops of ensemble

$$\frac{d\tilde{v}_k}{dM_k} > 0$$



## 2-Moment Model

Assume self preserving size distribution

$f(D) = n_0 D^{\mu} \exp(-\lambda D)$ with $\mu = const.$

Use **2** prognostic moments $M_j$ and $M_k$, any $j$, $k$ possible.

$\longrightarrow M_j,\ M_k = fct(\lambda, n_0)$

2 coupled quasi-linear PDE:

$$\frac{\partial M_j}{\partial t} - \frac{\partial F_j(M_j, M_k)}{\partial z} = 0$$

$$\frac{\partial M_k}{\partial t} - \frac{\partial F_k(M_j, M_k)}{\partial z} = 0$$

Conservation conditions for $M_j$, $M_k$.

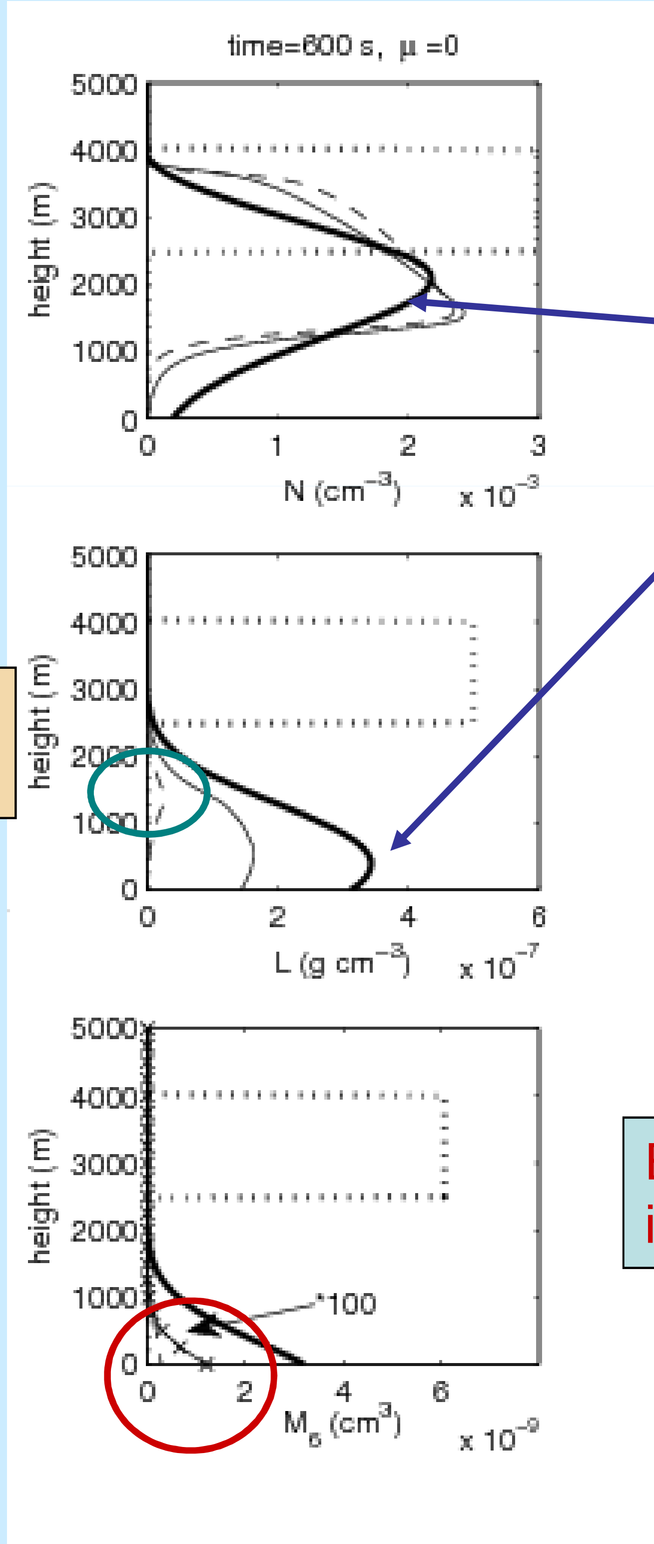
Any moment $M_l = fct(M_j, M_k)$!

2-Mom. Model

_____ $M_0$, $M_3$ progn.

-------- $M_0$, $M_6$ progn.

—— detaill. Model




Higher moment → faster propagation

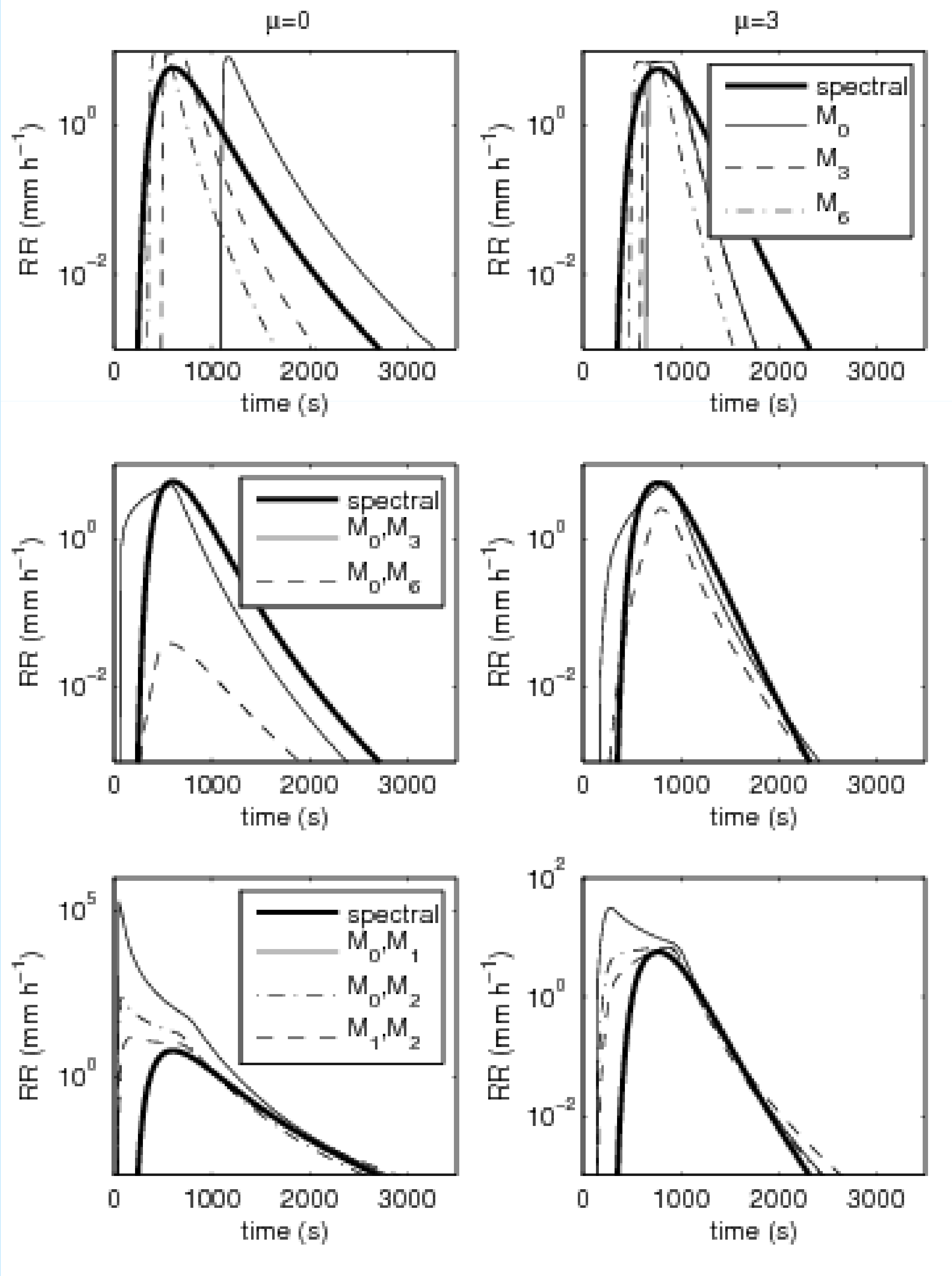
Reduced $M_3$–values in $M_0M_6$-model

Excessive $M_6$–values in $M_0M_3$-model

1 MOM

Rain Rate at z=0

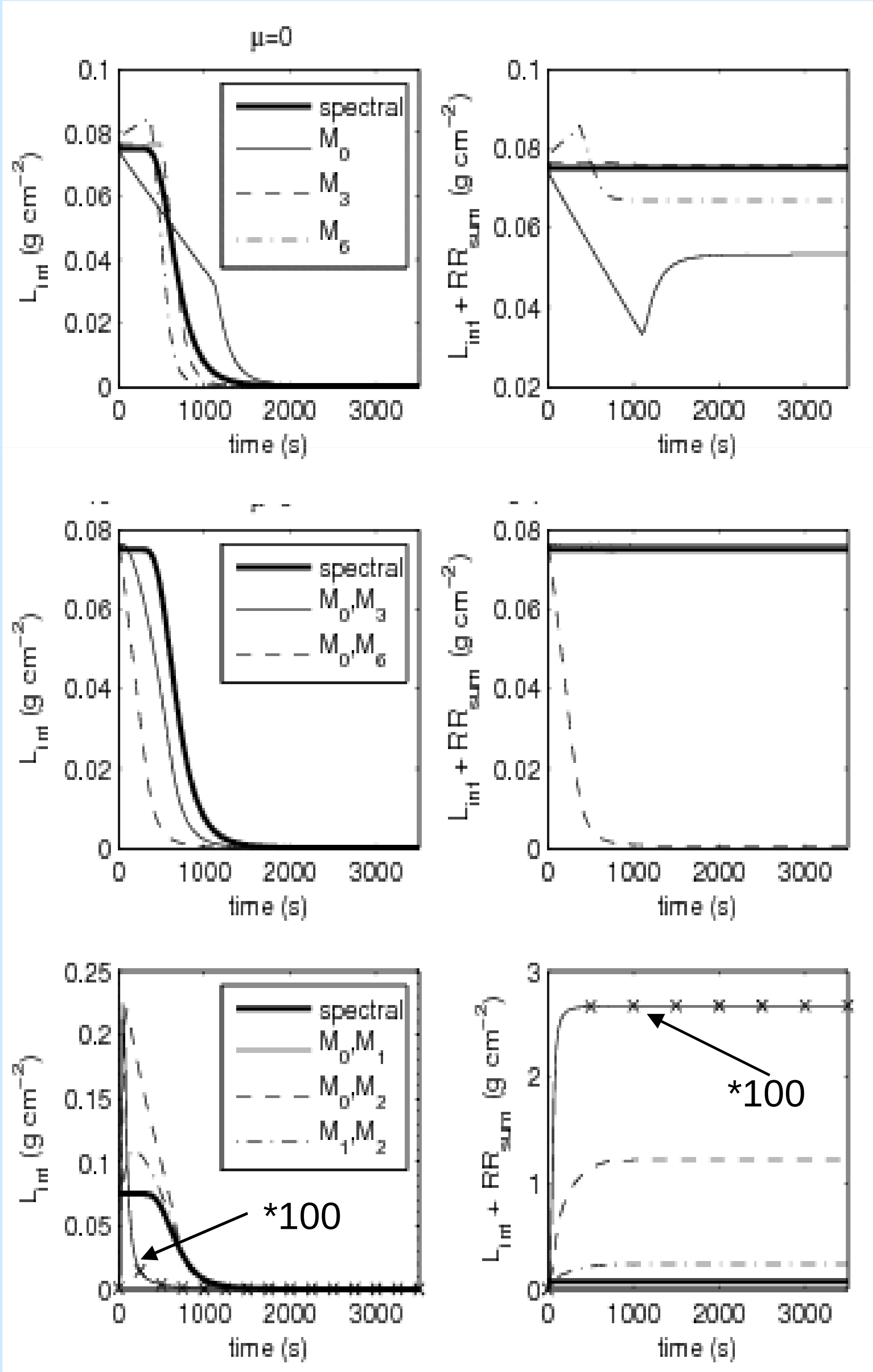
2 MOM



Test of mass conservation

$L_{int}$ = vertically integrated liquid water mass

$RR_{sum}$ = accumulated rain at z=0




1 MOM

2 MOM

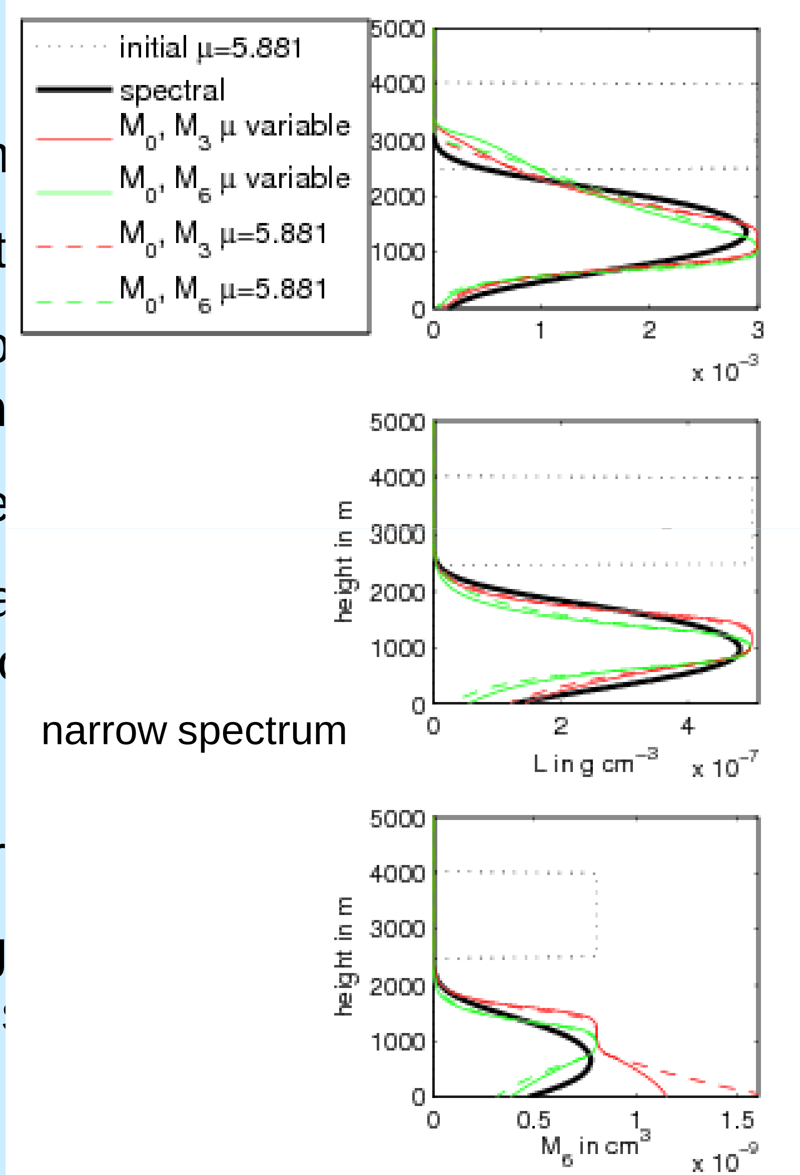
Summ

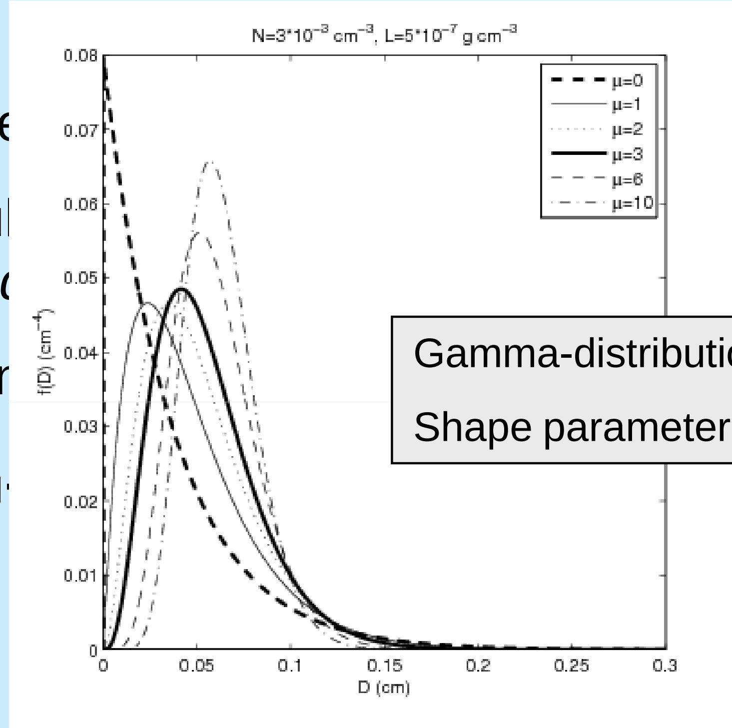
Result … e selecte

Reaso … sp. resul … **a**
**progn** … *over-/unc*

Conse … ne progr

Full pa … nks ~ $M_l$.
If $M_l$ n … tion.




Gamma-distribution

Shape parameter μ

Backgr

Averag … k of kth moment increases with k.
Effect i … ze distribution.

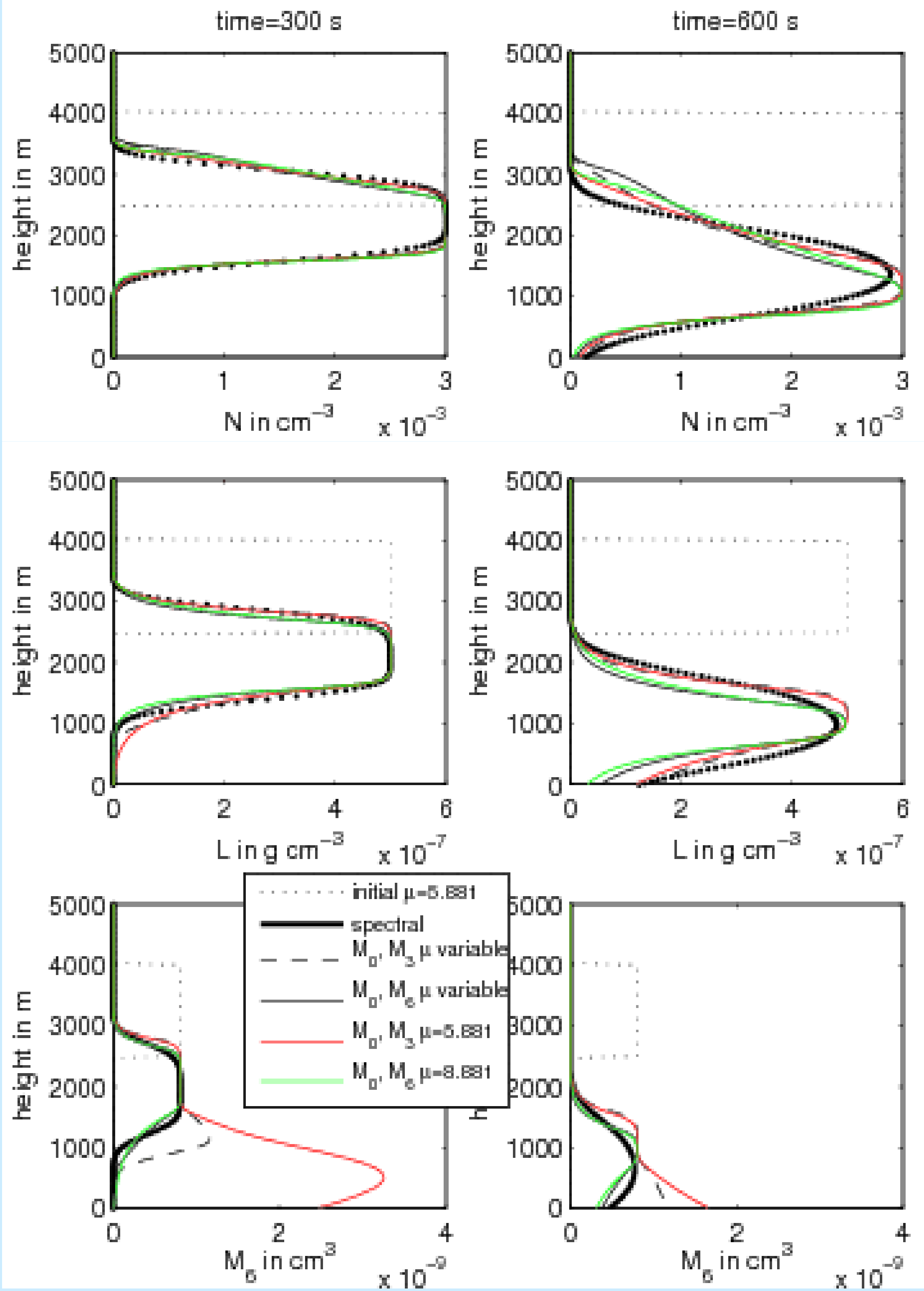
Outlook:

Use a narrow f(D)?

Test: Parameterize shape parameter ( -> Milbrandt and Yau, 2005)? Else????

Parameterize shape parameter

(Milbrandt + Yau, 2005)



Prognostic moments $M_j$, $M_k$ with $j < k$

Diagnosis $$M_l = \Gamma(l+\mu+1)\left[\frac{M_k}{\Gamma(k+\mu+1)}\right]^{(l-j)/(k-j)}\left[\frac{M_j}{\Gamma(j+\mu+1)}\right]^{(k-l)/(k-j)}$$

j=0, k=3; l=6

$j < k < l:$ $\quad M_l \propto M_k^{(l-j)/(k-j)}\, M_j^{-(l-k)/(k-j)}$

$\rightarrow$ $M_l$ small for $M_k$ small; $M_l$ huge for $M_j$ small [$M_{j,min} \neq 0$ required!]

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$j < l < k:$ $\quad M_l \propto M_k^{(l-j)/(k-j)}\, M_j^{(k-l)/(k-j)}$

j=0, k=6; l=3

$\rightarrow$ $M_l$ small for $M_k$ small or $M_j$ small

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j=3, k=6; l=0

$l < j < k:$ $\quad M_l \propto M_k^{-(j-l)/(k-j)}\, M_j^{(k-l)/(k-j)}$

$\rightarrow$ $M_l$ small for $M_j$ small; $M_l$ huge for $M_k$ small [$M_{k,min} \neq 0$ required!]

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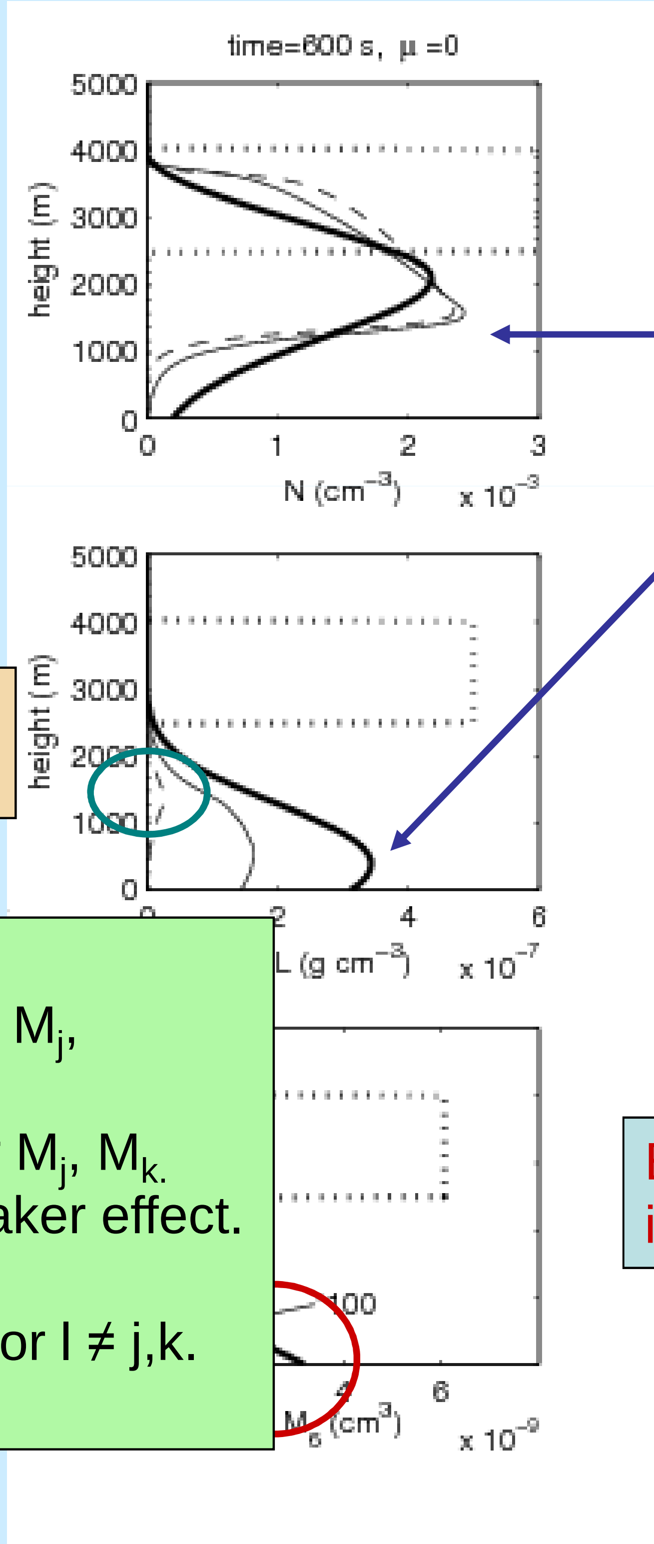


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*Problems:*

Acceptable only for progn. moment $M_j$, $M_k(z,t)$.

Conservation cond. fulfilled only for $M_j$, $M_k$.

Shock wave present, but much weaker effect.

Rain rate evtl. problematic.

Source rate $\sigma = fct(M_l)$ erroneous for $l \neq j,k$.

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