Development of Turbulence and Shallow-Convection Scheme for COSMO

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8th International SRNWP Workshop on Nonhydrostatic Modelling

Bad Orb, Germany, 26 – 28 October 2009

Outline

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- Towards a unified description of turbulence and shallow convection – possible alternatives
- A non-local second-order closure
- Performance in dry and cloudy convective PBLs
- Stably stratified PBL over temperature-heterogeneous surface – LES and prospects for improving parameterisations
- Conclusions and outlook

Motivation

Quoting Arakawa (2004, The Cumulus Parameterization Problem: Past, Present, and Future. *J. Climate*, **17**, 2493-2525), where, among other things,

"Major practical and conceptual problems in the conventional approach of cumulus parameterization, which include artificial separations of processes and scales, are discussed."

"It is rather obvious that for future climate models the scope of the problem must be drastically expanded from "cumulus parameterization" to "unified cloud parameterization" or even to "unified model physics". This is an extremely challenging task, both intellectually and computationally, and the use of multiple approaches is crucial even for a moderate success."

Motivation (cont'd): Recall …

Transport equation for a generic quantity *f*

$$d\overline{f}/dt = -\partial\overline{u_i' f'}/\partial x_i + \ldots$$

Splitting the sub-grid scale flux divergence (artificial separations of processes and scales!)

$$\partial\overline{u_i' f'}/\partial x_i = (\partial\overline{u_i' f'}/\partial x_i)_{conv} + (\partial\overline{u_i' f'}/\partial x_i)_{turb}$$

Convection (quasi-organised)

mass-flux closure

Turbulence (random)

ensemble-mean closure

Motivation (cont'd)

The tasks of developing a "unified cloud parameterization" and eventually a "unified model physics" seem to be too ambitious, at least at the moment.

However,

a unified description of boundary-layer turbulence and shallow convection

seems to be feasible. There are several ways to do so, but it is not a priory clear which way should be preferred (see Mironov 2009, for a detailed discussion).

Towards a Unified Description of Turbulence and Shallow Convection – Possible Alternatives

• ***Extended mass-flux schemes***

built around the top-hat updraught-downdraught representation of fluctuating quantities (ADHOC, Lappen and Randall 2001, 2005, 2006)

• ***Hybrid schemes***

where the mass-flux closure ideas and the ensemble-mean second-order closure ideas have roughly equal standing (EDMF, Soares et al. 2004, Siebesma and Teixeira 2000)

•***Non-local second-order closure schemes***

with transport equations for scalar variances and skewness-dependent parameterisations of the third-order transport (Abdella and McFarlane 1997, 1999, Zilitinkevich et al. 1999, Mironov et al. 1999, Abdella and Petersen 2000, Golaz et al. 2002, Gryanik and Hartmann 2002, Gryanik et al. 2005)

TKE-Scalar Variance Closure Model

- Transport (prognostic) equations for TKE and for variances of scalars (liquid water potential temperature, total water specific humidity) including third-order transport
- Algebraic (diagnostic) formulations for scalar fluxes, for the Reynolds-stress components, and for turbulence length scale
- Statistical SGS cloud scheme, either Gaussian (e.g. Sommeria and Deardorff 1977), or with exponential tail to account for the effect of cumulus clouds (e.g. Bechtold et al. 1995)
- Optionally, prognostic equations for scalar skewness (mass-flux ideas recast in terms of ensemble-mean quantities)

NB! A scheme should be reasonably inexpensive in terms of computation cost (hence diagnostic treatment of Reynolds stress and scalar fluxes)

Treatment of Scalar Variances

Prognostic equations for <$u_i'^2$> (kinetic energy of SGS motions) and for <θ'^2> (potential energy of SGS motions)

Convection/stable stratification =

Potential Energy ↔ Kinetic Energy

No reason to prefer one form of energy over the other!

The TKE equation

$$\frac{\partial e}{\partial t} = -\left(\overline{w'u'}\frac{\partial \bar{u}}{\partial z} + \overline{w'v'}\frac{\partial \bar{v}}{\partial z} \right) + g\alpha\overline{w'\theta'} - \frac{\partial}{\partial z}\left(\frac{1}{2}\overline{w'u_i'^2} + \overline{w'p'} \right) - \varepsilon$$

The scalar-variance equation

$$\frac{1}{2}\frac{\partial \overline{\theta'^2}}{\partial t} = -\overline{w'\theta'}\frac{\partial \bar{\theta}}{\partial z} - \frac{1}{2}\frac{\partial}{\partial z}\overline{w'\theta'^2} - \varepsilon_\theta$$

Comparison with One-Equation Models
(Draft Horses of Geophysical Turbulence Modelling)

Equation for $<\theta'^2>$

$$\frac{1}{2}\frac{\partial \overline{\theta'^2}}{\partial t} = -\overline{w'\theta'}\frac{\partial \overline{\theta}}{\partial z} - \frac{1}{2}\frac{\partial}{\partial z}\overline{w'\theta'^2} - \varepsilon_\theta$$

Production = Dissipation (implicit in all models that carry the TKE equations only).

Equation for $<w'\theta'>$

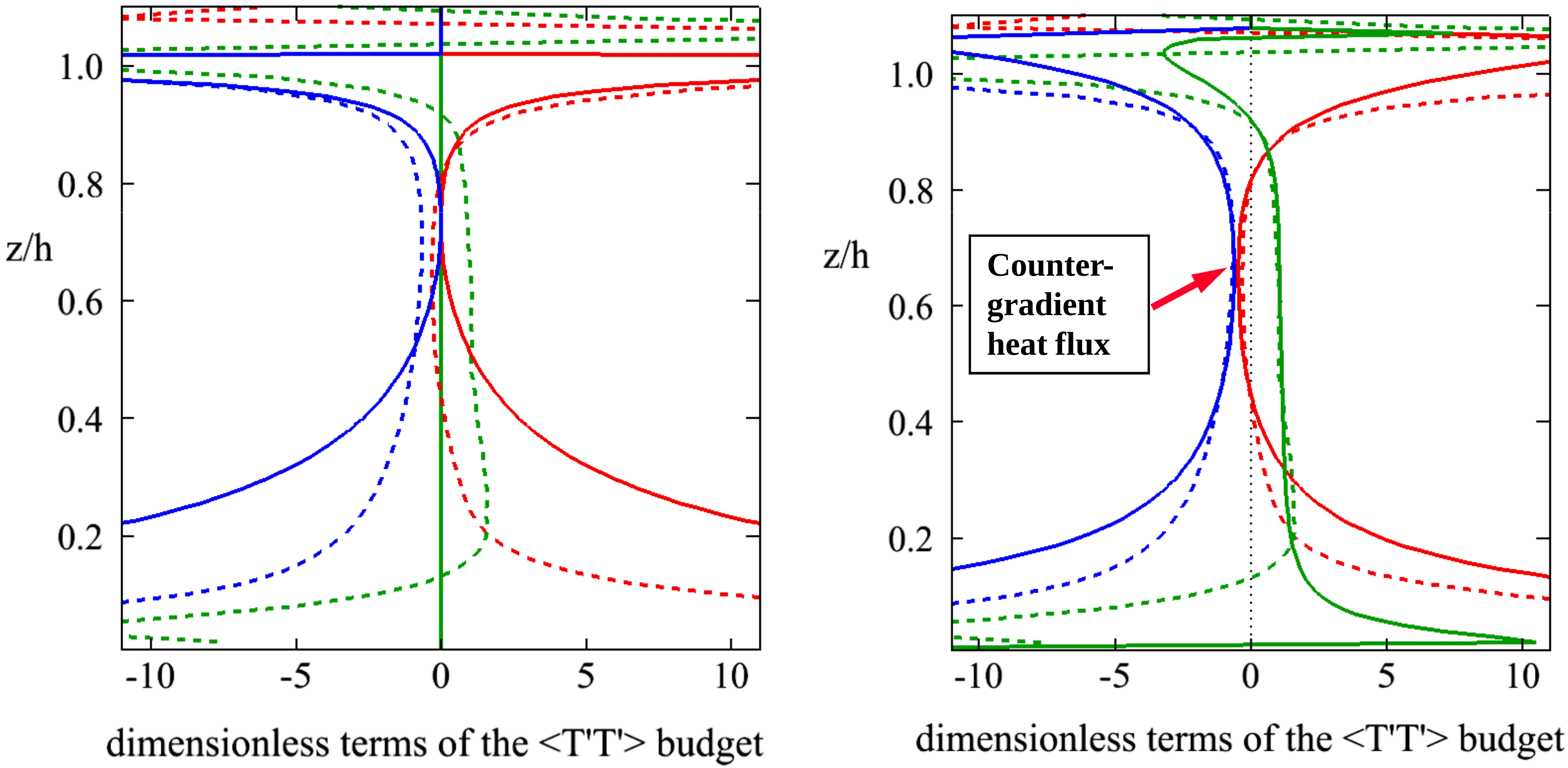
$$\overline{w'\theta'} = -C_{\theta g}\tau_\varepsilon e\frac{\partial \overline{\theta}}{\partial z} + C_g\tau_\varepsilon g\alpha\overline{\theta'^2}$$

No counter-gradient term (cf. turbulence models using "counter-gradient corrections" heuristically).

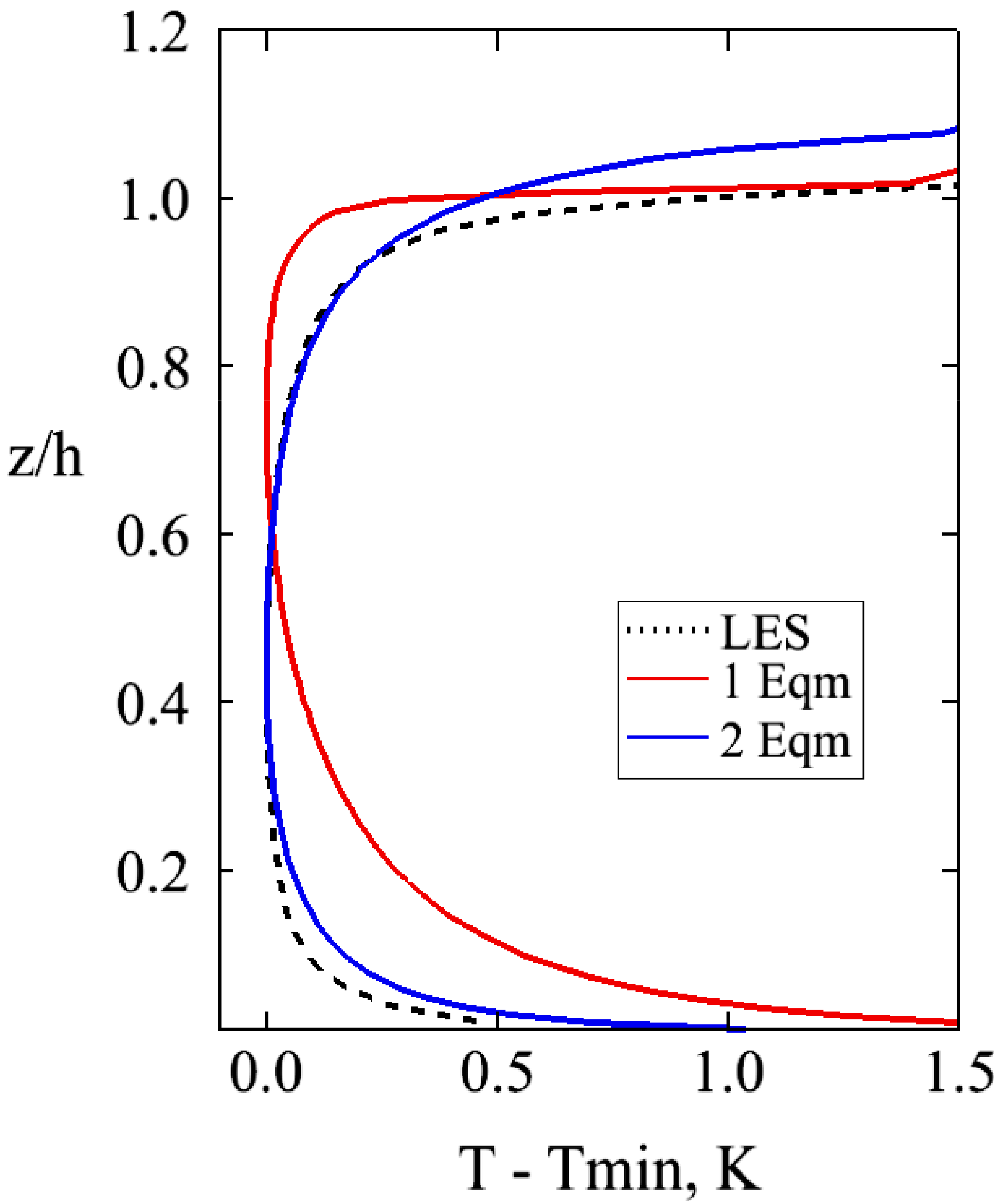
Importance of Scalar Variances

- Potential energy ↔ kinetic energy conversion in turbulent flows
- No way to get counter-gradient scalar fluxes in convective flows unless third-order scalar-variance transport is included
- Scalar variances are a crucial input of SGS cloud scheme
- The major effect of horizontal temperature heterogeneity in stably stratified flows is accounted for through the third-order transport of temperature variance

Budget of Potential-Temperature Variance in Dry Convective PBL
One-Equation and Two-Equation Models vs. LES Data



Dotted curves – LES data (Mironov et al. 2000), solid curves – model results. Left panel – one-equation model, right panel – two-equation model. Red – mean-gradient production/destruction, green – third-order transport, blue – dissipation. The budget terms are made dimensionless with $w_* \theta_*^2/h$.



Mean Potential Temperature in Dry Convective PBL

One-Equation and Two-Equation Models vs. LES Data

Potential temperature minus its minimum value within the PBL. **Black dotted** curve shows LES data (Mironov et al. 2000), red – one-equation model, blue – two-equation model.

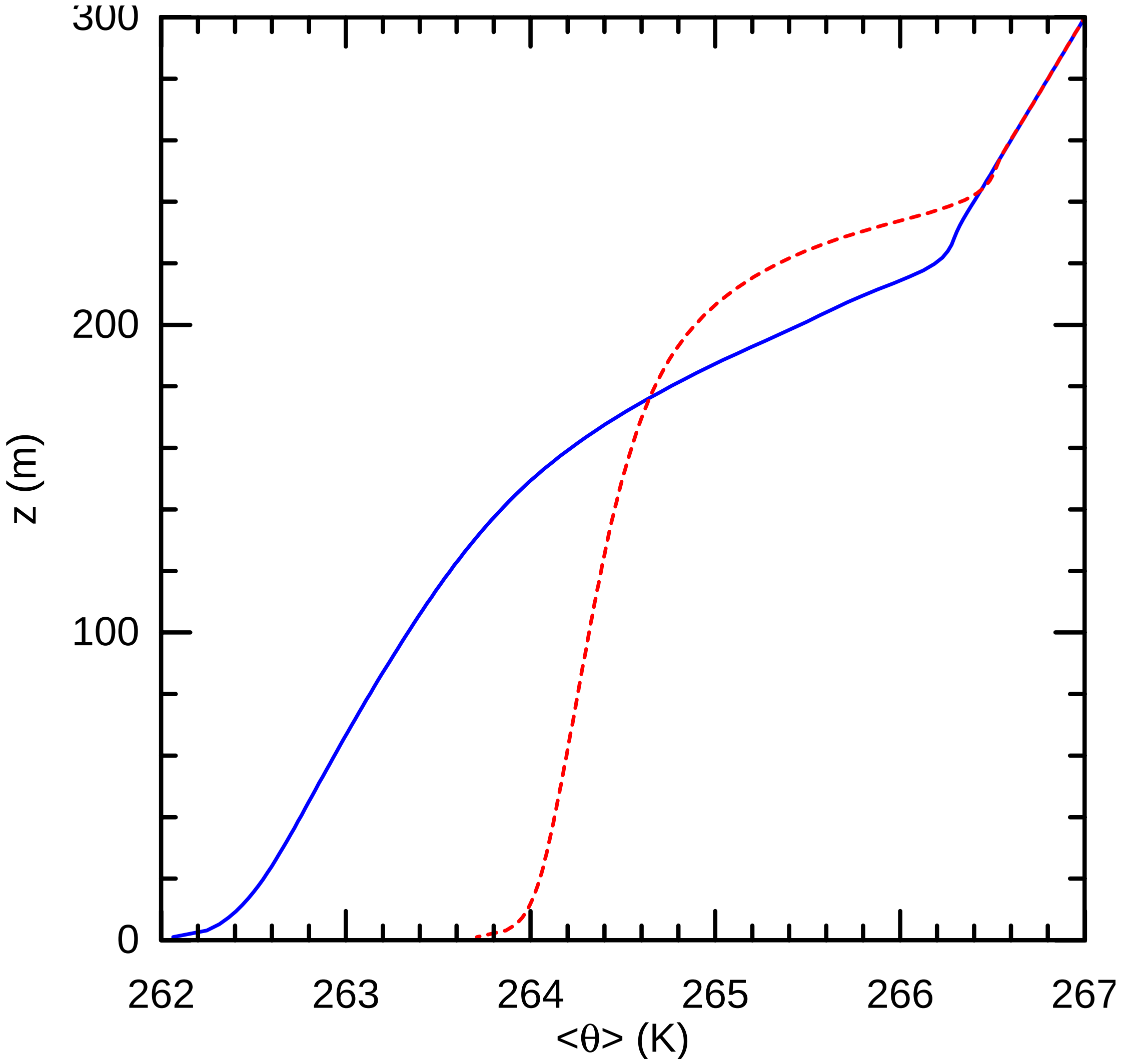
LES of Stably Stratified PBL (SBL)

- Traditional PBL (surface layer) models do not account for many SBL features (static stability increases → turbulence is quenched → sensible and latent heat fluxes are zero → radiation equilibrium at the surface → too low surface temperature)

- No comprehensive account of second-moment budgets in SBL
- Poor understanding of the role of horizontal heterogeneity in maintenance of turbulent fluxes (hence no physically sound parameterisation)

- LES of SBL over horizontally-homogeneous vs. horizontally-heterogeneous surface [the surface cooling rate varies sinusoidally in the streamwise direction such that the horizontal-mean surface temperature is the same as in the homogeneous cases, cf. Stoll and Porté-Agel (2009)]
- Mean fields, second-order and third-order moments
- Budgets of velocity and temperature variance and of temperature flux with due regard for SGS contributions (important in SBL even at high resolution)

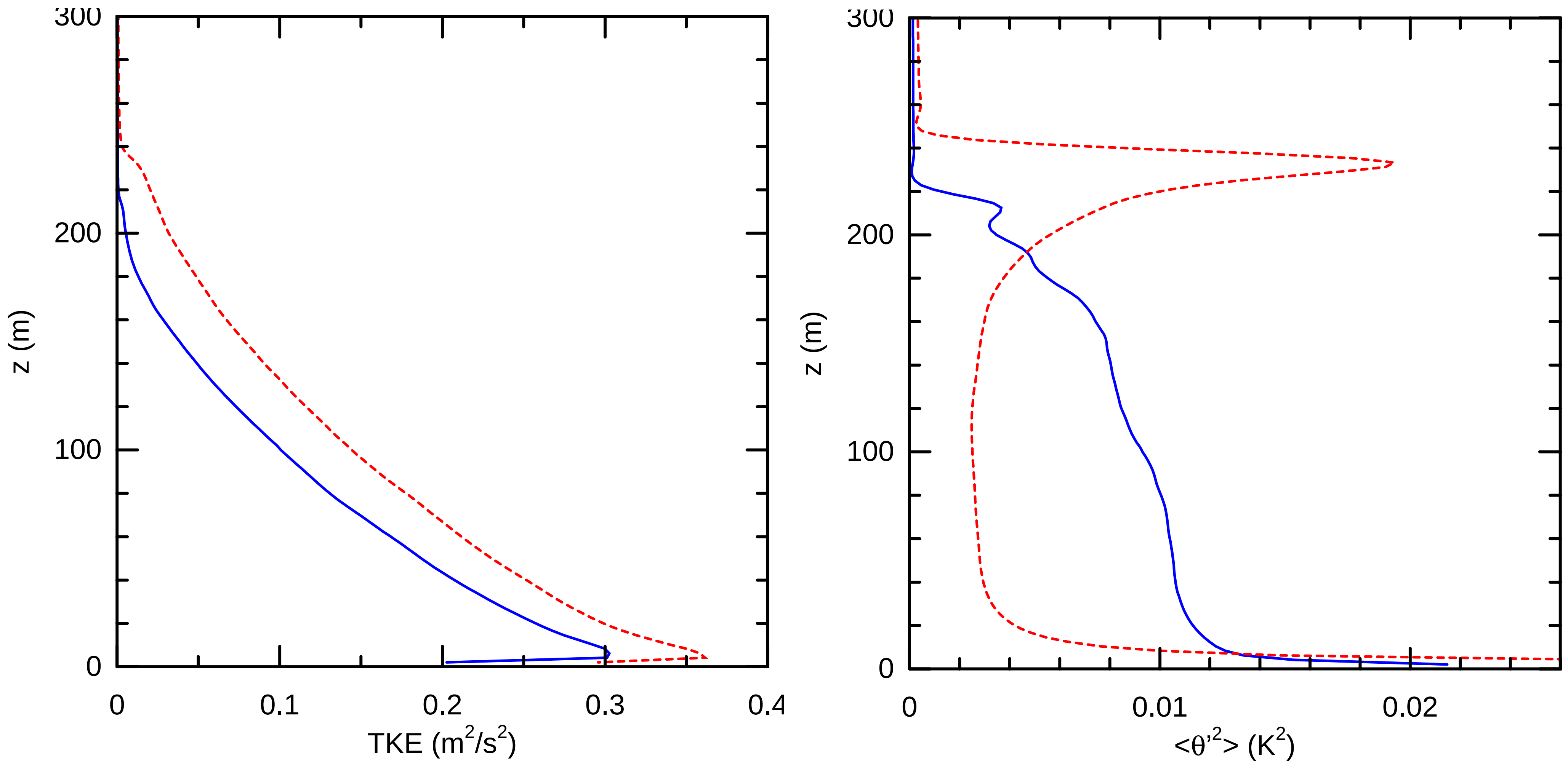
Mean Potential Temperature



Blue – horizontally-homogeneous SBL,

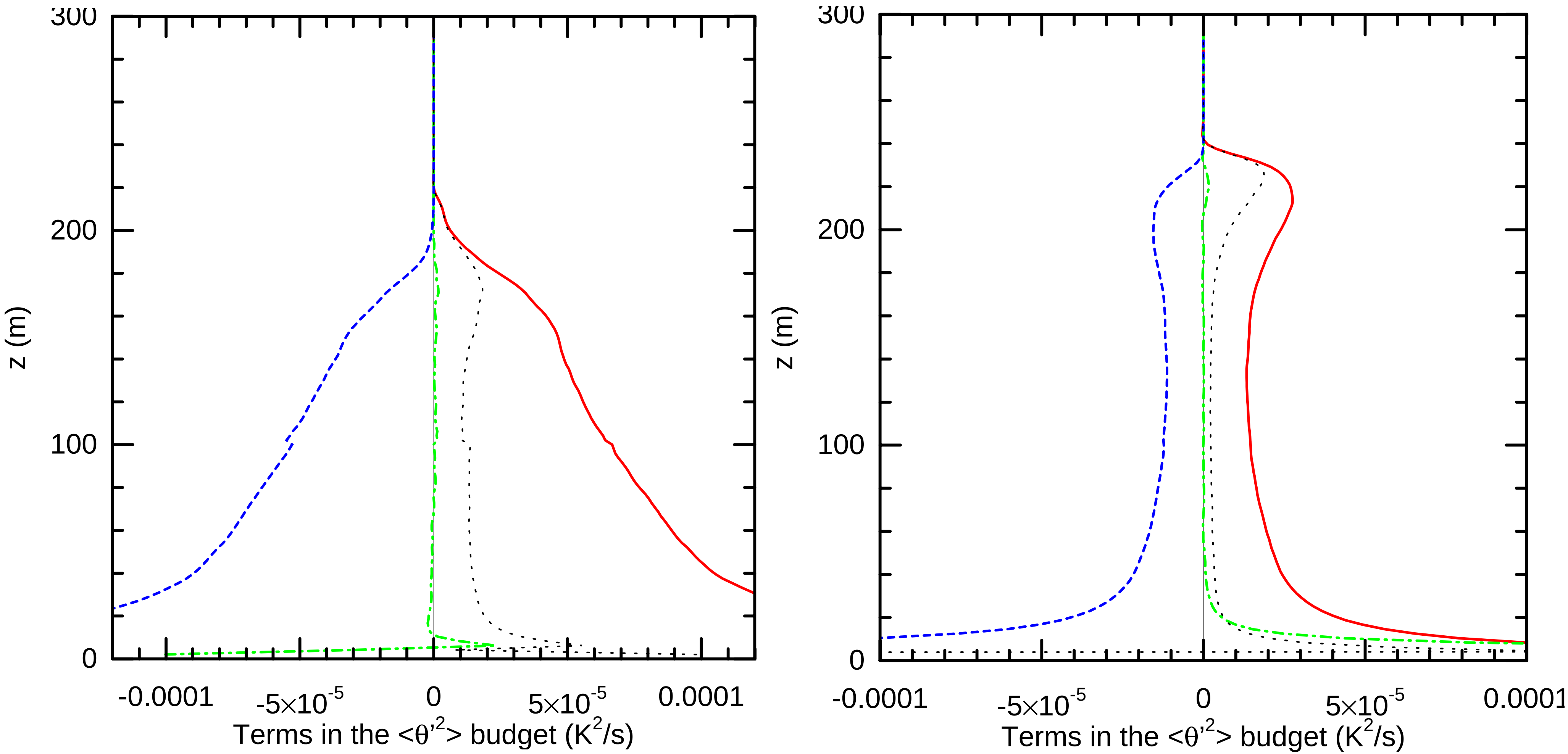
red – horizontally-heterogeneous SBL.

TKE and Temperature Variances



Blue – horizontally-homogeneous SBL, red – horizontally-heterogeneous SBL.

Budget of Temperature Variance



Left panel – horizontally-homogeneous SBL, right panel – horizontally-heterogeneous SBL. Red – mean-gradient production/destruction, green – third-order transport, blue – dissipation, black (thin dotted) – tendency .

Key Point: Third-Order Transport of Temperature Variance

LES estimate of **$<w'\theta'^2>$** (resolved plus SGS)

$$\left\langle \overline{w}''\overline{\theta}''^{2} \right\rangle + \left\langle \overline{w}''\overline{\overline{\theta'^{2}}}'' \right\rangle + 2\left\langle \overline{\theta}''\overline{\overline{w'\theta'}}'' \right\rangle + \left\langle \overline{w'\theta'^{2}} \right\rangle$$

Surface temperature variations modulate local static stability and hence the surface heat flux $\rightarrow$ net production/destruction of $<\theta'^2>$ due to divergence of third-order transport term!

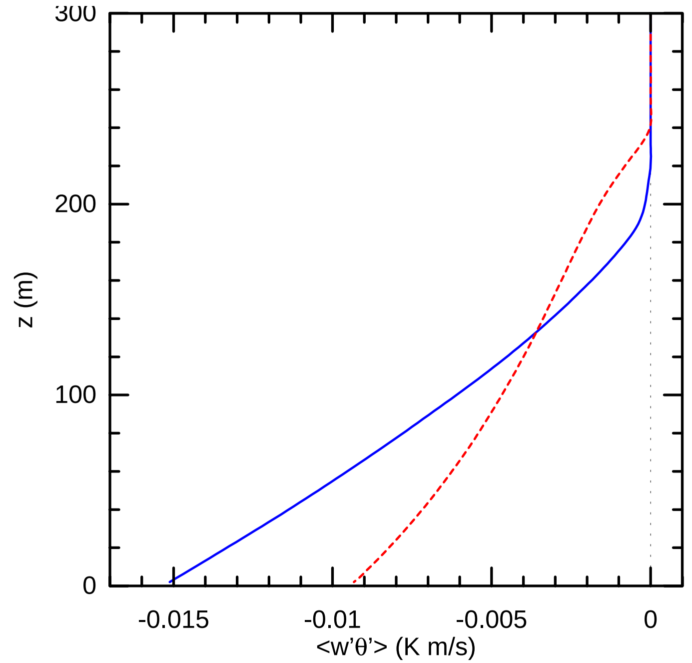
In heterogeneous SBL, the third-order transport of temperature variance is non-zero at the surface

Enhanced Mixing in Horizontally-Heterogeneous SBL An Explanation

$$\overline{w'\theta'} = -C_{\theta g}\tau_{\varepsilon} e \frac{\partial \overline{\theta}}{\partial z} + C_{\theta b}\tau_{\varepsilon} g\alpha\overline{\theta'^2}$$

Mean-gradient production of downward (negative) heat flux

Buoyancy production of upward (positive) heat flux



Due to non-zero flux of <θ'^2> at the surface

in horizontally-heterogeneous SBL:

increased <θ'^2> near the surface → reduced magnitude of downward heat flux → less work against the gravity → increased TKE → stronger mixing

Conclusions and Outlook

- A way towards a unified description of turbulence and shallow convection within the second-order closure framework is outlined
- Turbulent transport of scalar variances is a crucial point (neglected in most operational turbulence schemes)
- LES of stably stratified PBL over horizontally-inhomogeneous vs. horizontally-homogeneous surface provide insight into the PBL turbulence structure and transport properties and suggest the way to improve stable PBL parameterisations

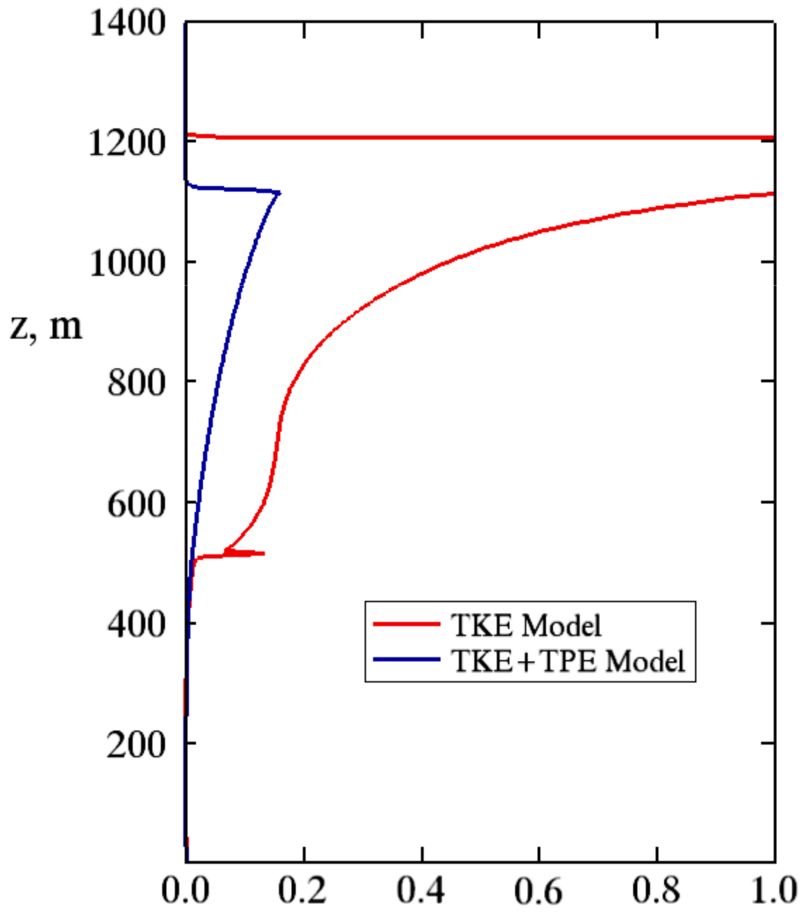
- Comprehensive testing in various PBL regimes (most notably, PBL with Cu clouds where problems are encountered)
- Improvements in terms of numerical stability and computational efficiency; implementation into NWP models

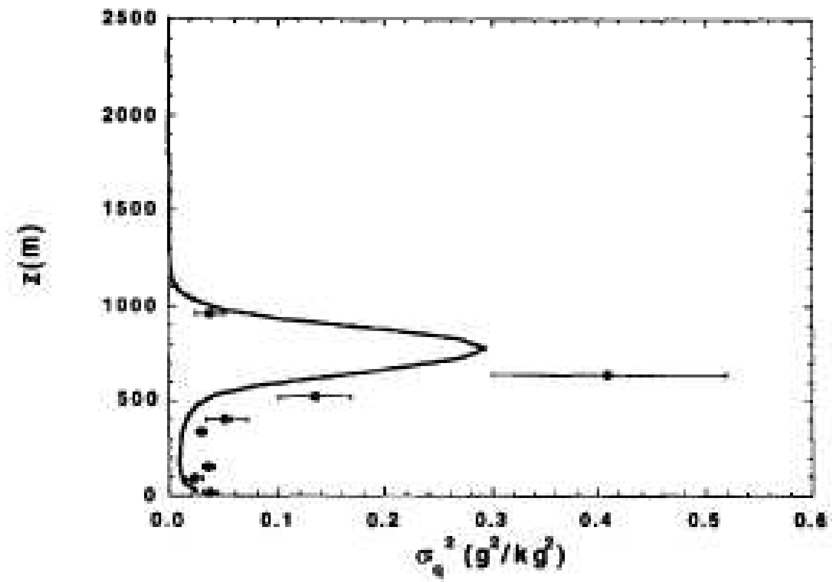
Thanks for your attention!

Acknowledgements. Thanks are due to Peter Bechtold, Vittorio Canuto, Sergey Danilov, Stephan de Roode, Evgeni Fedorovich, Jean-François Geleyn, Andrey Grachev, Vladimir Gryanik, Erdmann Heise, Martin Köhler, Friedrich Kupka, Cara-Lyn Lappen, Donald Lenschow, Vasily Lykossov, Pedro Miranda, Chin-Hoh Moeng, Jean-Marcel Piriou, David Randall, Matthias Raschendorfer, Bodo Ritter, Pier Siebesma, Pedro Soares, Joao Teixeira, Jeffrey Weil, Jun-Ichi Yano, and Sergej Zilitinkevich for discussions.

Unused

PBL with Cumulus Clouds, BOMEX



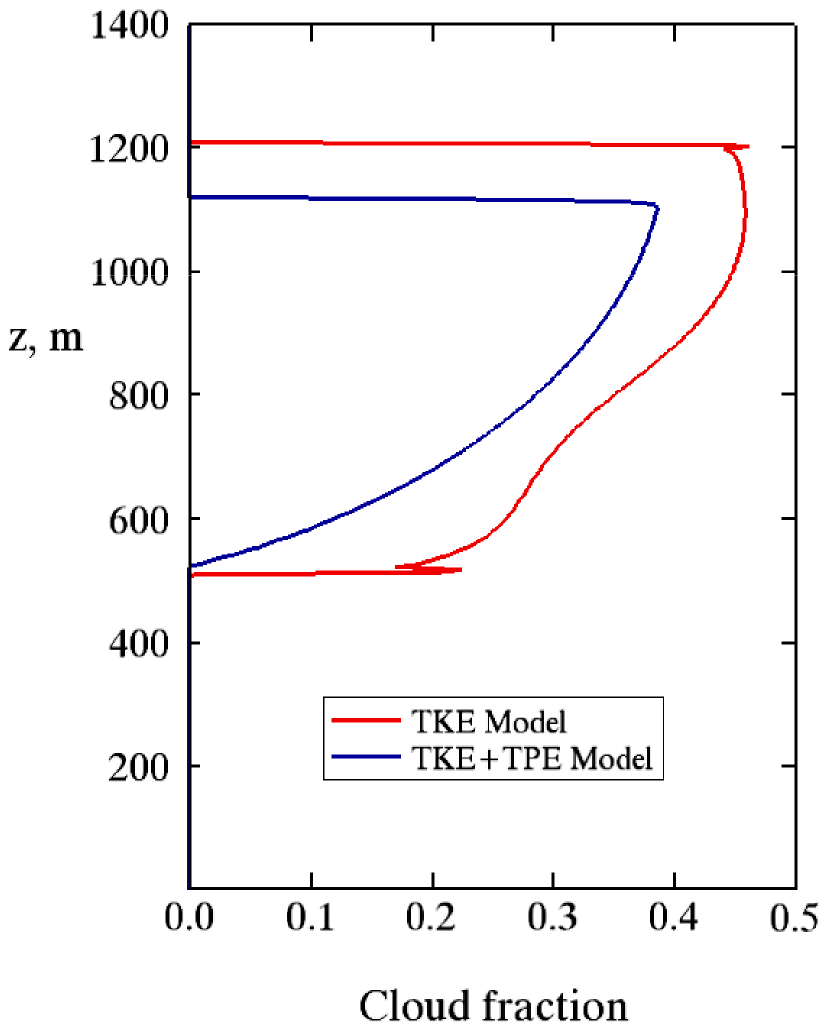


Variance of the total water specific humidity.

Left: TKE model (red) vs. TKE-Scalar Variance (TKE-TPE) model (blue).

Right: LES data (Cuijpers et al. 1996)

PBL with Cumulus Clouds, BOMEX(cont'd)

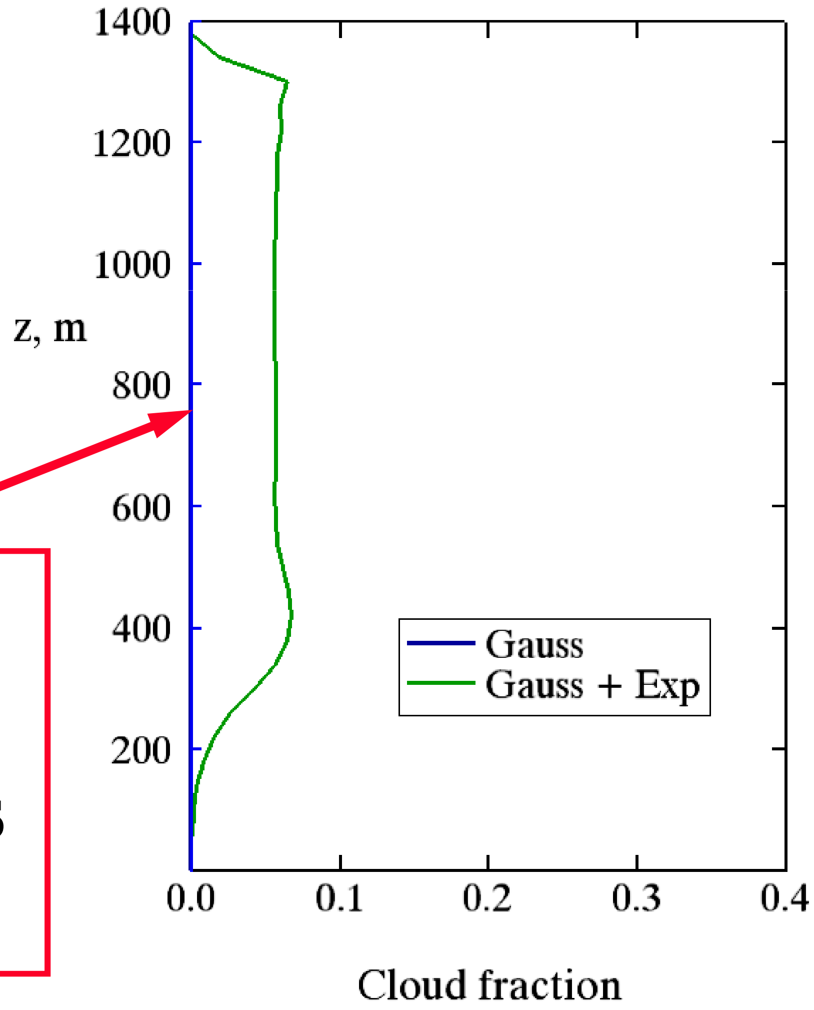


Data suggest cloud fraction of order 0.1

Fractional cloud cover with Gaussian SGS statistical cloud scheme.

TKE model (red) vs. TKE-Scalar Variance (TKE-TPE) model (blue).

PBL with Cumulus Clouds, BOMEX (cont'd)



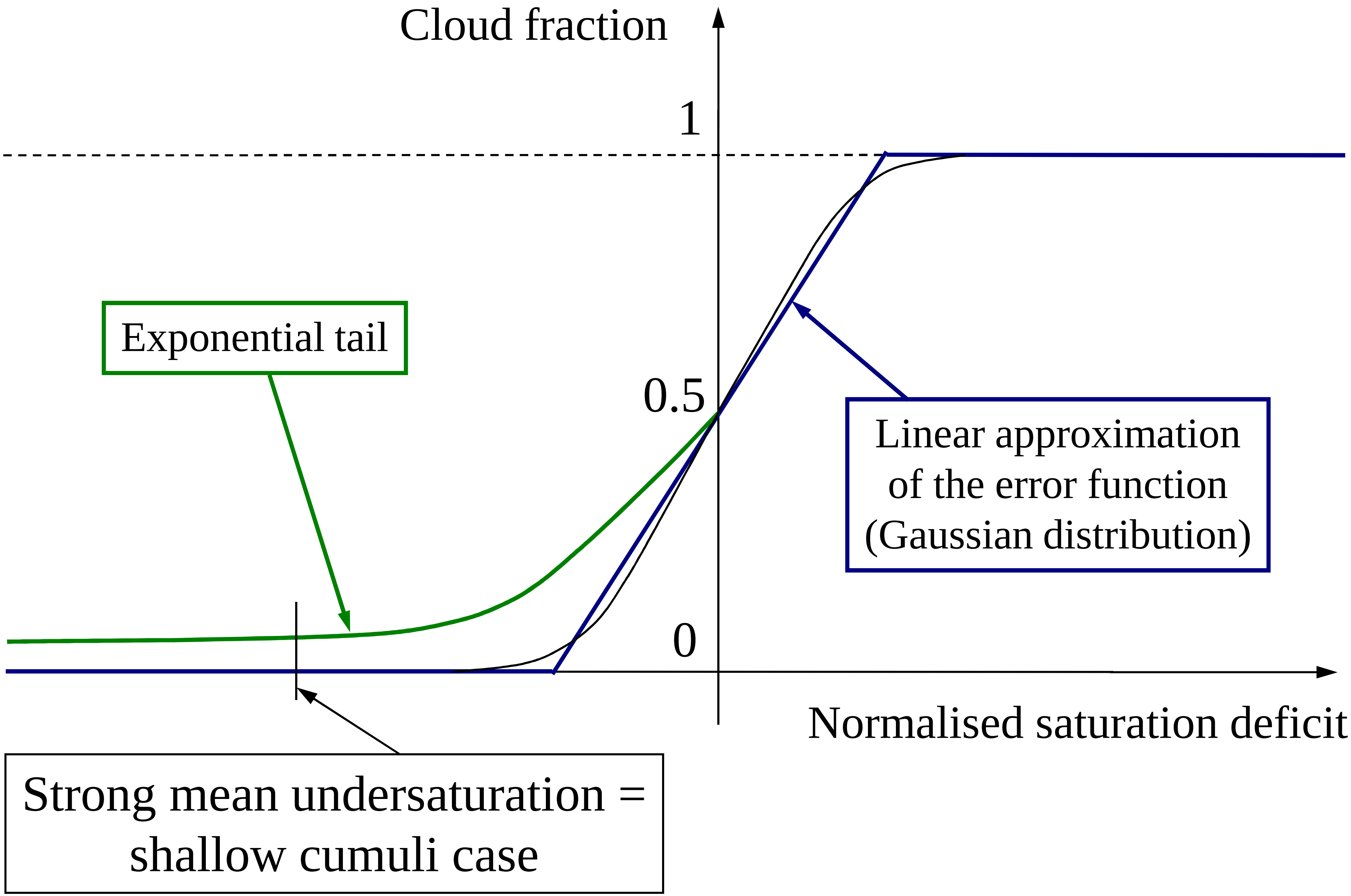
Gaussian SGS cloud scheme gives no clouds!

Data suggest cloud fraction of order 0.1

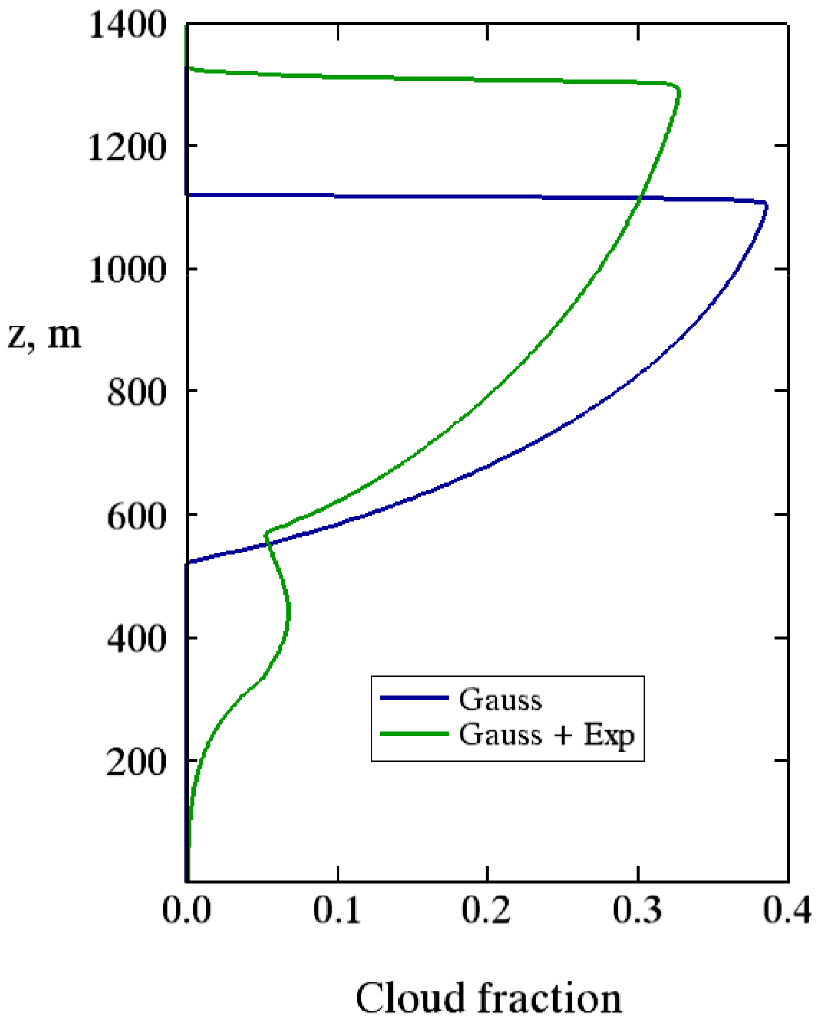
Fractional cloud cover with the θ_l and q_t profiles taken from LES.

Curves shows cloud cover with non-Gaussian SGS statistical cloud scheme (includes exponential tail to account for the effect of cumuli).

Cu Case (cont'd)



Cu Case (cont'd)



Fractional cloud cover simulated by the TKE-Scalar Variance model using Gaussian (blue) and non-Gaussian (green) SGS statistical cloud scheme.

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Skewness-Dependent Parameterisation of Third-Order Transport

$$\overline{u_i'\theta'^2} = -K\frac{\partial\overline{\theta'^2}}{\partial x_i} + S_\theta \overline{\theta'^2}^{1/2}\,\overline{u_i'\theta'}, \qquad S_\theta = \frac{\overline{\theta'^3}}{\overline{\theta'^2}^{3/2}}$$

Down-gradient term (diffusion)

Non-gradient term (advection)

Accounts for non-local transport due to coherent structures (convective plumes or rolls) – mass-flux ideas!

Skewness-Dependent Parameterisation of Third-Order Transport (cont'd)

$$S_\theta \overline{\theta'^2}^{1/2}\,\overline{u_i'\theta'} = \left(\frac{S_\theta \overline{u_i'\theta'}}{\overline{\theta'^2}^{1/2}}\right)\overline{\theta'^2} = w_i\,\overline{\theta'^2}$$

Plume/roll scale "advection" velocity

Closure for Skewness

In order to determine skewness, we make use of the transport equation for the potential-temperature triple correlation

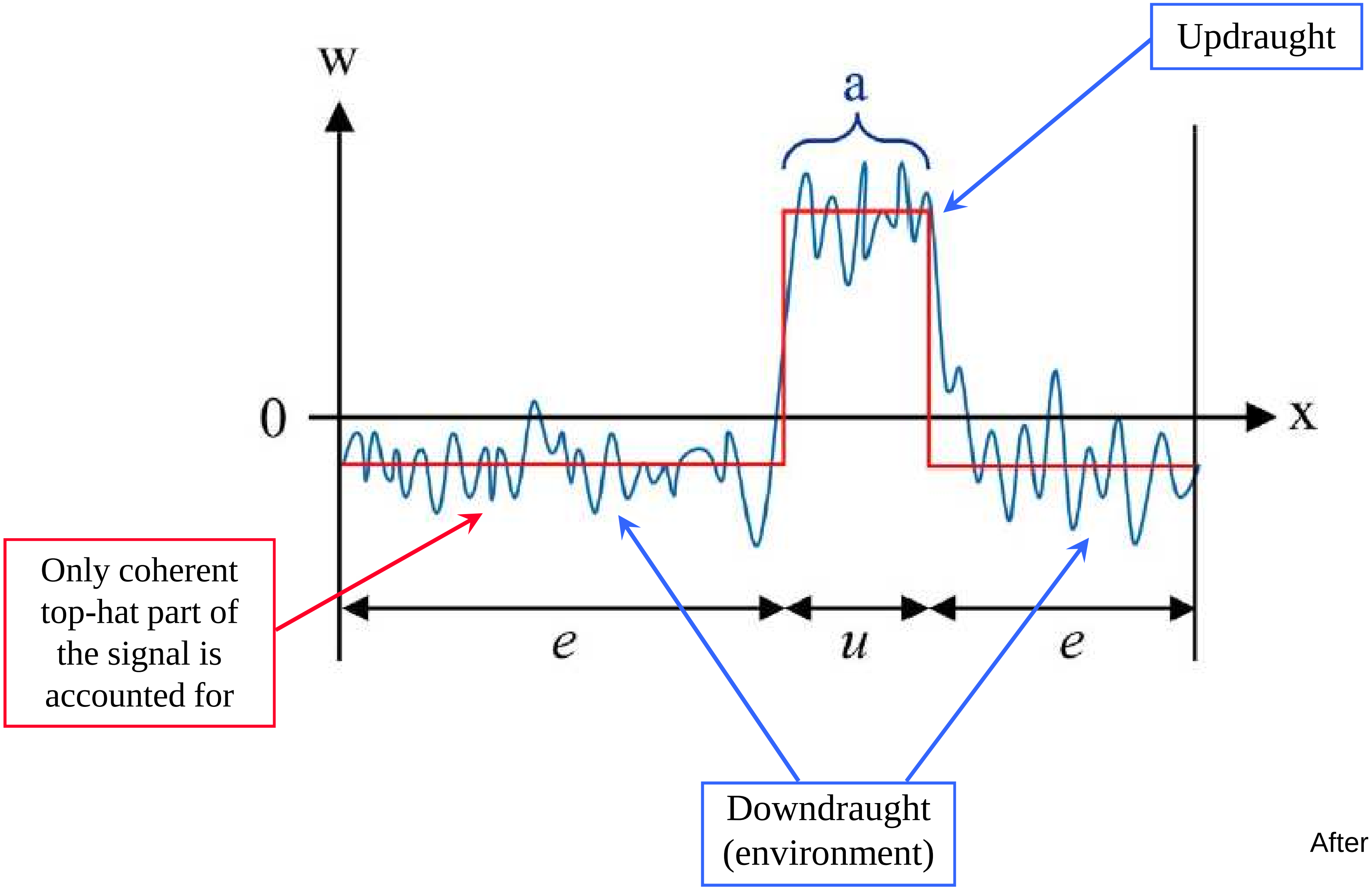
$$\frac{1}{3}\left(\frac{\partial}{\partial t}+\overline{u}_i\frac{\partial}{\partial x_i}\right)\overline{\theta'^3}=-\overline{u_i'\theta'^2}\frac{\partial\overline{\theta}}{\partial x_i}+\overline{\theta'^2}\frac{\partial}{\partial x_i}\overline{u_i'\theta'}-\underline{\frac{1}{3}\frac{\partial}{\partial x_i}\overline{u_i'\theta'^3}}-\varepsilon_{\theta^3}$$

Using the mass-flux ideas, the fourth-order moment is closed through the temperature skewness (Gryanik and Hartmann 2002) – no need for equations of higher order!

$$\boxed{\overline{u_i'\theta'^3}=3\left(1+\frac{1}{3}S_\theta^2\right)\overline{\theta'^2}\,\overline{u_i'\theta'}}$$

Analogies to Mass-Flux Approach

A top-hat representation of a fluctuating quantity



After M. Köhler (2005)

Analogies to Mass-Flux Approach (cont'd)

Two-delta-function mass-flux framework. Averaging rule

$$\overline{w'^m X'^n} = a(1-a)\left[(1-a)^{m+n-1} - (-a)^{m+n-1}\right](w_u - w_d)^m (X_u - X_d)^n$$

Second-order moments

$$\overline{w'^2} = a(1-a)(w_u - w_d)^2, \quad \overline{X'^2} = a(1-a)(X_u - X_d)^2,$$

$$\overline{w'X'} = a(1-a)(w_u - w_d)(X_u - X_d)$$

Third-order moments

$$\overline{w'^3} = a(1-a)(1-2a)(w_u - w_d)^3, \quad \overline{X'^2} = a(1-a)(1-2a)(X_u - X_d)^3,$$

$$\overline{w'X'^2} = a(1-a)(1-2a)(w_u - w_d)(X_u - X_d)^2$$

Notice the factor (1-2*a*)!

Analogies to Mass-Flux Approach (cont'd)

Skewness

$$S_x \equiv \frac{\overline{X'^3}}{\overline{X'^2}^{3/2}} = \frac{1-2a}{[a(1-a)]^{1/2}}$$

S_x tends to ∞ (-∞) as a tends to 0 (1). Then

$$\overline{w'X'^2} = a(1-a)(1-2a)(w_u - w_d)(X_u - X_d)^2$$
$$= S_X \overline{X'^2}^{1/2} \overline{w'X'^2}$$

The mass-flux formulation

recast in terms of the ensemble-mean quantities!

Sensitivity to Filter Scale (Resolution)

$$\overline{u_i'\theta'^2} = -K\frac{\partial\overline{\theta'^2}}{\partial x_i} + S_\theta\overline{\theta'^2}^{1/2}\overline{u_i'\theta'}, \quad S_\theta = \frac{\overline{\theta'^3}}{\overline{\theta'^2}^{3/2}}$$

As the resolution is refined, the SGS motions are (expected to be) increasingly Gaussian.

Then, $S\rightarrow 0$ and the parameterisation of the third-order transport term reduces to the down-gradient diffusion approximation.

Relation to Scale Separation Ideas

We apply a triple decomposition, using (i) a low-pass filter whose characteristic horizontal scale, Δ, is much less than the domain size, *L*, and (ii) a horizontal averaging operator over *L*. A fluctuating quantity *f* may then be represented as a sum of the horizontal mean filtered part, a deviation of the filtered quantity from the horizontal mean, and a sub-filter fluctuation,

$$f = \langle \bar{f} \rangle + \bar{f}'' + f',$$

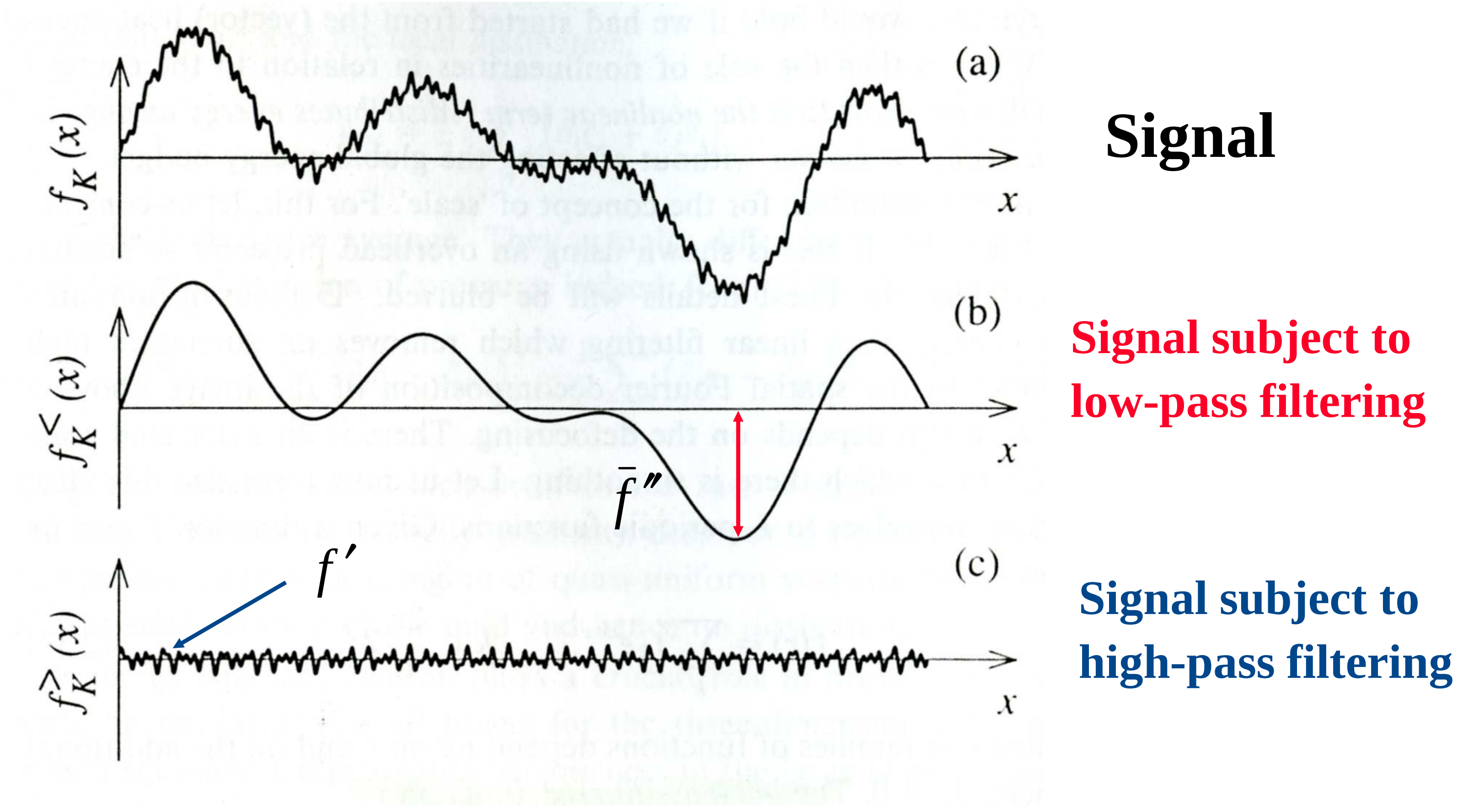
where an overbar denotes a low-pass filtered quantity, and a prime denotes a deviation therefrom. Angle brackets denote averaging over the horizontal, and a double prime denotes a fluctuation about a horizontal mean.

There is nothing really new in it, cf.

- Mean flow-wave-turbulence decomposition (Hussein and Reynolds 1970, 1972, Reynolds and Hussein 1972)
- A procedure routinely used in LES studies to compute (approximations to) ensemble-mean statistical moments as a sum of resolved scale and sub-grid scale contributions (e.g. Brown 1995, Mironov et al. 2000, Mironov 2001)
- Energy budget scale-by-scale (Frisch 1995, section 2.4)

Relation to Scale Separation Ideas (cont'd)

Low-pass filtered and high-pass filtered quantity



$$total\ variance = \left\langle \bar{f}''^2 \right\rangle + \left\langle \overline{f'^2} \right\rangle$$

Relation to Scale Separation Ideas (cont'd)

Variance budget of low-pass filtered scalar quantity

$$\frac{1}{2}\frac{d\left\langle \bar{f}''^2 \right\rangle}{dt} = -\left\langle \bar{u}_i''\bar{f}'' \right\rangle \frac{\partial \left\langle \bar{f} \right\rangle}{\partial x_i} - \frac{1}{2}\frac{\partial}{\partial x_i}\left\langle \bar{u}_i''\bar{f}''^2 \right\rangle - \left\langle \bar{f}'' \frac{\partial \overline{u_i' f'}''}{\partial x_i} \right\rangle$$

Spectral transfer through the filter scale

Variance budget of high-pass (sub-filter) scale quantity,

$$\frac{1}{2}\frac{d\left\langle \overline{f'^2} \right\rangle}{dt} = -\left\langle \overline{u_i' f'} \right\rangle \frac{\partial \left\langle \bar{f} \right\rangle}{\partial x_i} - \frac{1}{2}\frac{\partial}{\partial x_i}\left(2\left\langle \bar{f}''\overline{u_i' f'}'' \right\rangle + \left\langle \bar{u}_i''\overline{f'^2}'' \right\rangle + \left\langle \overline{u_i' f'^2} \right\rangle \right) - \left\langle \kappa \overline{\left(\frac{\partial f'}{\partial x_i} \right)^2} \right\rangle + \left\langle \bar{f}'' \frac{\partial \overline{u_i' f'}''}{\partial x_i} \right\rangle,$$

Dissipation

Relation to Scale Separation Ideas (cont'd)

Adding the two budgets, we get the total variance budget

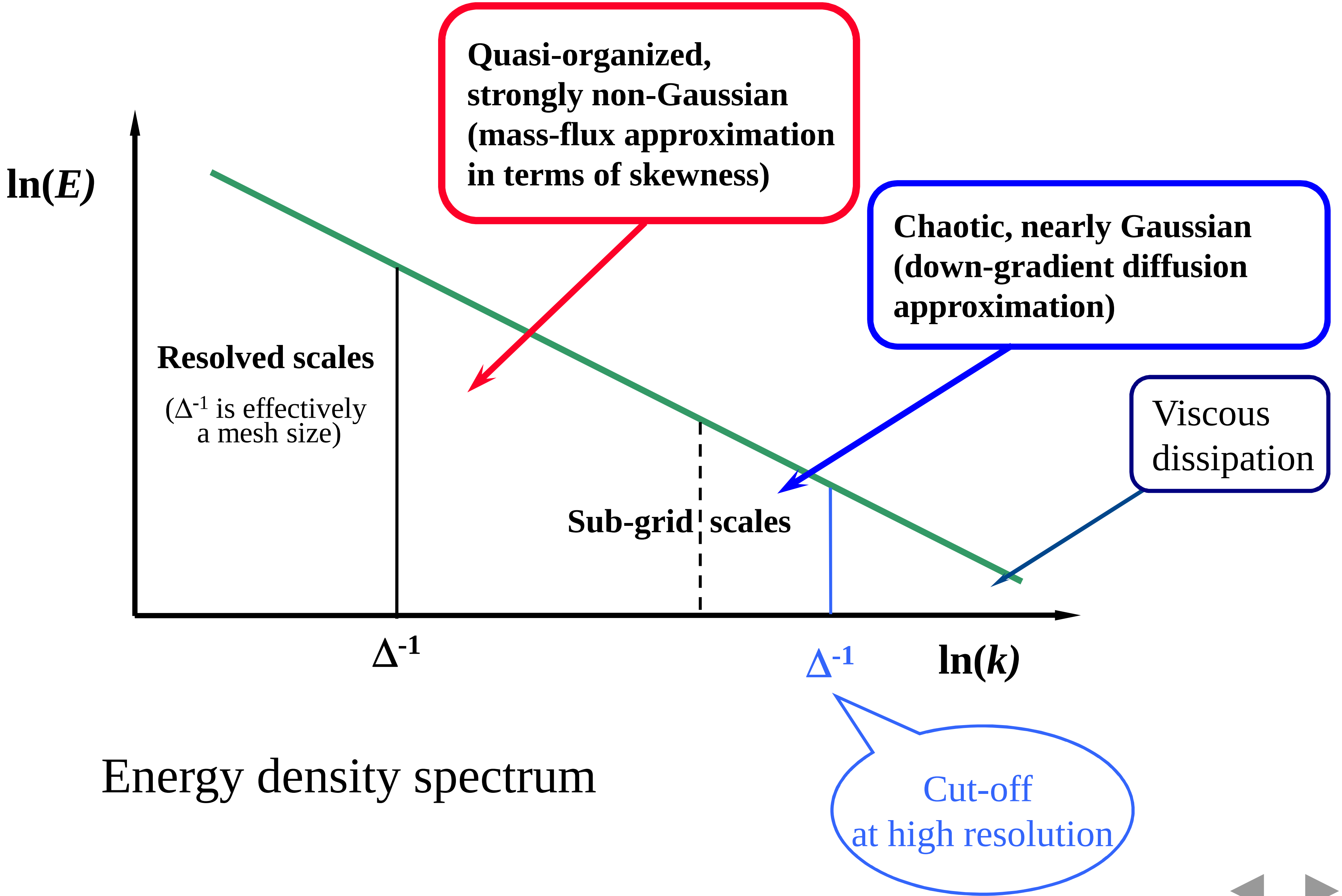
$$\frac{1}{2}\frac{d}{dt}\left(\left\langle \bar{f}''^2 \right\rangle + \left\langle \overline{f'^2} \right\rangle\right) = -\left(\left\langle \bar{u}_i''\bar{f}'' \right\rangle + \left\langle \overline{u_i' f'} \right\rangle\right)\frac{\partial \left\langle \bar{f} \right\rangle}{\partial x_i} - \varepsilon_{f^2}$$

$$-\frac{1}{2}\frac{\partial}{\partial x_i}\left(\left\langle \bar{u}_i''\bar{f}''^2 \right\rangle + 2\left\langle \bar{f}''\overline{u_i' f'}'' \right\rangle + \left\langle \bar{u}_i''\overline{f'^2}'' \right\rangle + \left\langle \overline{u_i' f'^2} \right\rangle\right)$$

Quasi-organised motions (advective transport)

Chaotic motions (down-gradient diffusion)

Relation to Scale Separation Ideas (cont'd)



Energy density spectrum

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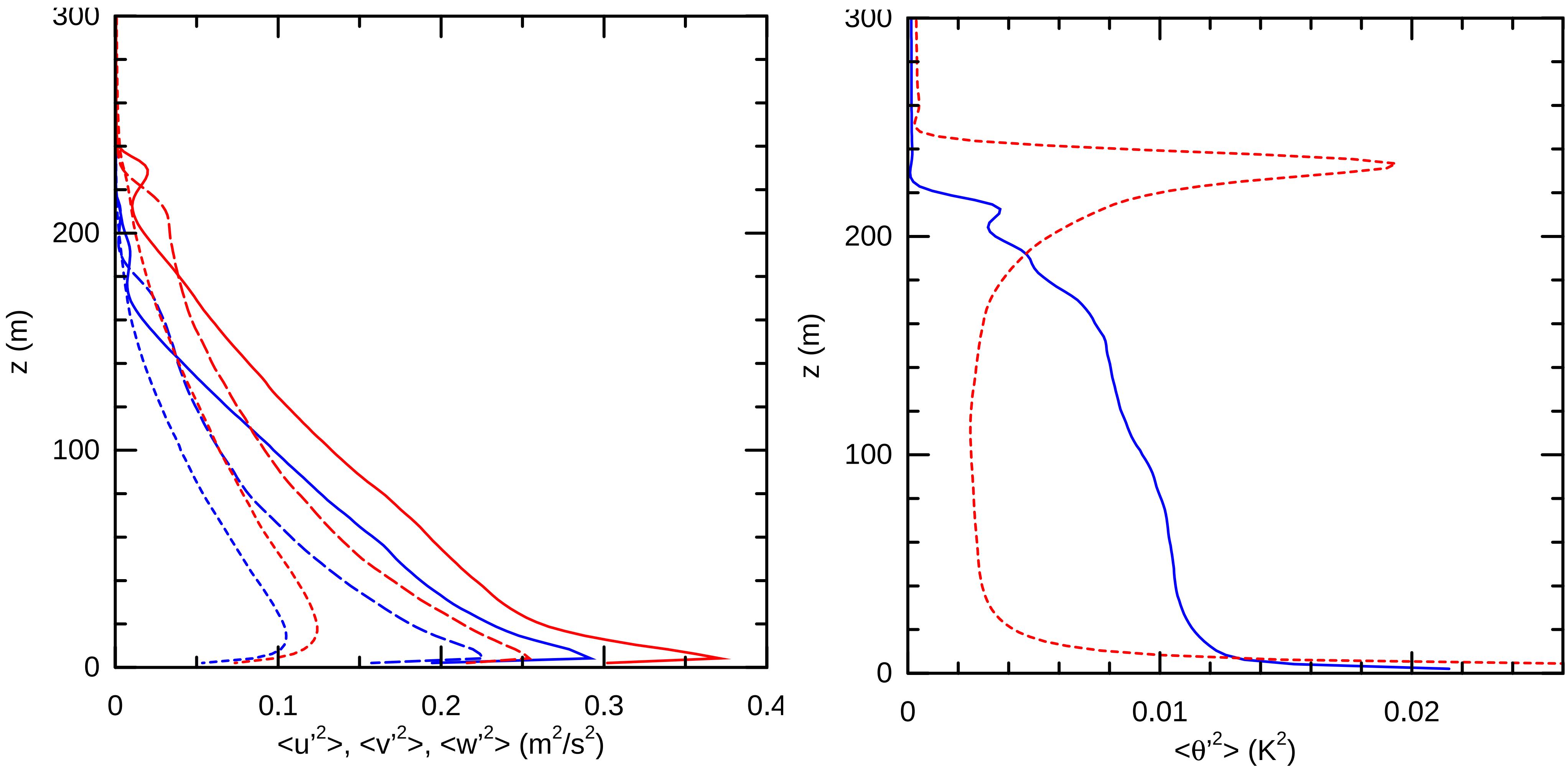
Velocity and Temperature Variances



Blue – horizontally-homogeneous SBL, red – horizontally-heterogeneous SBL.

Left panel: short-dashed – $<w'^2>$, long-dashed – $<v'^2>$, solid – $<u'^2>$.

Budget of the Vertical Temperature Flux

Left panel – horizontally-homogeneous SBL, right panel – horizontally-heterogeneous SBL. **Red** – mean-gradient production/destruction, **black** – buoyancy production, **green** – third-order transport, **blue** – pressure gradient-temperature covariance, thin dotted black – tendency .

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Conclusions and Outlook

- A way towards a unified description of turbulence and shallow convection within the second-order closure framework is outlined
- A non-local closure model (prognostic equations for the TKE and for the scalar variances) is developed and tested through single-column numerical experiments
- LES of stably stratified PBL over horizontally-inhomogeneous vs. horizontally-homogeneous surface provide insight into the PBL turbulence structure and transport properties and suggest the way to improve stable PBL parameterisations

- Comprehensive testing in various PBL regimes (most notably, PBL with Cu clouds where problems are encountered)
- Improvements in terms of numerical stability and computational efficiency; implementation into NWP models