Predicting the Solid-to-Liquid Ratio of Precipitation using a Bulk Microphysics Scheme

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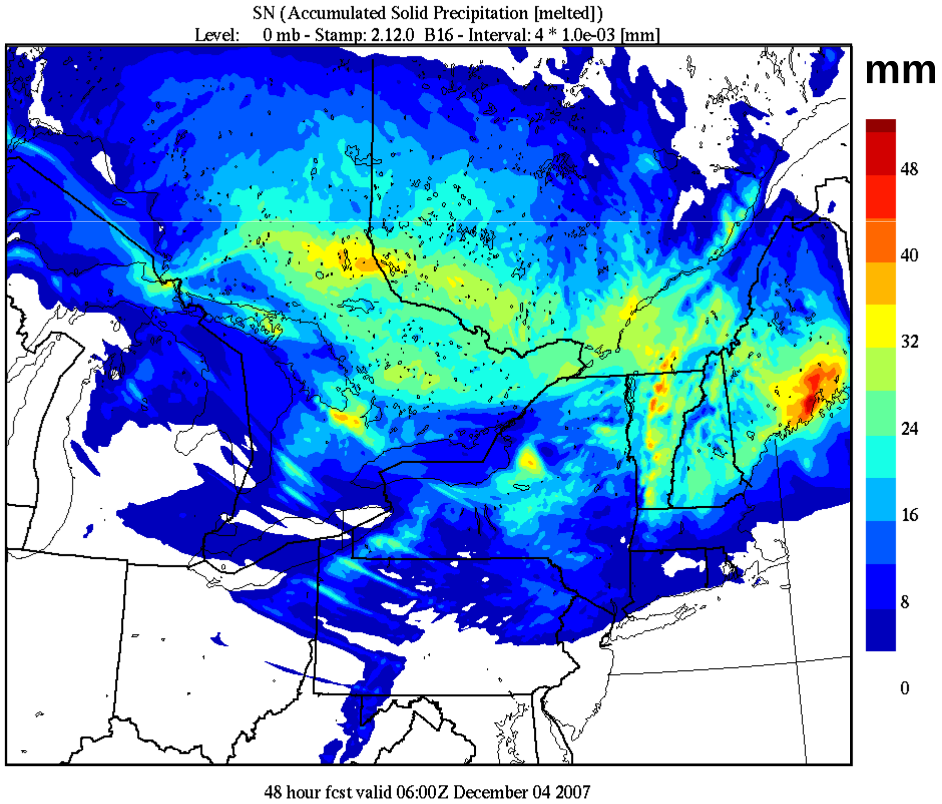
RPN Service through science

Standard forecast parameter:

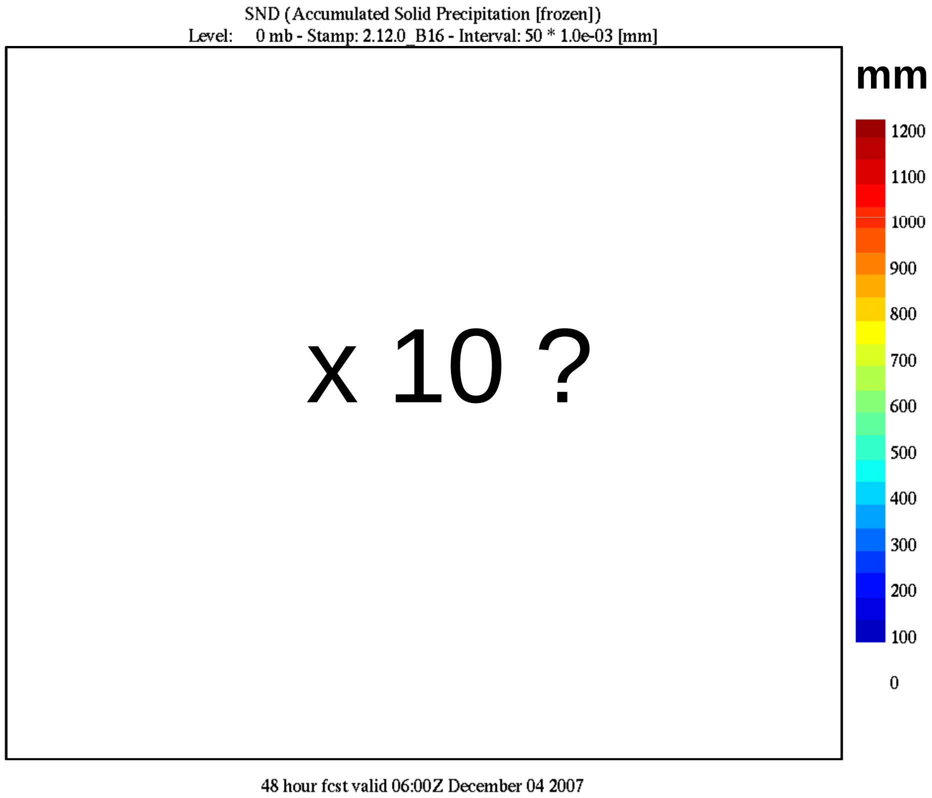
(directly from model QPF)



Accumulated Precipitation

(Liquid-Equivalent)

Desired forecast parameter:



Accumulated Precipitation

(Unmelted - i.e. *Snow Depth*)

OUTLINE

1. Background
2. Proposed method
3. Preliminary results
4. Conclusion

DEFINITIONS

SNOW DENSITY – bulk density of freshly falling snow **at the surface**

SOLID-TO-LIQUID ratio – ratio of the depth of the unmelted precipitation to its liquid-equivalent value

e.g. for precipitating snow with ρ_S = 100 kg m^{-3} (ρ_L = 1000 kg m^{-3}),

$$\text{Solid:Liquid} = (1/\rho_S) / (1/\rho_L)$$

$$= 1000 \text{ kg m}^{-3} / 100 \text{ kg m}^{-3}$$

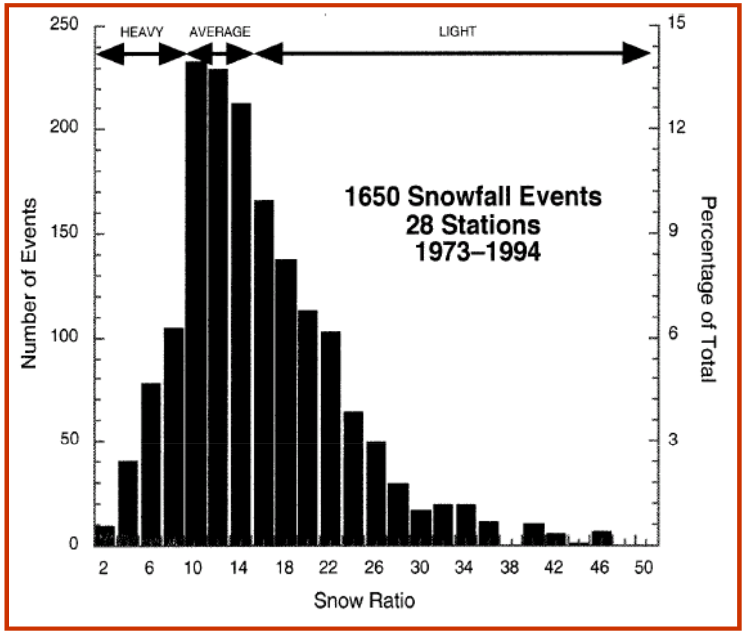
$$= 10:1$$

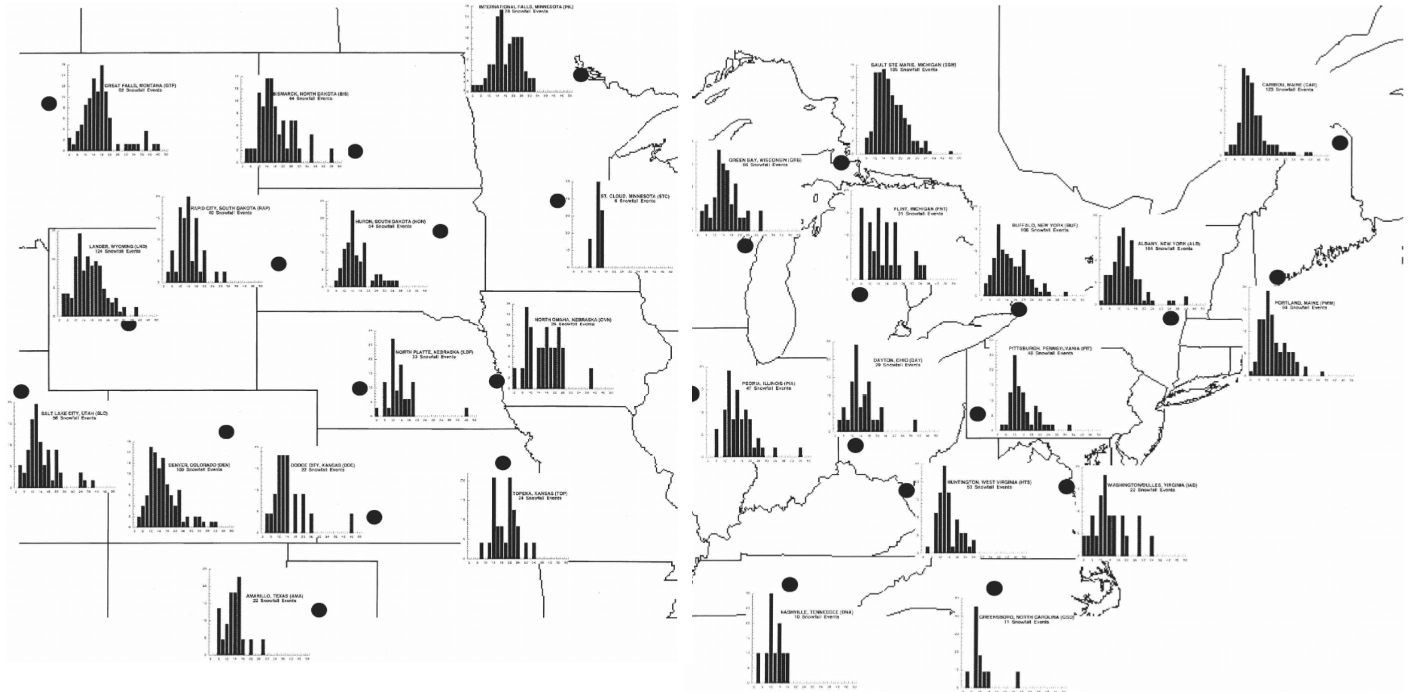
FACTORS AFFECTING SNOW DENSITY

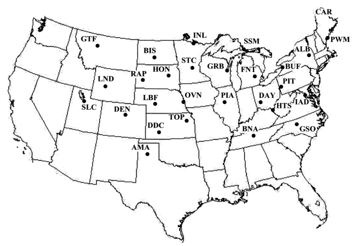
1. In-cloud processes (affect crystal habit and size)
   - supersaturation
   - liquid water content
2. Sub-cloud processes
   - sublimation
   - melting
3. Compaction upon impact

Observed SOLID-LIQUID ratios:

- can range from 3:1 to 100:1
- average value approximately 10:1
- varies geographically







Source: Ware et al. (2006), *Weather and Forecasting*

APPROACHES TO PREDICTION:

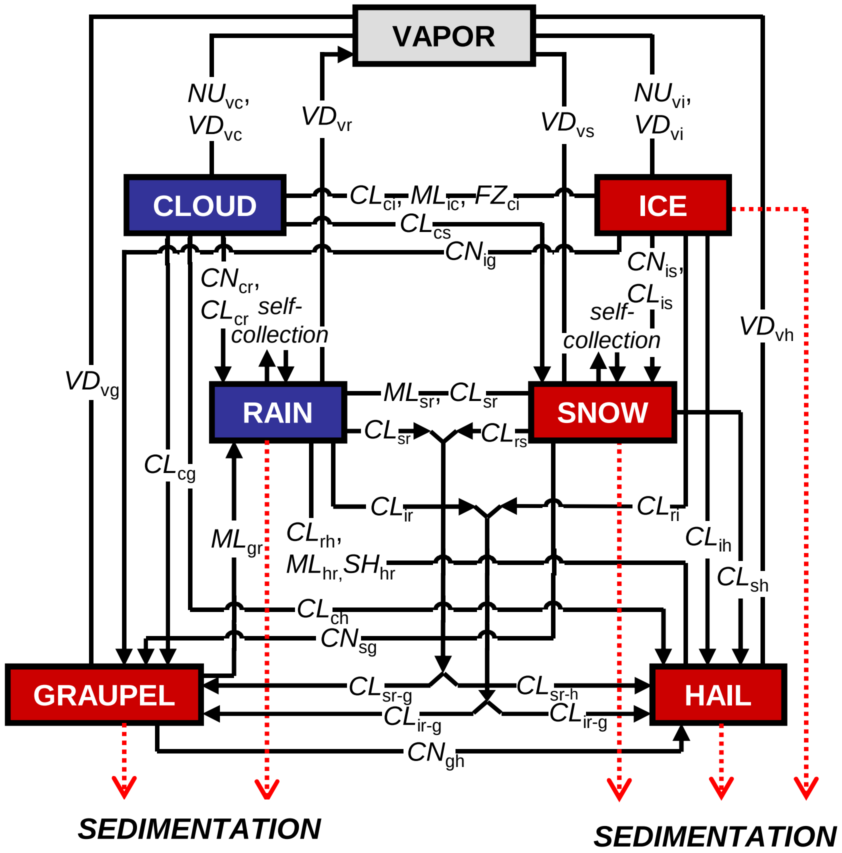
- 10:1 rule
- Climatology
- Neural network diagnostic (statistics of environmental conditions)

  e.g. Roebber et al. (2003)
- Decision tree algorithm (based on physical principles and environment)

  e.g. Dubé (2006)
- **Predicted from the microphysics scheme**

Cloud Microphysics Scheme:*

2-moment, 6-categories



Size distribution of each category *x*:

$$N_x(D) = N_{0x} D^{\alpha_x} e^{-\lambda_x D}$$

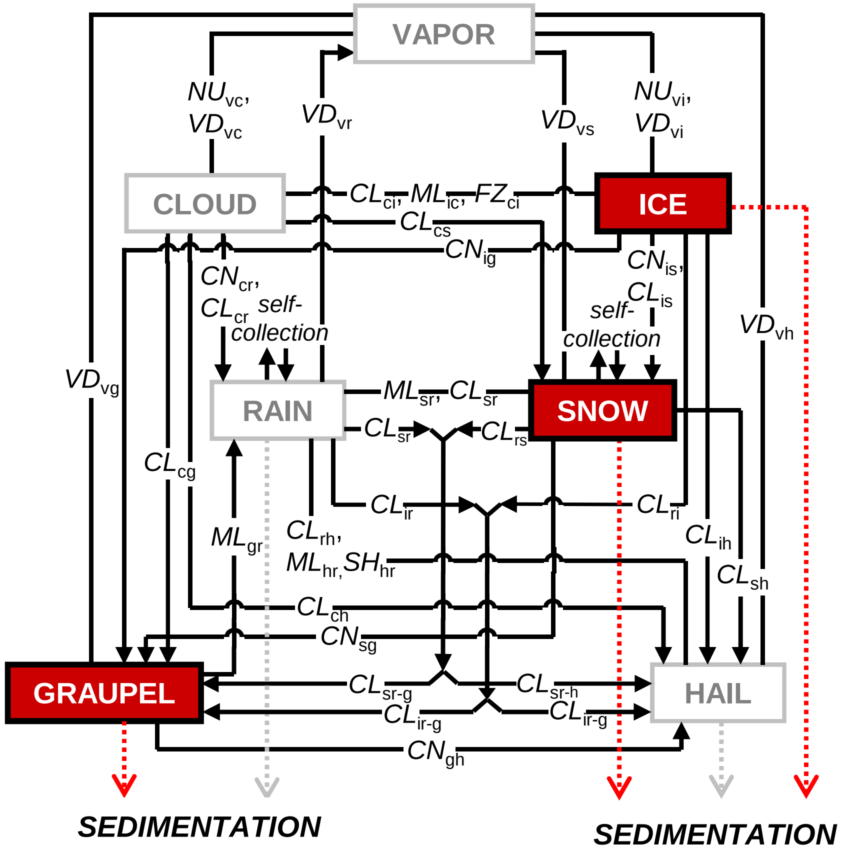
Prognostic quantities:

- mass mixing ratio (***q*$_x$**)
- total number concentration (***N*$_x$**)

*Milbrandt and Yau, 2005a,b (*J. Atmos. Sci.*)

Cloud Microphysics Scheme:

Representation of "snow": (i.e. solid, white precipitation at ground)



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Prognostic quantities:

- mass mixing ratio ($\boldsymbol{q_x}$)
- total number concentration ($\boldsymbol{N_x}$)

"Snow" is represented by 3 categories:

ICE (pristine crystals),

GRAUPEL (rimed crystals),

SNOW (large crystals / aggregates)

"<u>Snow" is represented by 3 categories</u>:

- ***ICE*** (pristine crystals), $\rho_i = 500 \text{ kg m}^{-3}$
- ***GRAUPEL*** (rimed crystals) $\rho_g = 400 \text{ kg m}^{-3}$
- ***SNOW*** (large crystals / aggregates) $\rho_s = f\,(D_s)$

"<u>Snow" is represented by 3 categories</u>:

ICE (pristine crystals),	ρ_i = 500 kg m^{-3}
GRAUPEL (rimed crystals)	*ρ_g = 400 kg m^{-3}
SNOW (large crystals / aggregates)	$\rho_s = f\,(D_s)$

* <u>For ***GRAUPEL***</u>

Recently changed to $\rho_g = f\,(R_i)$, where $R_i = \dfrac{r_{drop} \cdot V_{impact}}{T_{sfc}}$

- following Heymsfield and Pflaum (1985); Cober and List (1993)

"Snow" is represented by 3 categories:

ICE (pristine crystals), $\rho_i = 500 \text{ kg m}^{-3}$

GRAUPEL (rimed crystals) $\rho_g = 400 \text{ kg m}^{-3}$

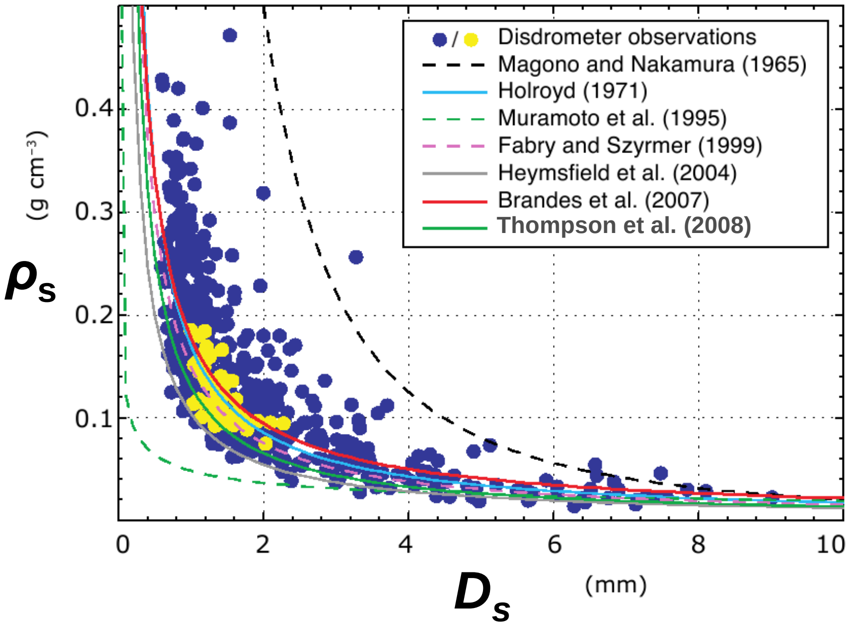
SNOW (large crystals / aggregates) *$\rho_s = f\,(D_s)$

*For ***SNOW*** :

Use of $m_s(D) = cD_s^{\,d}$

$$\Rightarrow \quad \rho_s(D) = eD_s^{\,f}$$

(for the bulk density of an equivalent-mass sphere)



Brandes et al. (2007) *J. Appl. Meteor. and Clim.*

"<u>Snow" is represented by 3 categories</u>:

ICE (pristine crystals), ρ_i = 500 kg m^{-3}

GRAUPEL (rimed crystals) ρ_g = 400 kg m^{-3}

SNOW (large crystals / aggregates) $\rho_s = f\,(D_s)$

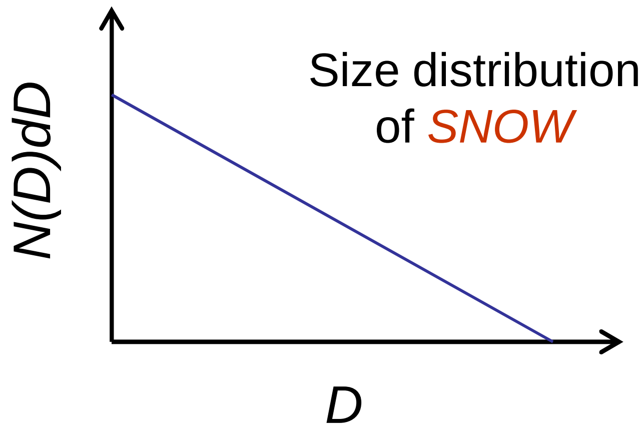
Thus, the density of the precipitating <u>total "snow"</u> is the mass-weighted mean of the 3 component densities:

$$\rho_{snow} = \frac{(q_i\rho_i) + (q_g\rho_g) + (q_s\overline{\rho}_s)^{*}}{q_i + q_g + q_s}$$

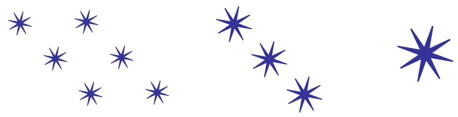
(in air)

$$* \; \overline{\rho} = \frac{\int_0^\infty \rho(D)N(D)dD}{\int_0^\infty N(D)dD}$$

BUT: $\bar{\rho}_{\text{instantaneous}} \neq \bar{\rho}_{\text{accumulated}}$



$$\bar{\rho} = \frac{\int_0^\infty \rho(D) N(D) dD}{\int_0^\infty N(D) dD}$$



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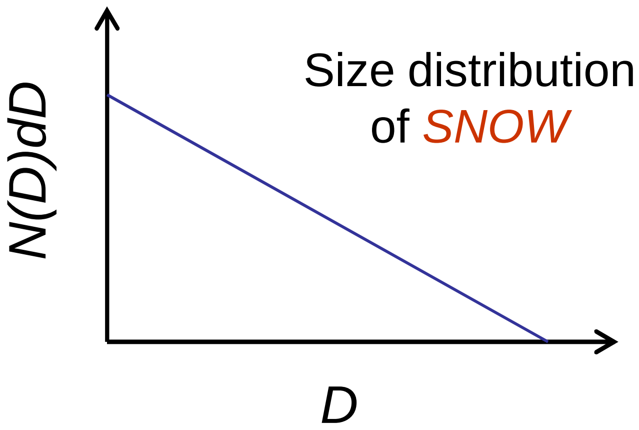

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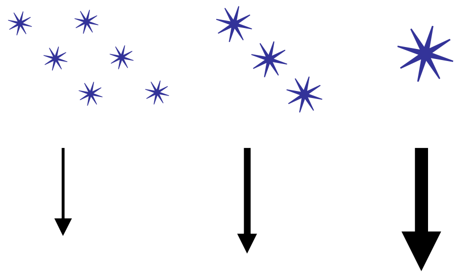
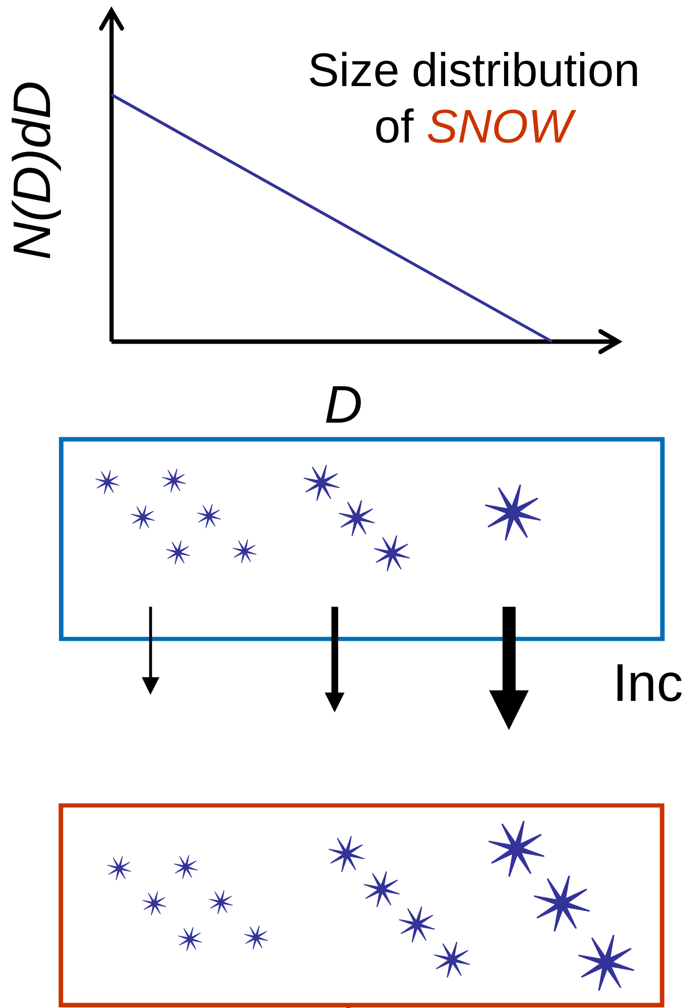


Increasing $V(D)$ with D
(fall velocity)

BUT: $\bar{\rho}_{\text{instantaneous}} \neq \bar{\rho}_{\text{accumulated}}$



$$\bar{\rho} = \frac{\int_0^\infty \rho(D) N(D) dD}{\int_0^\infty N(D) dD}$$

Snapshot

Increasing $V(D)$ with D
(fall velocity)

$$\bar{\rho} = ?$$

Accumulated over Δt

This determines the snow depth

Formal Approach:

For each category *x* (*x* = *i, g, s*):

Compute solid (unmelted) volume fluxes, F_{v_x}

$$\frac{F_{v\_x}}{F_{m\_x}} = \frac{\int_0^\infty V(D)\cdot vol(D)\cdot N(D)dD}{\int_0^\infty V(D)m(D)N(D)dD} = \frac{\int_0^\infty V(D)\cdot \frac{m(D)}{\boxed{\rho(D)}}\cdot N(D)dD}{\int_0^\infty V(D)m(D)N(D)dD} = \underbrace{\frac{\frac{1}{\rho_x}\int_0^\infty V(D)m(D)N(D)dD}{\int_0^\infty V(D)m(D)N(D)dD} = \frac{1}{\rho_x}}$$

$$F_{v\_x} = \frac{F_{m\_x}}{\rho_x}$$

BUT – only true for constant ρ_x

(OK for *ICE* and *GRAUPEL*)

For *SNOW*, $\boxed{\rho = \rho(D)}$ - must compute F_v directly (from integral)

$$\longrightarrow \quad F_{v\_s} = \int_0^\infty V(D)\cdot vol(D)\cdot N(D)dD$$

Instantaneous precipitation rates are given by:

$$F_{v\_liq} = \frac{F_{m\_i}}{\rho_L} + \frac{F_{m\_g}}{\rho_L} + \frac{F_{m\_s}}{\rho_L}$$

→ total solid (liquid-equivalent) precipitation rate

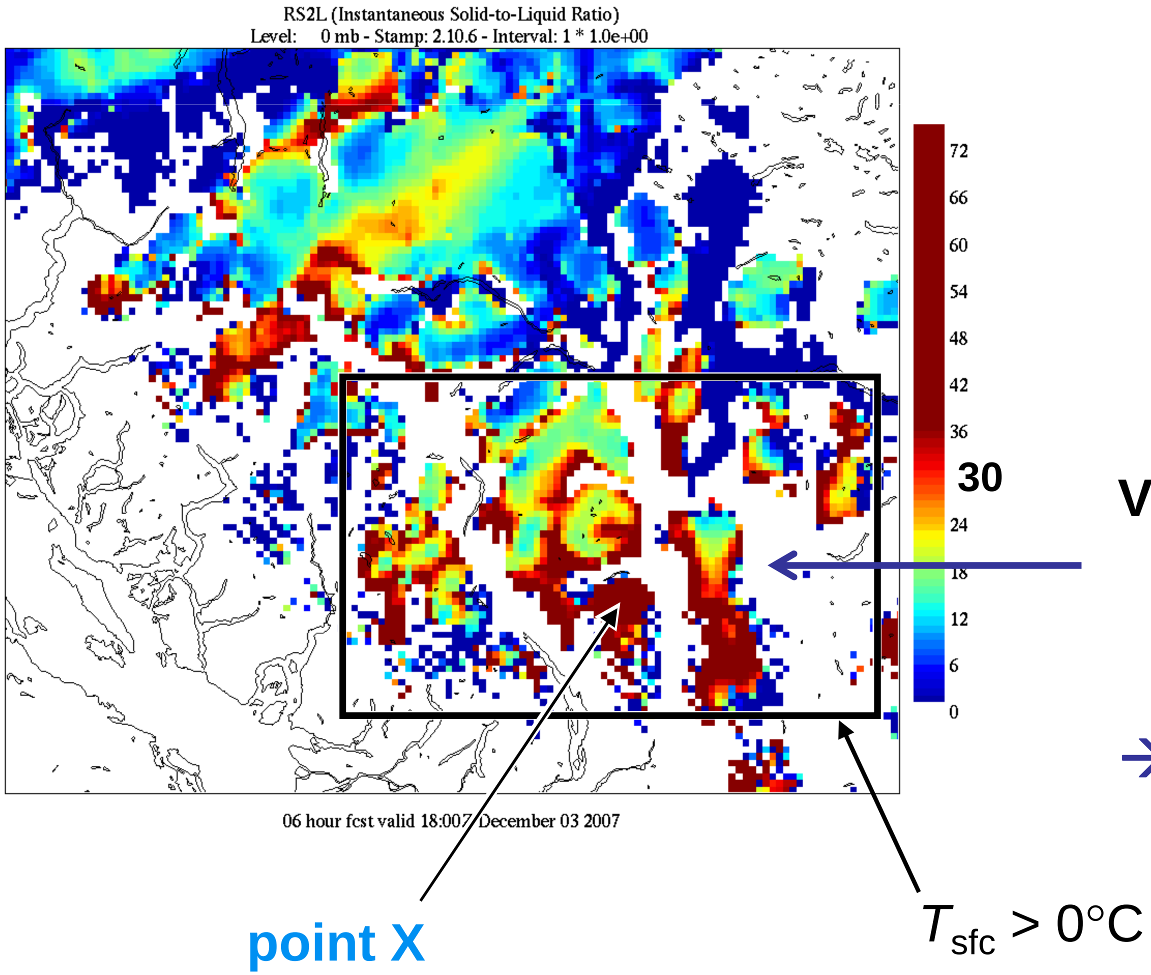
$$F_v = \frac{F_{m\_i}}{\rho_i} + \frac{F_{m\_g}}{\rho_g} + \int_0^\infty V_s(D) \cdot vol_s(D) \cdot N_s(D) dD$$

→ total solid (unmelted) precipitation rate

→ $$SOLID - to - LIQUID_{inst} = \frac{F_v}{F_{v\_liq}}$$

BUT:

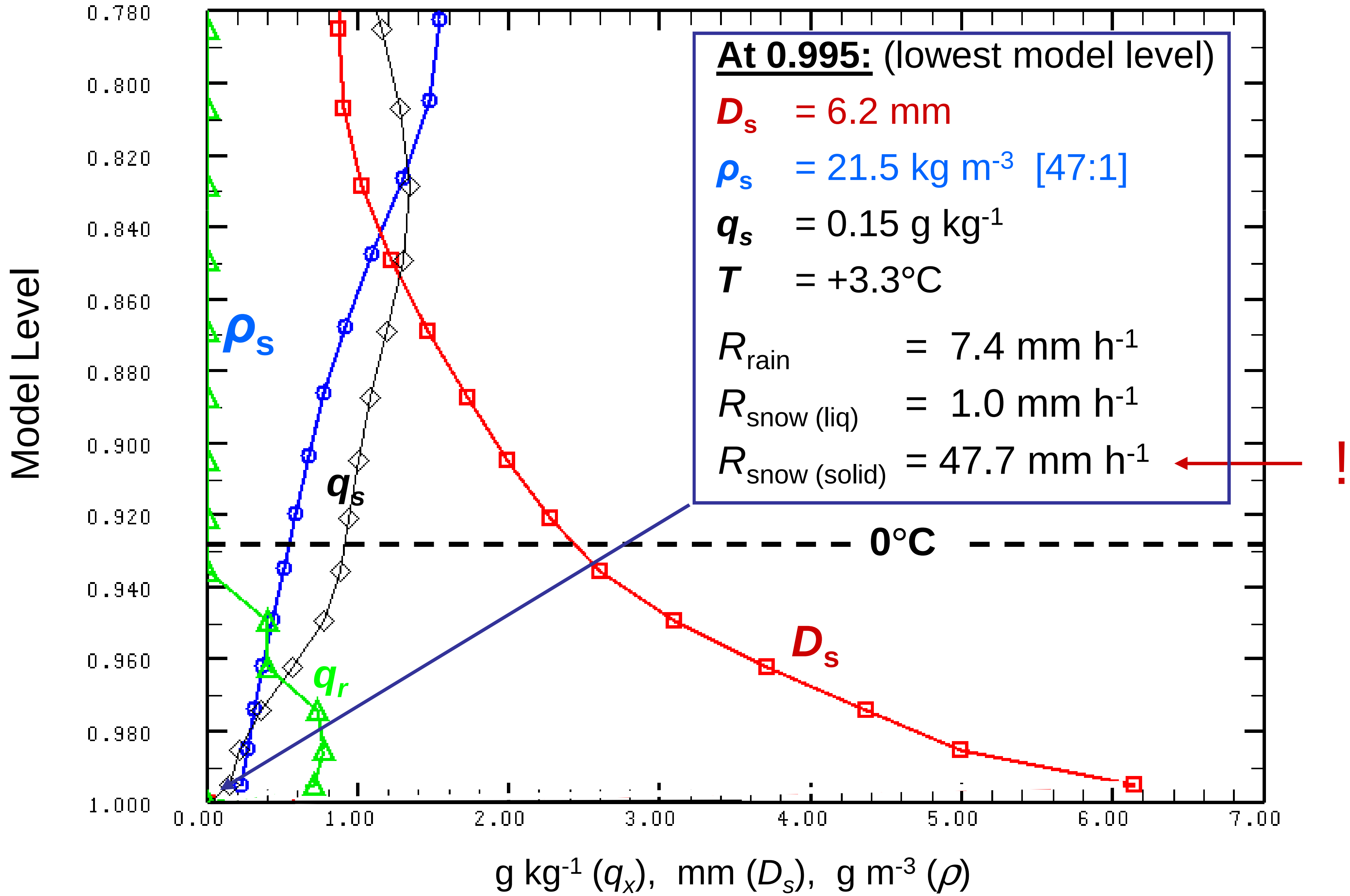
Solid-to-Liquid Ratio (instantaneous)



Very large solid:liquid ratios (low density) for melting snow

→ **Unrealistically large snow depths**

Vertical Profile at point X



Source of Problem (for too small ρ_s / too large snow rates)

- melting snow (q_s) can be have large D_s and thus the diagnostic $\rho_s(D_s)$ is unrealistically small
- increased ρ_s due to partial melting (increasing liquid fraction of melting snowflake) is not accounted for

– Need to adjust the bulk density of melting snow –

(tending towards density of water with increasing degree of melting)

Proposed Solution: 1

- Imposing a **MINIMUM** λ_s (e.g. 700 m^{-1}) – Heymsfield et al., 2008
- Very large D_s is controlled (e.g. max. D_s = 1.43 mm)

BUT:

Scheme still sees relatively large, dry, low-density snow

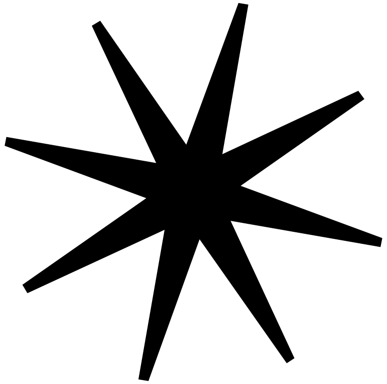
Proposed Solution: 2

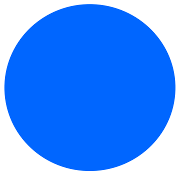
- Approximate **liquid fraction** of melting snow by $q_r / (q_r + q_s)$
- Use mass-weighted density to approximate density of melting snow
- Physically justifiable
  - unless warm-rain coalescence is active, q_r in melting layer comes from melting q_s (and q_g)

Estimation of liquid fraction:

Actual model representation:

$\rho_S = f\,(D_s)$ $\rho_L = 1000 \text{ kg m}^{-3}$



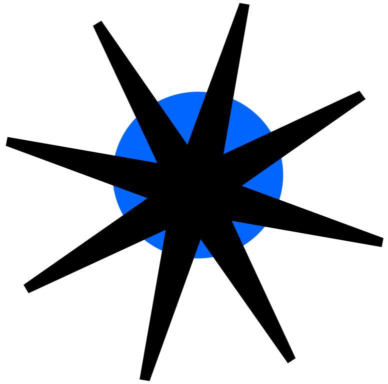


q_s q_r

(N_s) (N_r)

Conceptual view of melting snow:

$\rho_{s\_melting}$

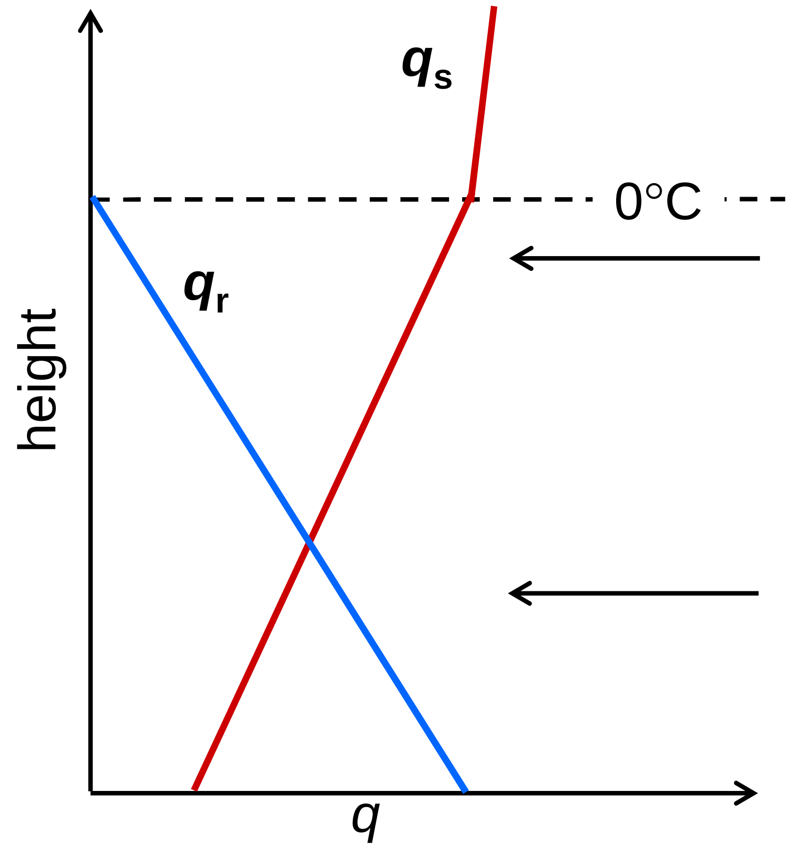


$$\frac{q_r}{q_r + q_s}$$ → liquid fraction of melting snow

Adjustments:

if $T < 0°C$: $$f_{liq} = \frac{q_r}{q_r + (q_i + q_g + q_s)}$$

$$\rho_{s\_melting} = (1 - f_{liq})\rho_s(D_s) + f_{liq}\rho_L$$



e.g. Assume D_s = 5 mm → $\rho_s(D_s)$ = 26 kg m^{-3} :

$$\rho_{s\_melting} = 0.95(26\text{ kg m}^{-3}) + 0.05(1000\text{ kg m}^{-3}) = 75\text{ kg m}^{-3}$$

$$\rho_{s\_melting} = 0.50(26\text{ kg m}^{-3}) + 0.50(1000\text{ kg m}^{-3}) = 513\text{ kg m}^{-3}$$

Adjustments:

$$F_v' = \frac{F_{m\_i}}{\rho_i} + \frac{F_{m\_g}}{\rho_g} + F_{v\_s}^{*}$$

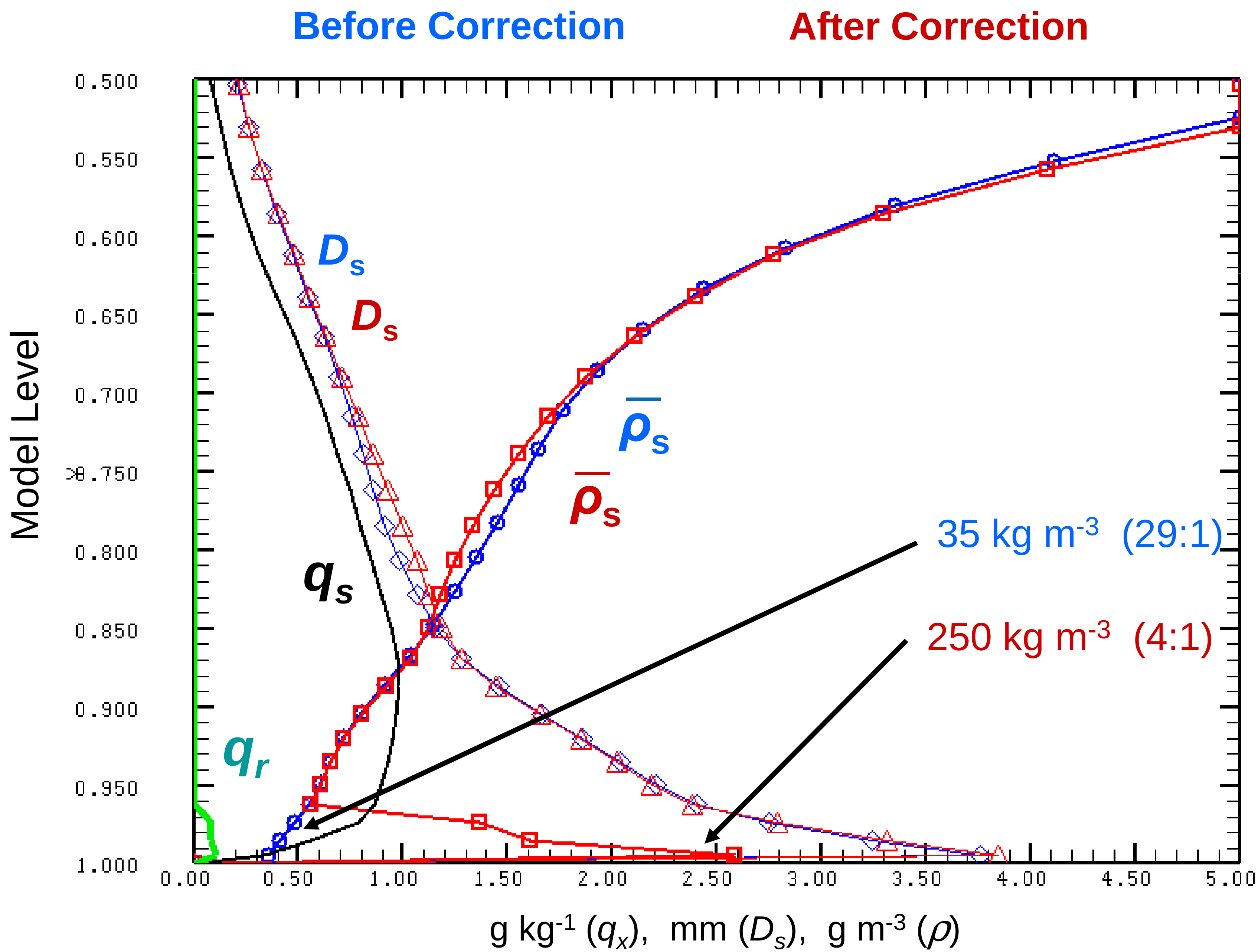
$$^{*}F_{v\_s} = \int_0^\infty V(D) \cdot vol(D) \cdot N(D) dD$$

Actual application:

if $T < 0°C$:

$$F_v = (1 - f_{liq}) \cdot F_v' + f_{liq} \cdot F_{v\_liq}$$

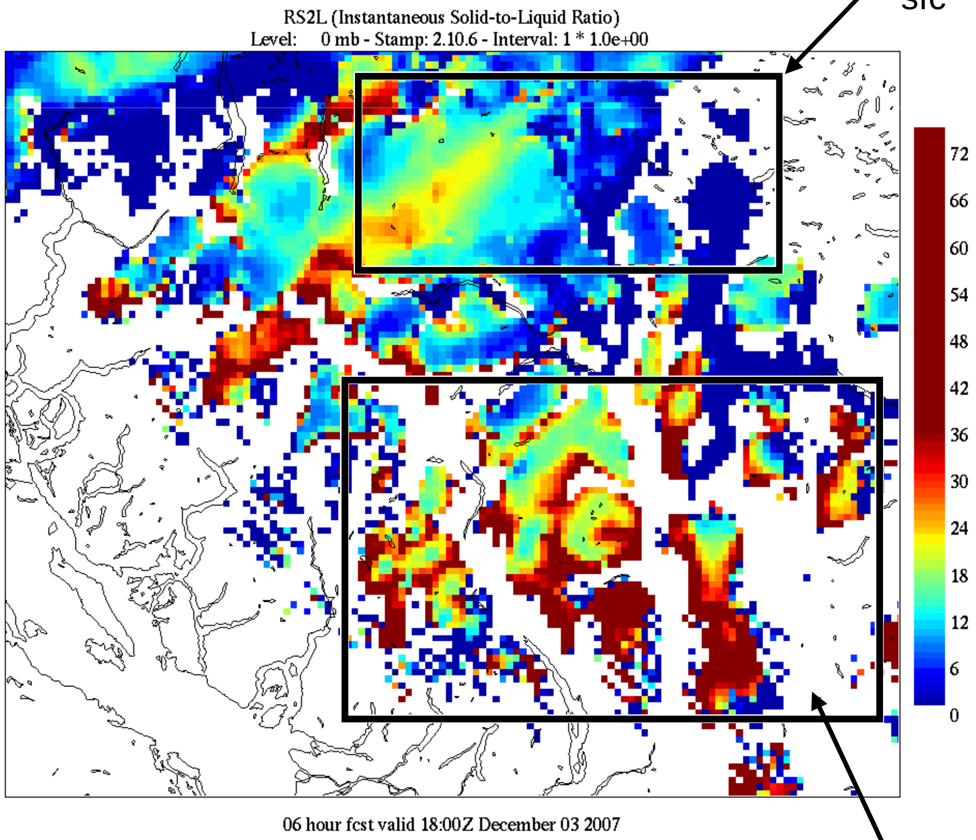
Thus, as the liquid fraction approaches 1, the <u>total volume flux</u> tends towards the <u>total liquid-equivalent volume flux</u>


Before Correction
After Correction
Model Level
0.500
0.550
0.600
0.650
0.700
0.750
0.800
0.850
0.900
0.950
1.000
0.00 0.50 1.00 1.50 2.00 2.50 3.00 3.50 4.00 4.50 5.00
D_s
D_s
$\overline{\rho}_s$
$\overline{\rho}_s$
q_s
q_r
35 kg m^{-3} (29:1)
250 kg m^{-3} (4:1)
g kg^{-1} (q_x), mm (D_s), g m^{-3} (ρ)

Solid-to-liquid ratio (instantaneous)

Before Correction

$T_{sfc} < 0°C$

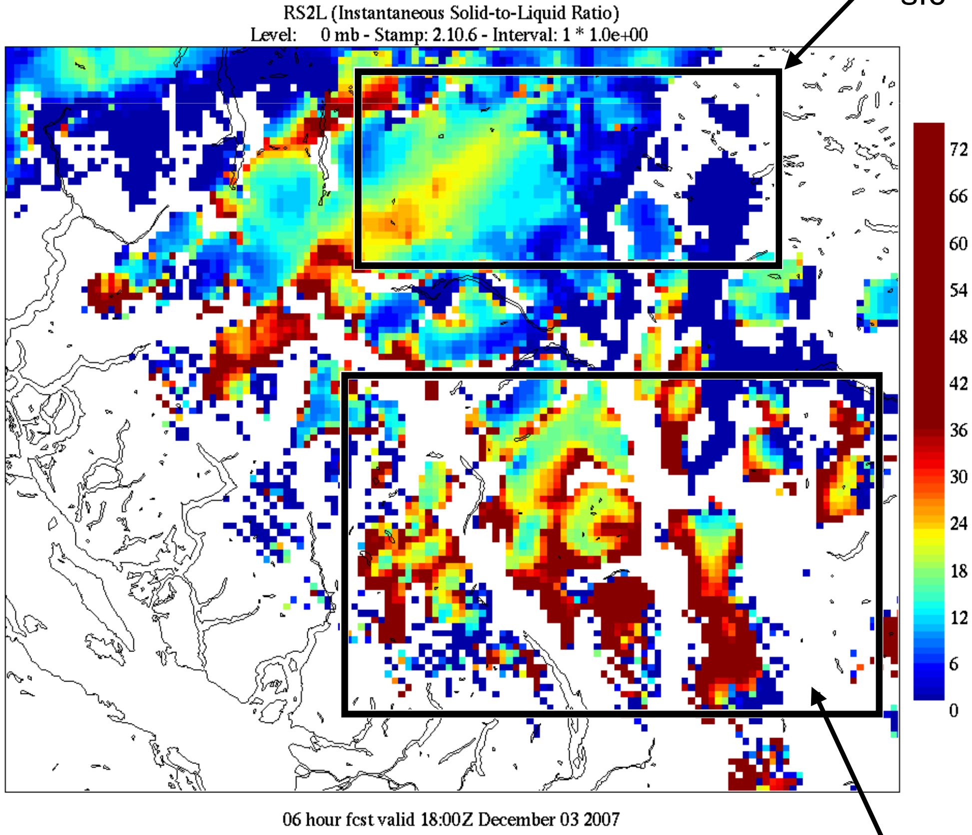


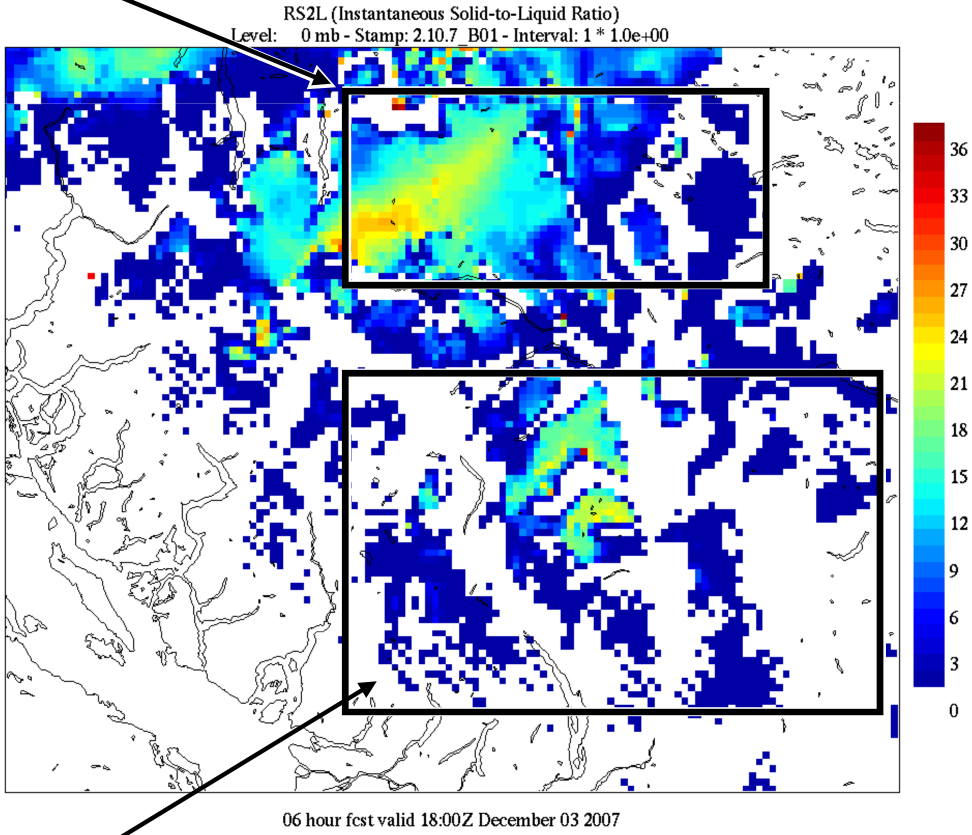
$T_{sfc} > 0°C$

Solid-to-liquid ratio (instantaneous)

Before Correction | **After Correction**

$T_{sfc} < 0°C$





$T_{sfc} > 0°C$

NOTE: <u>Same</u> color scales

Finally, Instantaneous precipitation rates are given by:

$$F_{v\_liq} = \frac{F_{m\_i}}{\rho_L} + \frac{F_{m\_g}}{\rho_L} + \frac{F_{m\_s}}{\rho_L}$$

→ total solid (liquid-equivalent) precipitation rate

$$F_v = \frac{F_{m\_i}}{\rho_i} + \frac{F_{m\_g}}{\rho_g} + \int_0^\infty V_s(D) \cdot vol_s(D) \cdot N_s(D) dD$$

→ $$SOLID - to - LIQUID_{inst} = \frac{F_v}{F_{v\_liq}}$$

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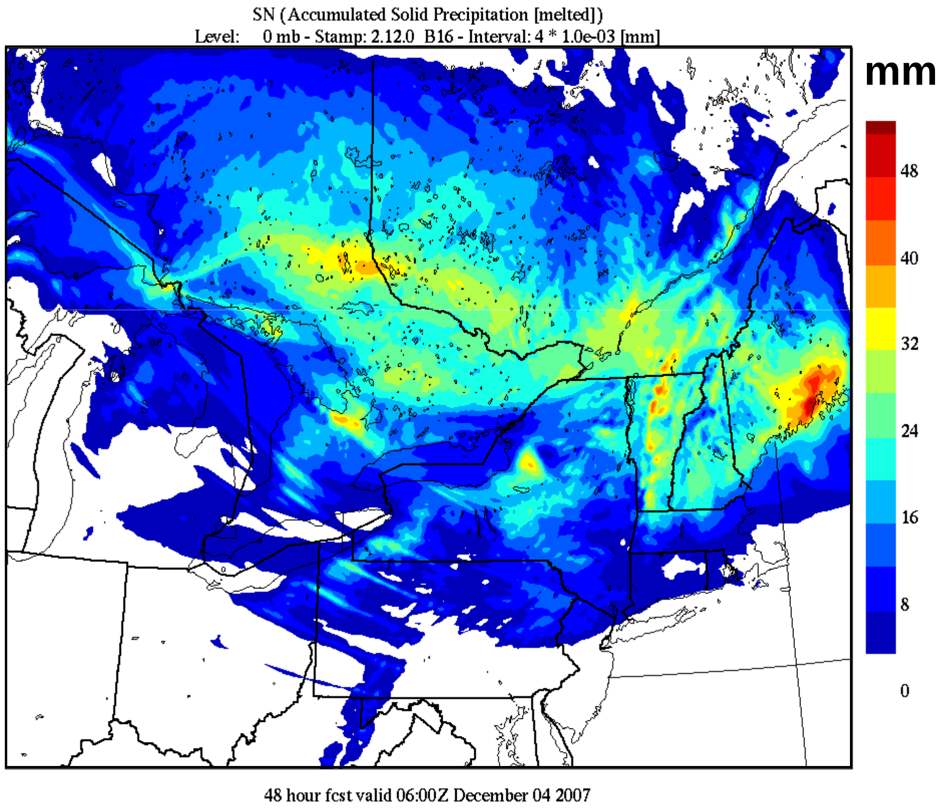
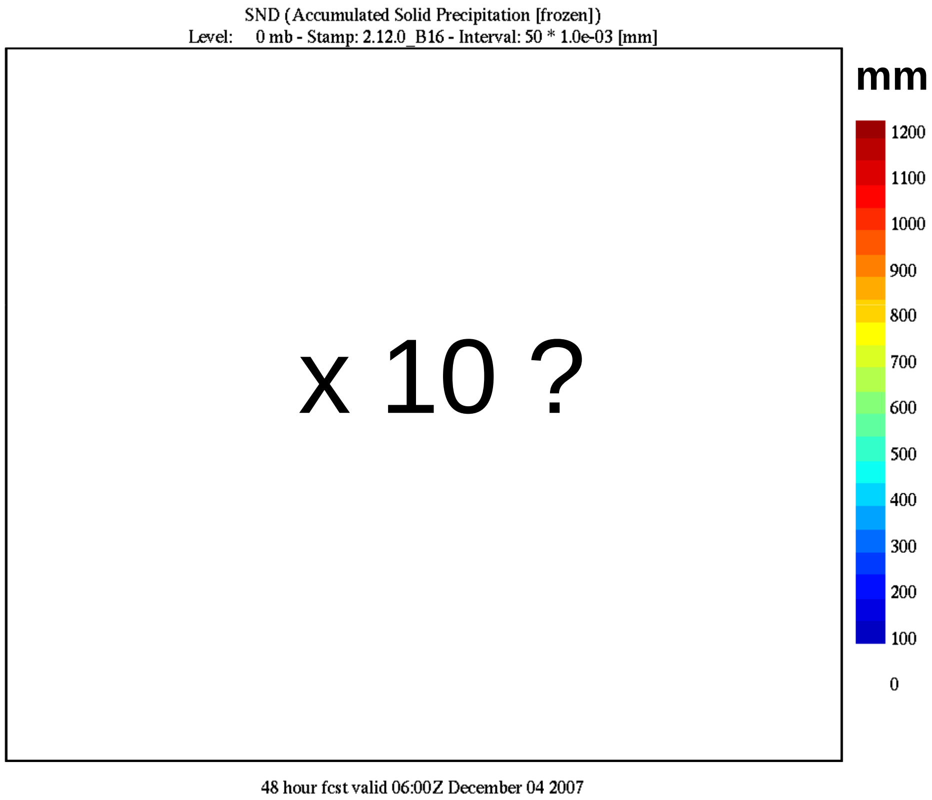
$$F_v = (1 - f_{liq}) \cdot F_v' + f_{liq} \cdot F_{v\_liq} \quad \text{(if } T < 0°\text{C)}$$

→ $$SOLID - to - LIQUID_{inst} = \frac{F_v}{F_{v\_liq}}$$





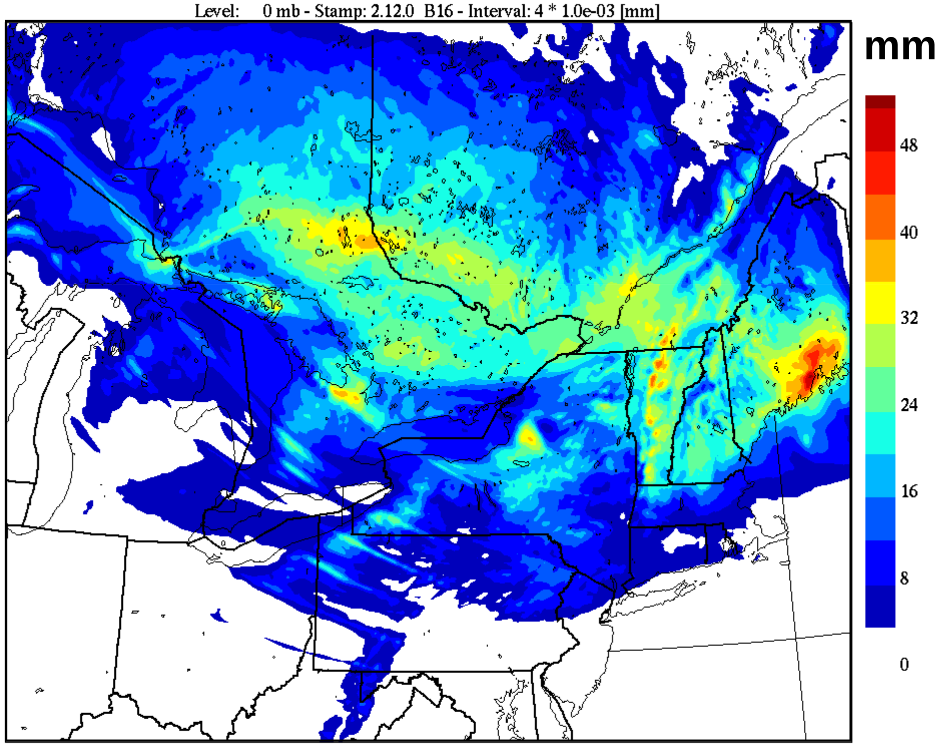
Accumulated Precipitation

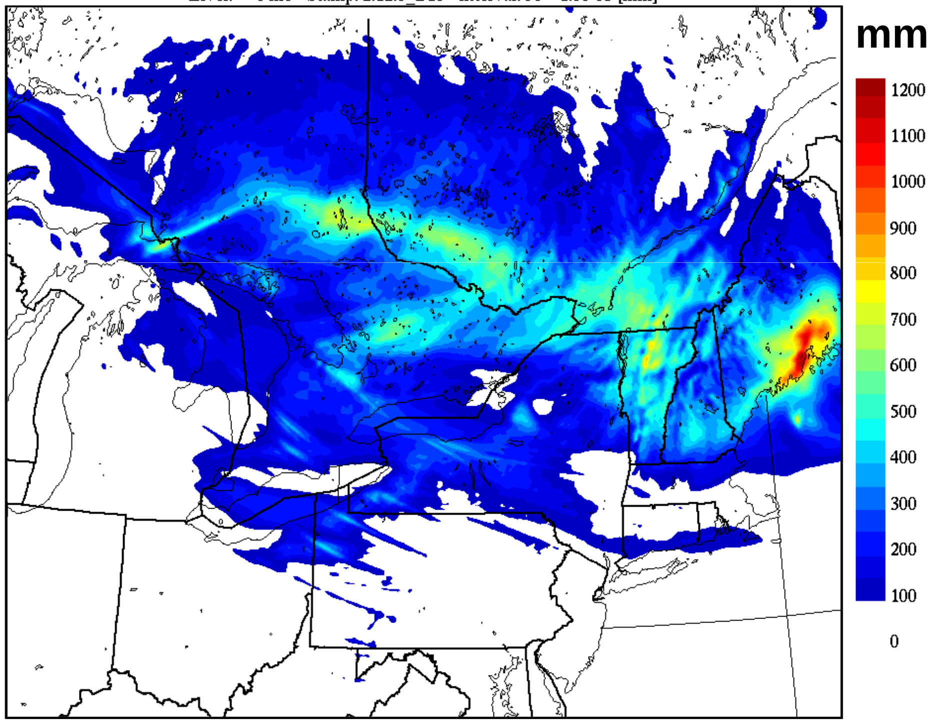
(liquid-equivalent)

Accumulated Precipitation

(unmelted)

i.e. snow depth





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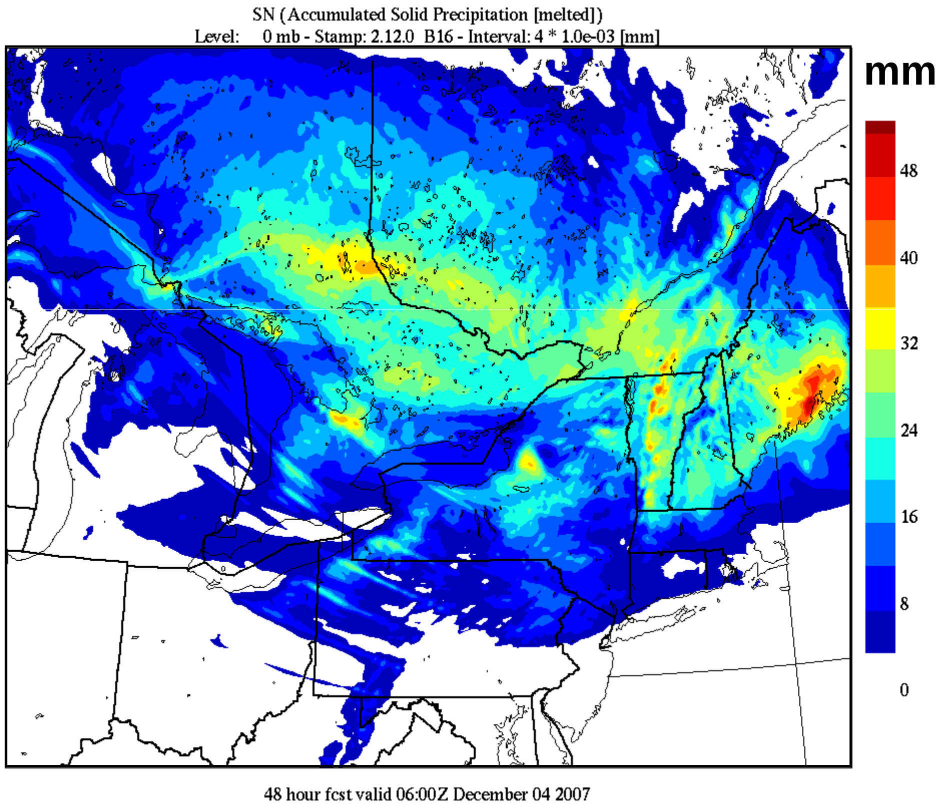
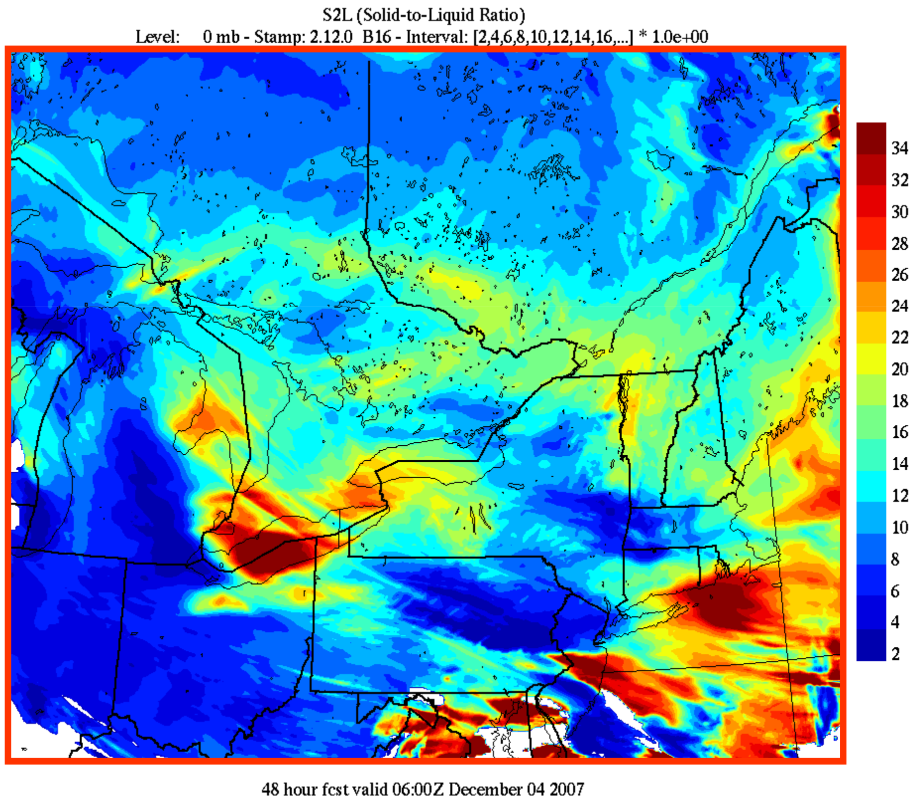


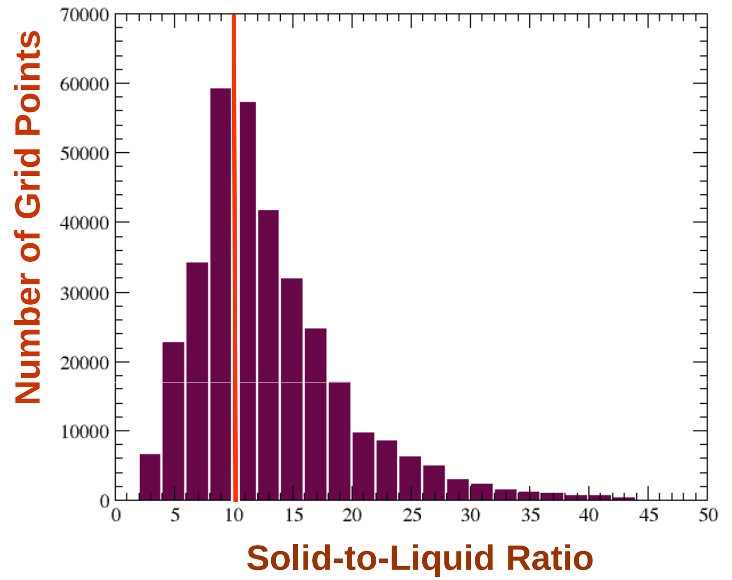


Accumulated Precipitation

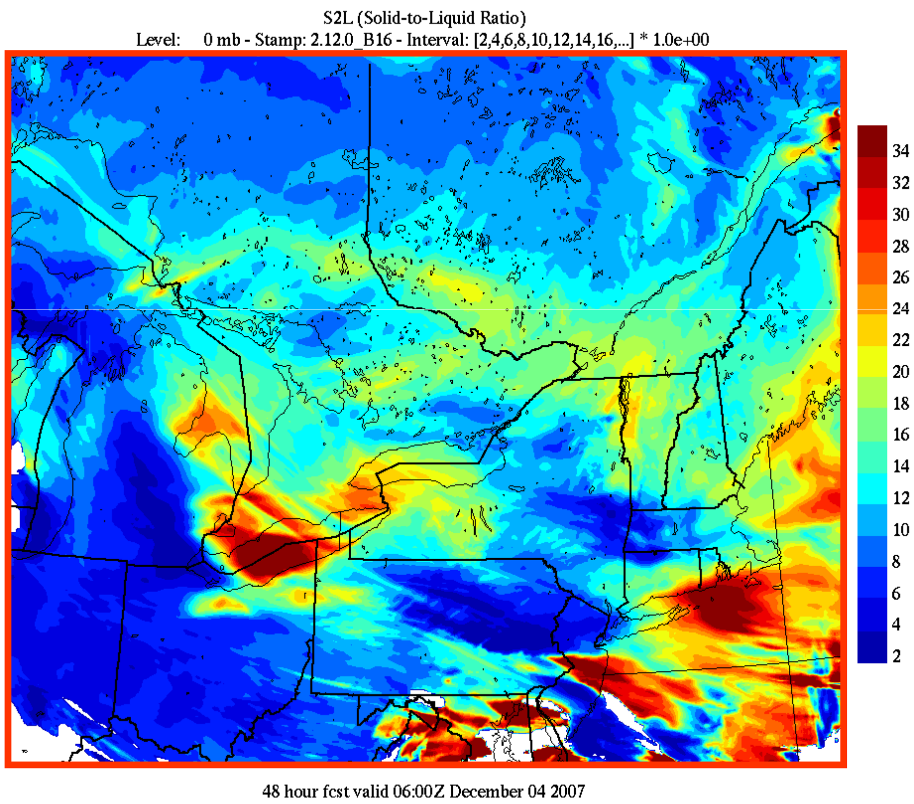
(liquid-equivalent)

Solid-to-Liquid Ratio

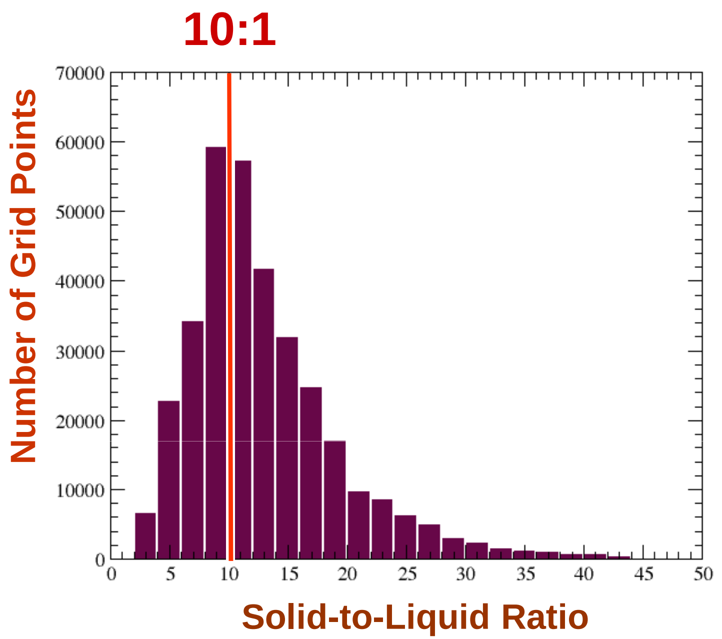
10:1



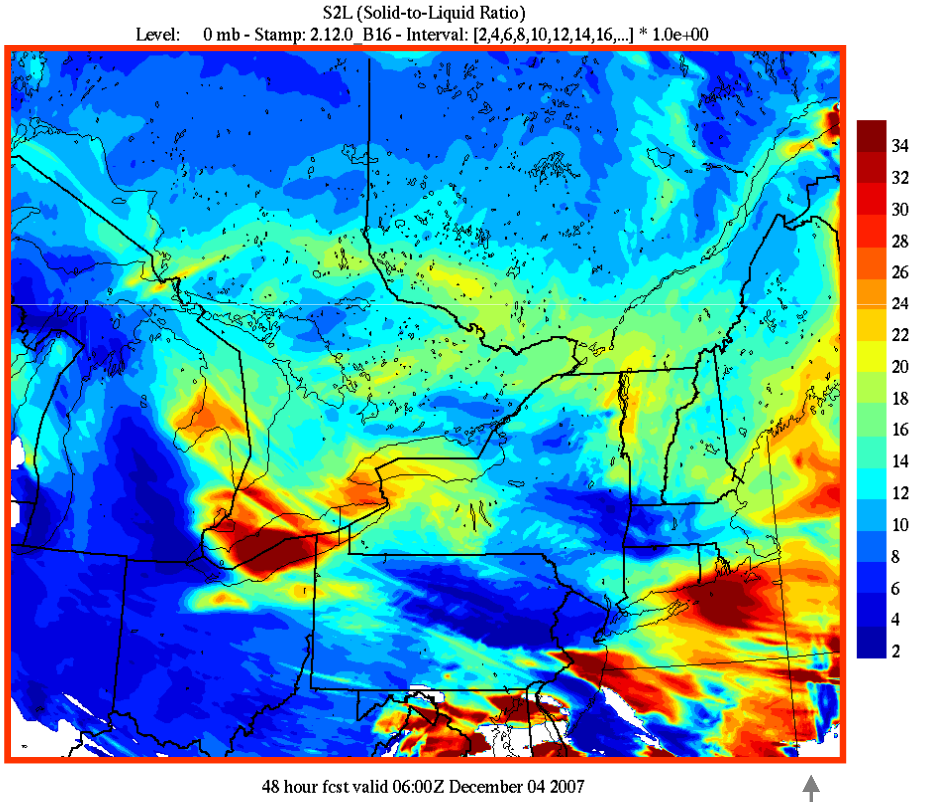
Distribution on grid

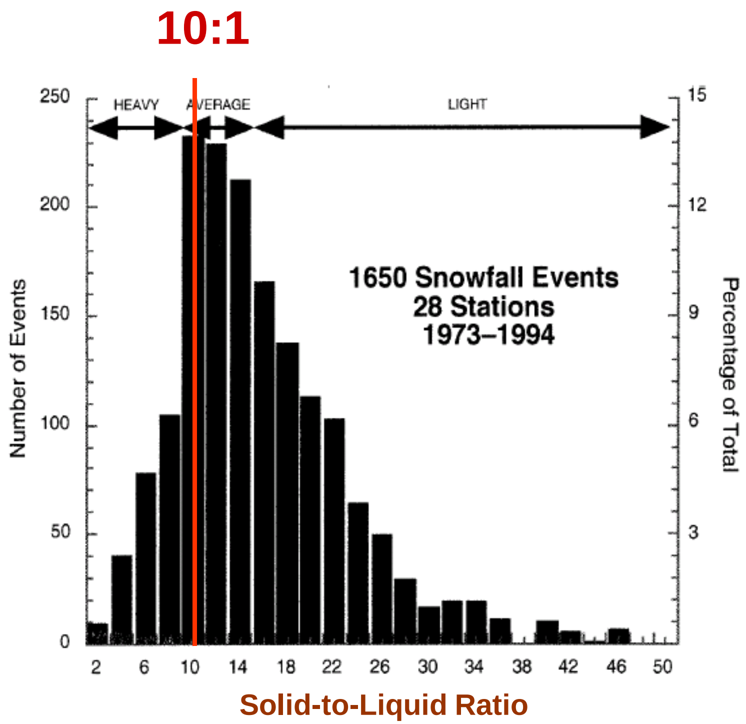


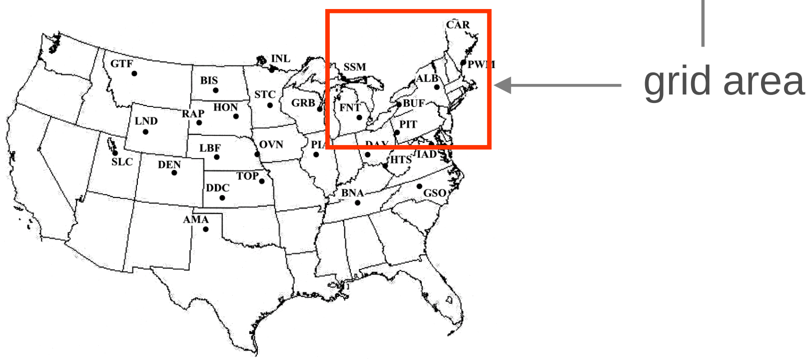
Solid-to-Liquid Ratio

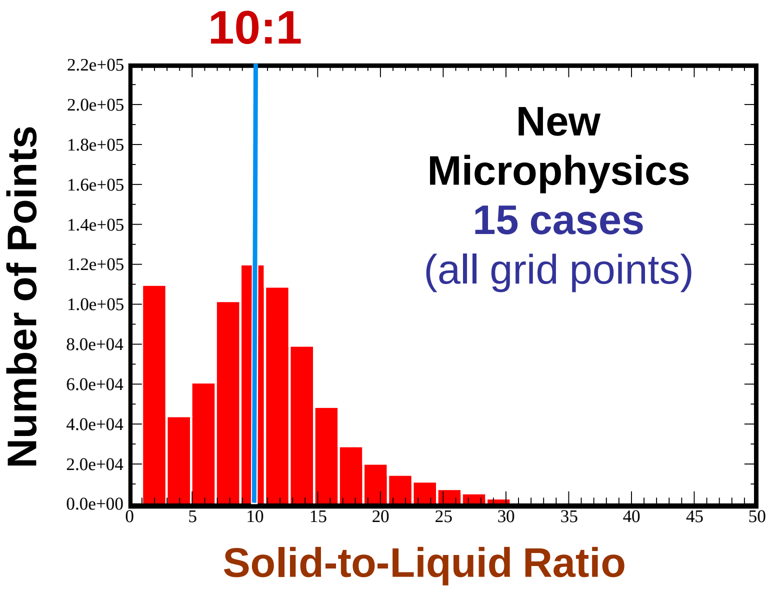


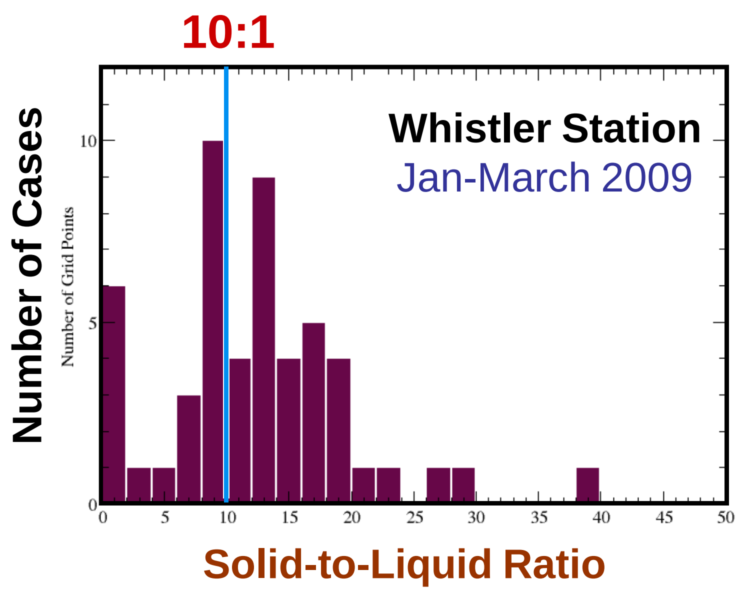
Source: Roebber et al. (2003), *Weather and Forecasting*



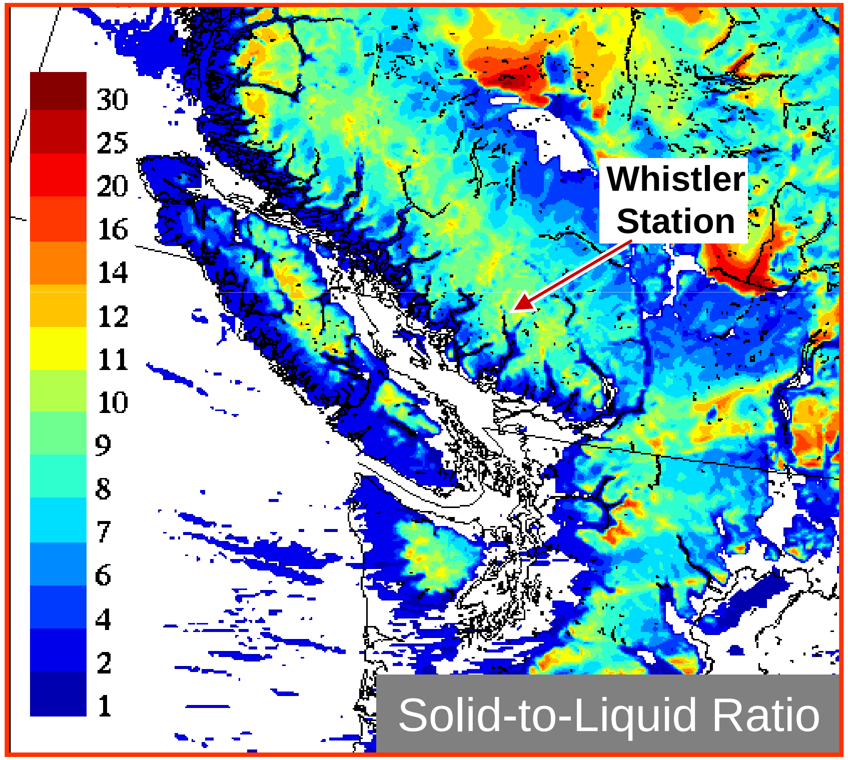


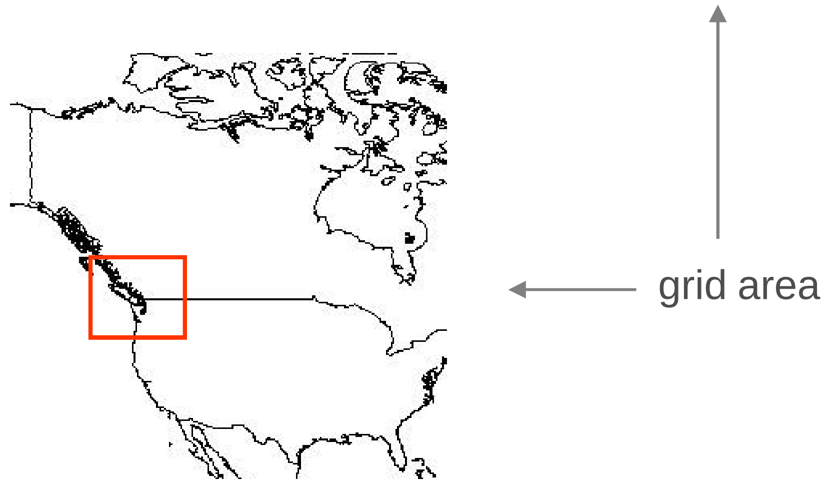






Case: 12 March 2009 (00 z)





CONCLUSION

- The cloud microphysics scheme predicts the individual quantities and size distributions of pristine crystals, aggregates, graupel

- This information can be exploited to compute the **instantaneous solid** (unmelted) **precipitation rate** → it need not be simply inferred

- Preliminary tests indicate that this method produces a realistic probability distribution of the solid-to-liquid ratio

THANK YOU