The RAW filter: an improvement to the Robert-Asselin filter

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Impact of time stepping in weather and climate prediction

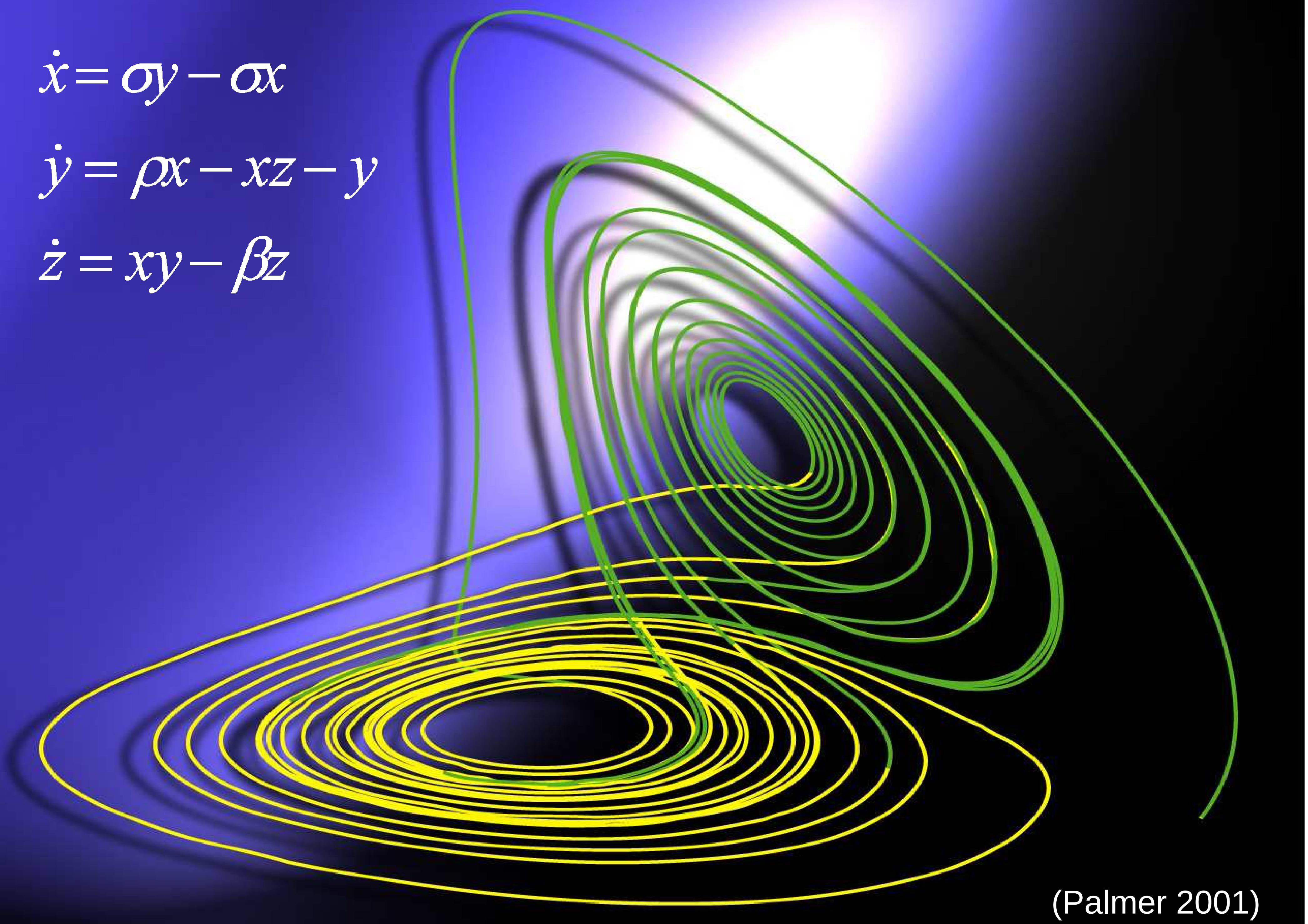
"In the weather and climate prediction community, when thinking in terms of model predictability, there is a tendency to associate model error with the physical parameterizations. In this paper, it is shown that time truncation error can be a substantial part of the total forecast error."

(Teixeira et al. 2007)

$$\dot{x} = \sigma y - \sigma x$$

$$\dot{y} = \rho x - xz - y$$

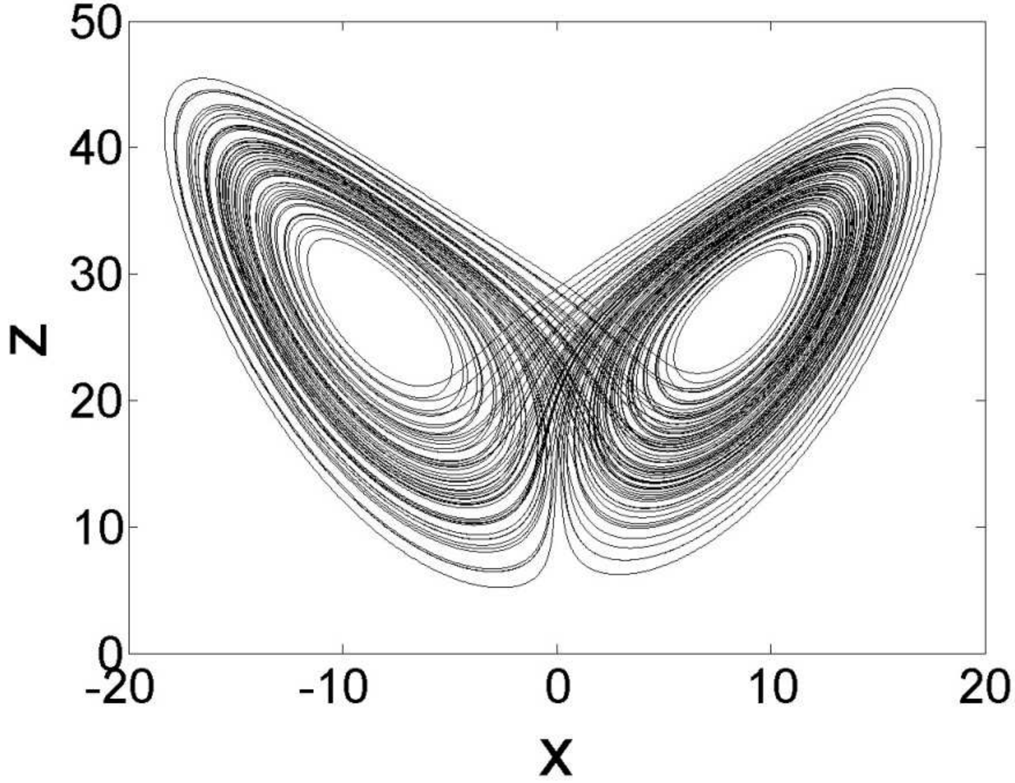
$$\dot{z} = xy - \beta z$$



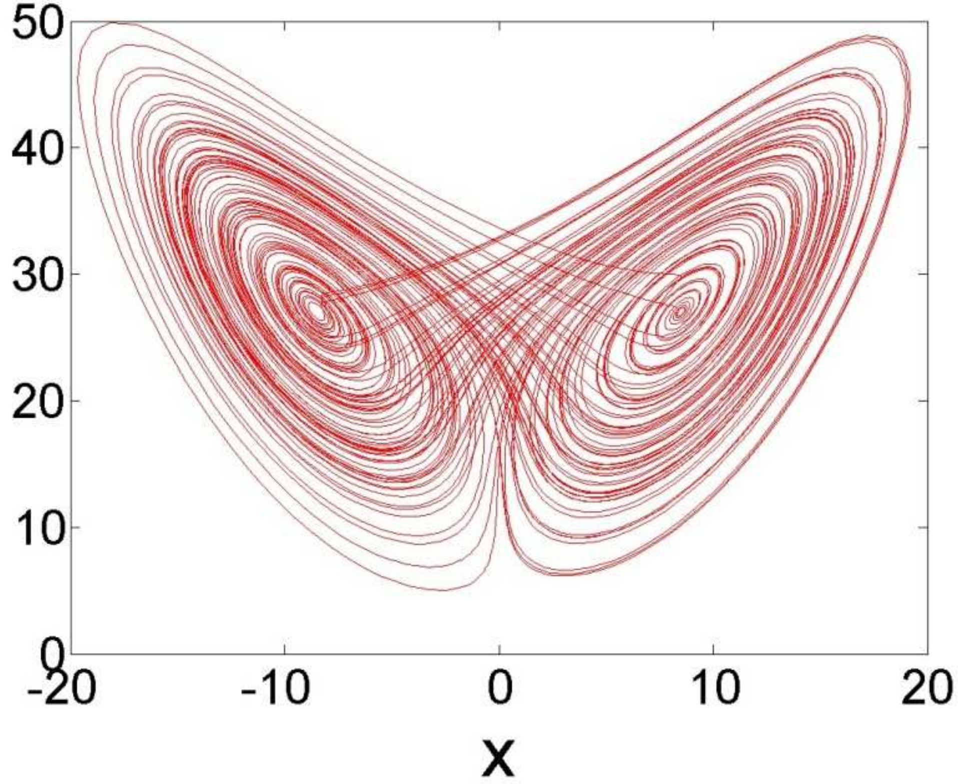
(Palmer 2001)

Impact of different time steps on the 'climate' of the Lorenz attractor

$\Delta t = 0.001$



$\Delta t = 0.01$

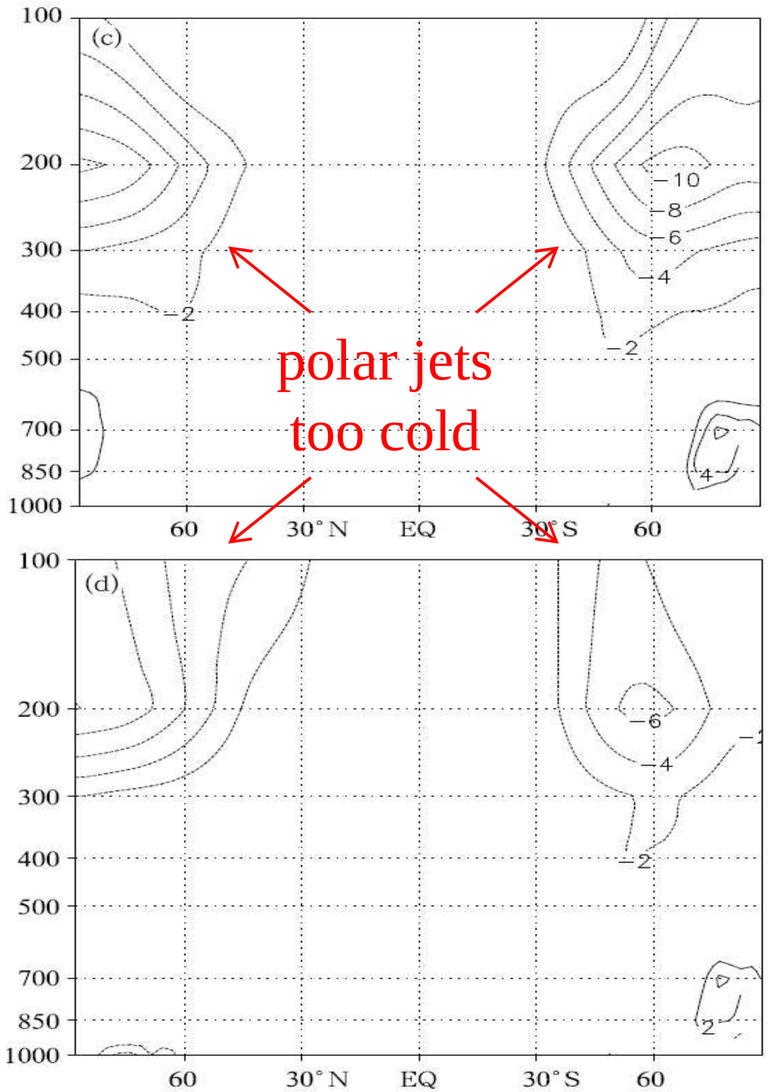


Using the explicit Euler forward scheme

Impact of different time-stepping schemes in CAM atmosphere GCM

zonal-mean temperature error (°C) relative to ERA40

pressure (hPa)



pressure (hPa)

leapfrog

2nd-order Adams-Bashforth

(Zhao & Zhong 2009)

Time-stepping methods

Method	Order	Formula	Storage factor	Efficiency factor	Amplitude error	Phase error	Maximum $\omega\Delta t$
Forward (Adams–Bashforth)	1	$\phi^{n+1} = \phi^n + hF(\phi^n)$	2	0	$1 + \frac{p^2}{2}$	$1 - \frac{p^2}{3}$	0
Backward (Adams–Moulton)	1	$\phi^{n+1} = \phi^n + hF(\phi^{n+1})$	Implicit	∞	$1 - \frac{p^2}{2}$	$1 - \frac{p^2}{3}$	∞
Matsuno	1	$\phi^* = \phi^n + hF(\phi^n)$ $\phi^{n+1} = \phi^n + hF(\phi^*)$	2	.5	$1 - \frac{p^2}{2}$	$1 + \frac{2}{3}p^2$	1
Asselin-filtered leapfrog	1	$\phi^{n+1} = \overline{\phi^{n-1}} + 2hF(\phi^n)$ $\overline{\phi^n} = \phi^n + \gamma(\overline{\phi^{n-1}} - 2\phi^n + \phi^{n+1})$	3	<1	$1 - \frac{\gamma}{2(1-\gamma)}p^2$	$1 + \frac{1+2\gamma}{6(1-\gamma)}p^2$	<1
Leapfrog	2	$\phi^{n+1} = \phi^{n-1} + 2hF(\phi^n)$	2	1	1	$1 + \frac{p^2}{6}$	1
Runge–Kutta (Williamson/Huen)	2	$q_1 = hF(\phi^n), \quad \phi_1 = \phi^n + q_1$ $q_2 = hF(\phi_1) - q_1, \quad \phi^{n+1} = \phi_1 + q_2/2$	2	0	$1 + \frac{p^4}{8}$	$1 + \frac{p^2}{6}$	0
Adams–Bashforth	2	$\phi^{n+1} = \phi^n + \frac{h}{2}[3F(\phi^n) - F(\phi^{n-1})]$	3	0	$1 + \frac{p^4}{4}$	$1 + \frac{5}{12}p^2$	0
Adams–Moulton (Trapizoidal)	2	$\phi^{n+1} = \phi^n + \frac{h}{2}[F(\phi^{n+1}) + F(\phi^n)]$	Implicit	∞	1	$1 - \frac{p^2}{12}$	∞
Leapfrog, then Adams–Bashforth (Magazenkov)	2	$\phi^n = \phi^{n-2} + 2hF(\phi^{n-1})$ $\phi^{n+1} = \phi^n + \frac{h}{2}[3F(\phi^n) - F(\phi^{n-1})]$	3	.67	$1 - \frac{p^4}{4}$	$1 + \frac{p^2}{6}$	.67
Leapfrog-Trapizoidal (Kurihara)	2	$\phi^* = \phi^{n-1} + 2hF(\phi^n)$ $\phi^{n+1} = \phi^n + \frac{h}{2}[F(\phi^n) + F(\phi^*)]$	3	.71	$1 - \frac{p^4}{4}$	$1 - \frac{p^2}{12}$	1.41
Young's method A	2	$\phi_1 = \phi^n + hF(\phi^n)/2$ $\phi_2 = \phi_1 + hF(\phi_1)/2$ $\phi^{n+1} = \phi^n + hF(\phi_2)$	3	0	$1 + \frac{p^6}{128}$	$1 + \frac{p^2}{24}$	0
Runge–Kutta (Williamson)	3	$q_1 = hF(\phi^n), \quad \phi_1 = \phi^n + q_1/3,$ $q_2 = hF(\phi_1) - 5q_1/9, \quad \phi_2 = \phi^1 + 15q_2/16$ $q_3 = hF(\phi_2) - 153q_2/128, \quad \phi^{n+1} = \phi_2 + 8q_3/15$	2	.58	$1 - \frac{p^4}{24}$	$1 + \frac{p^4}{30}$	1.73
ABM predictor-corrector	3	$\phi^* = \phi^n + \frac{h}{2}[3F(\phi^n) - F(\phi^{n-1})]$ $\phi^{n+1} = \phi^* + \frac{5h}{12}[F(\phi^*) - 2F(\phi^n) + F(\phi^{n-1})]$	4	.60	$1 - \frac{19}{144}p^4$	$1 + \frac{1243}{8640}p^4$	1.20
Adams–Moulton	3	$\phi^{n+1} = \phi^n + \frac{h}{12}[5F(\phi^{n+1}) + 8F(\phi^n) - F(\phi^{n-1})]$	Implicit	0	$1 + \frac{p^4}{24}$	$1 - \frac{11}{720}p^4$	0
Adams–Bashforth	3	$\phi^{n+1} = \phi^n + \frac{h}{12}[23F(\phi^n) - 16F(\phi^{n+1}) + 5F(\phi^{n-2})]$	4	.72	$1 - \frac{3}{8}p^4$	$1 + \frac{289}{720}p^4$	0.72
Runge–Kutta (Classical)	4	$k_0 = hF(\phi^n)$ $k_1 = hF(\phi^n + k_0/2)$ $k_2 = hF(\phi^n + k_1/2)$ $k_3 = hF(\phi^n + k_2)$ $\phi^{n+1} = \phi^n + \frac{1}{6}(k_0 + 2k_1 + 2k_2 + k_3)$	3*	.70	$1 - \frac{p^6}{144}$	$1 - \frac{p^4}{120}$	2.82
ABM predictor-corrector	4	$\phi^* = \phi^n + \frac{h}{12}[23F(\phi^n) - 16F(\phi^{n-1}) + 5F(\phi^{n-2})]$ $\phi^{n+1} = \phi^* + \frac{3h}{8}[F(\phi^*) - 3F(\phi^n) + 3F(\phi^{n-1}) - F(\phi^{n-2})]$	5	.59	$1 - \frac{265}{1536}p^6$	$1 - \frac{329}{2880}p^4$	1.18
Adams–Moulton	4	$\phi^{n+1} = \phi^n + \frac{h}{24}[9F(\phi^{n+1}) + 19F(\phi^n) - 5F(\phi^{n-1}) + F(\phi^{n-2})]$	Implicit	0	$1 + \frac{p^6}{48}$	$1 + \frac{19}{720}p^4$	0
Adams–Bashforth	4	$\phi^{n+1} = \phi^n + \frac{h}{24}[55F(\phi^n) - 59F(\phi^{n-1}) + 37F(\phi^{n-2}) - 9F(\phi^{n-3})]$	5	.43	$1 - \frac{13}{24}p^6$	$1 - \frac{251}{720}p^4$	.43

increasing accuracy

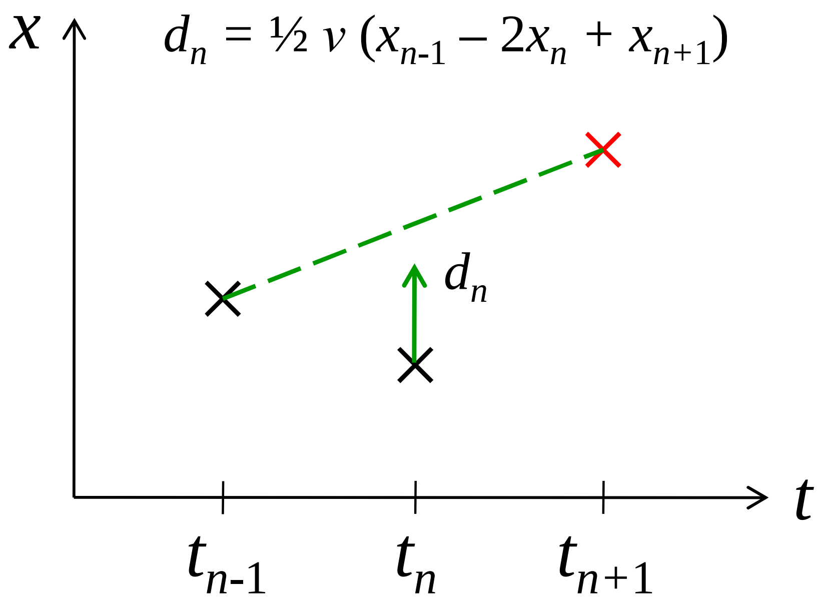
(Durran 1991)

Leapfrog with Robert-Asselin filter

- Widely used in current numerical models
  - atmosphere: ECHAM, MAECHAM, MM5, CAM, MESO-NH, HIRLAM, KMCM, LIMA, SPEEDY, IGCM, PUMA, COSMO, FSU-GSM, FSU-NRSM, NCEP-GFS, NCEP-RSM, NSEAM, NOGAPS, RAMS, CCSR/NIES-AGCM
  - ocean: OPA, ORCA, NEMO, HadOM3, DieCAST, TIMCOM, GFDL-MOM, POM, MICOM, HYCOM, POSEIDON, NCOM, ICON, OFES, SOM
  - coupled: HiGEM (oce), COAMPS (atm), PlaSim (atm), ECHO (atm), MIROC (atm), FOAM (oce), NCAR-CCSM (atm), BCM (oce), NCEP-CFS (atm/oce), QESM (oce), CHIME (oce), FORTE (atm)
  - others: GTM, ADCIRC, QUAGMIRE, MORALS, SAM, ARPS, CASL, CReSS, JTGCM, ECOMSED, UKMO-LEM, MPI-REMO
- Asselin (1972) has received over 450 citations
- Has many problems
  - "*The Robert-Asselin filter has proved immensely popular, and has been widely used for over 20 years. However, it is not the last word…*" (Lynch 1991)
  - "*Replacement of the Asselin time filter… can be a feasible way to improve the ability of climate models*" (Zhao & Zhong 2009)
  - "*The Robert-Asselin filter can produce slewing frequency as well as the well-known damping and phase errors*" (Thrastarson & Cho 2011)

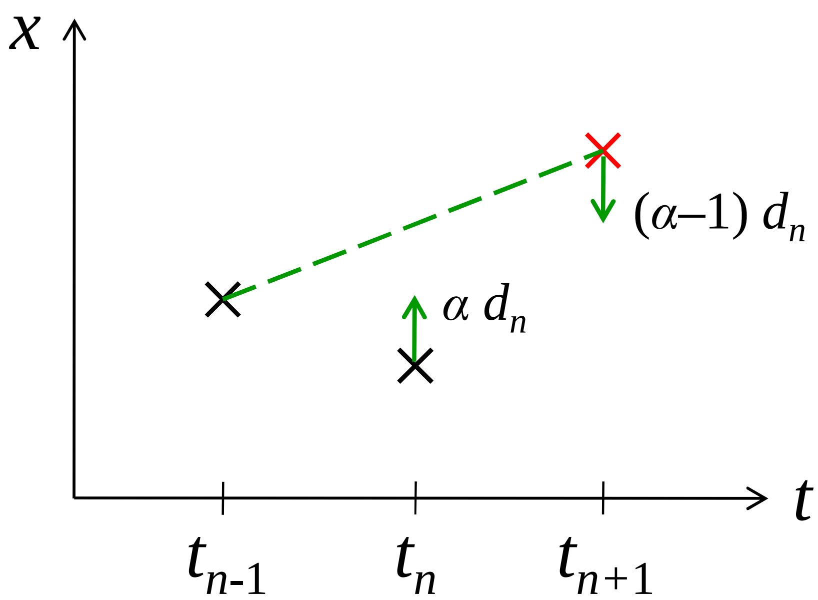
A proposed improvement

<u>LF+RA</u>
(Robert 1966, Asselin 1972)



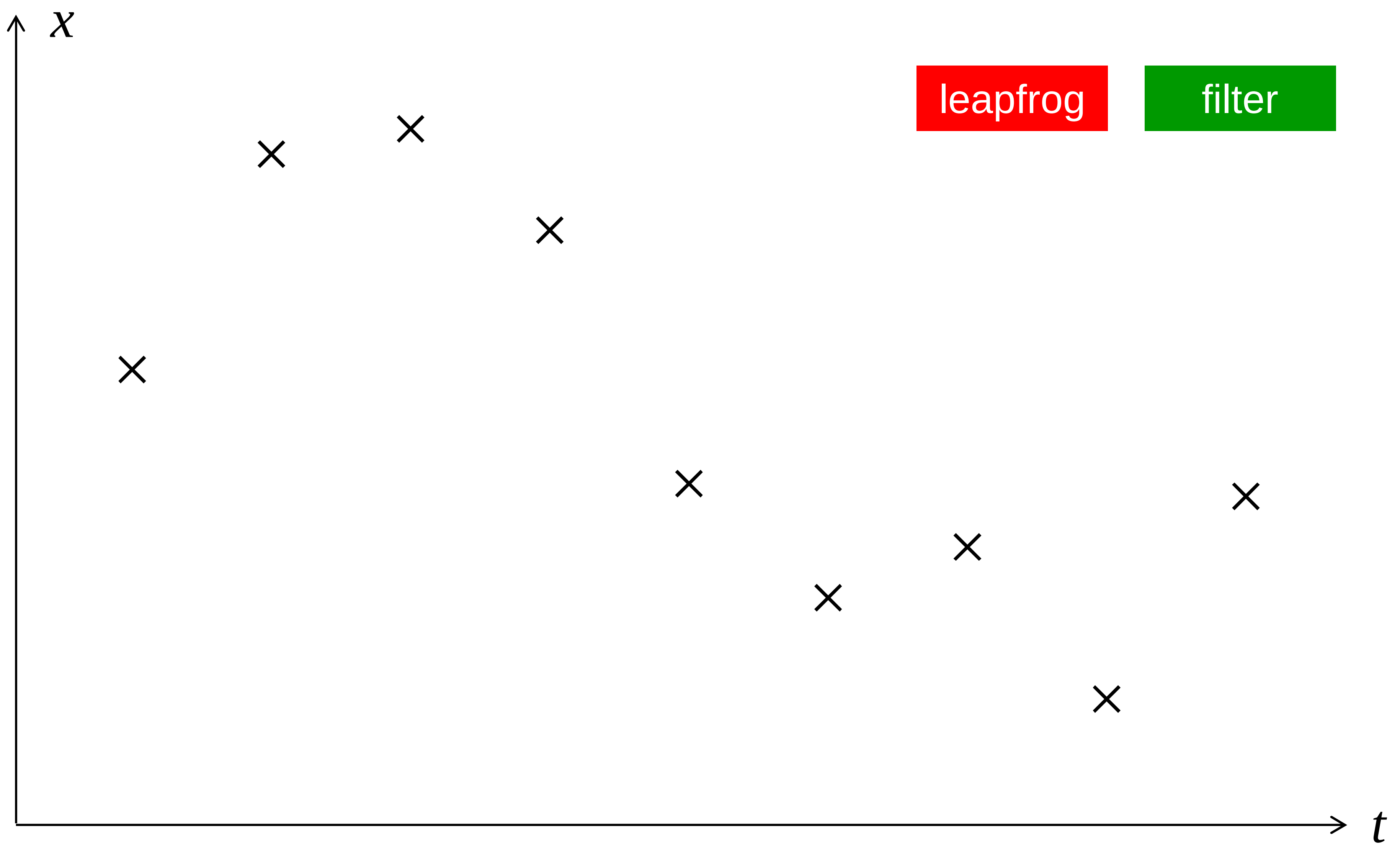
- use leapfrog to calculate x_{n+1}
- RA filter nudges x_n
- reduces curvature <u>but does not conserve mean</u>
- amplitude accuracy is 1st order

<u>LF+RAW</u>
(Williams 2009, 2011)



- use leapfrog to calculate x_{n+1}
- RAW filter nudges x_n <u>and</u> x_{n+1}
- reduces curvature <u>and conserves mean</u> (for α=½)
- amplitude accuracy is 3rd order

A proposed improvement



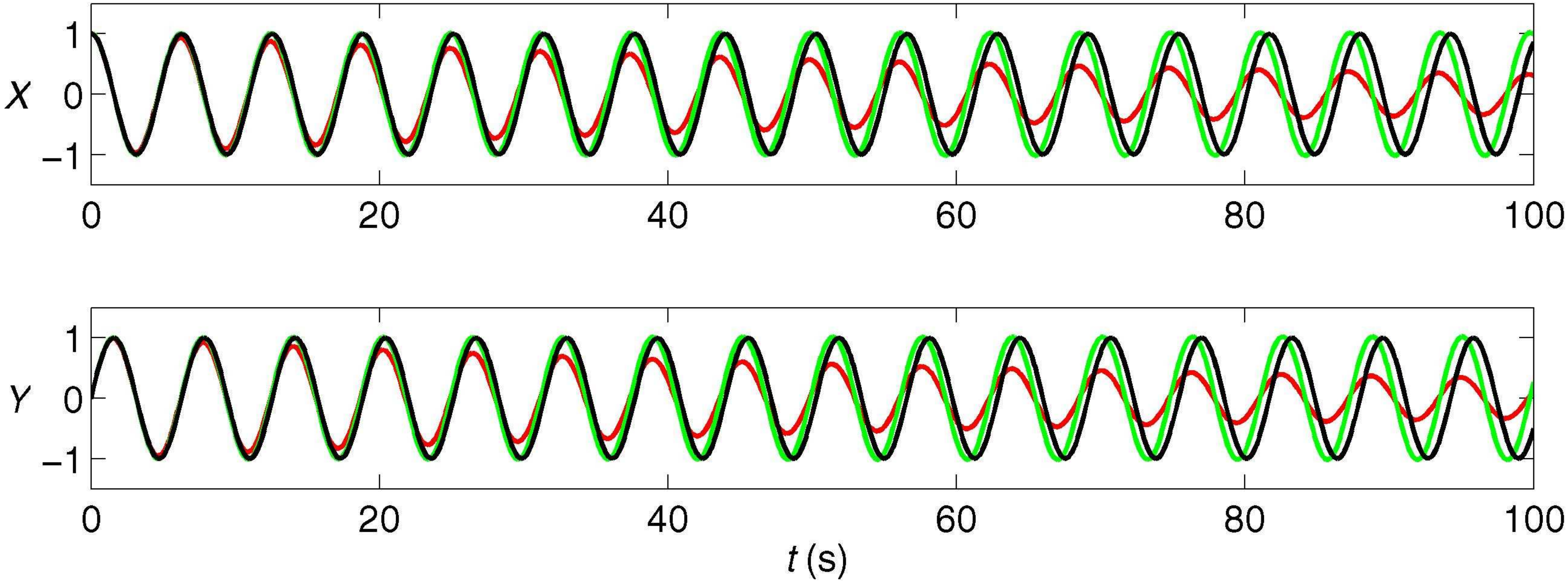
Simple test integration

$$\frac{\mathrm{d}X}{\mathrm{d}t} = -\omega Y$$
$$\frac{\mathrm{d}Y}{\mathrm{d}t} = +\omega X$$

$$\omega = 1 \text{ rad s}^{-1}$$

exact
LF+RA
LF+RAW$_{\alpha=1/2}$

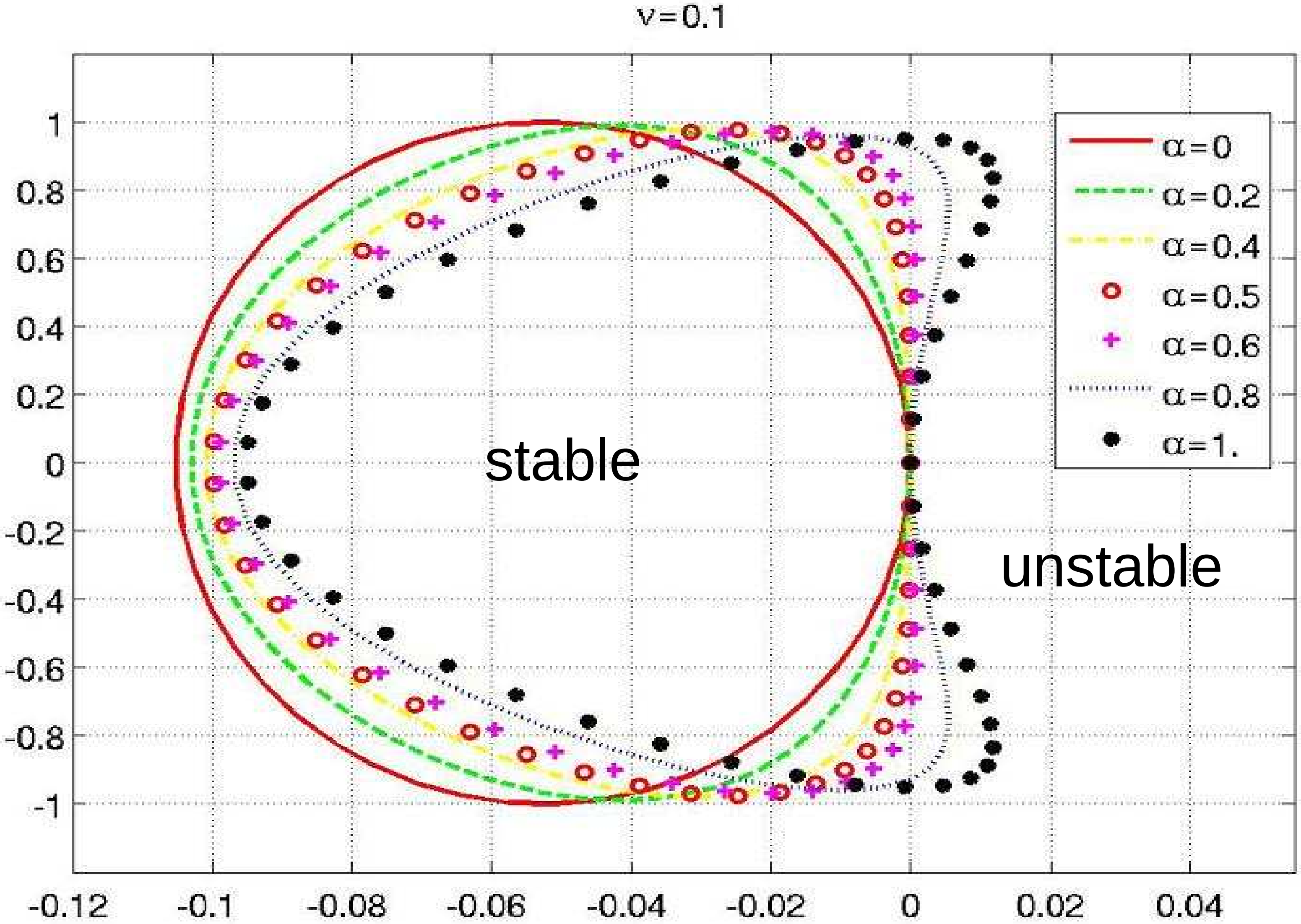
$$\left.\begin{matrix}\text{LF+RA} \\ \text{LF+RAW}_{\alpha=1/2}\end{matrix}\right\} \begin{matrix}\Delta t = 0.2 \text{ s} \\ \nu = 0.2\end{matrix}$$



(Williams 2009)

Analysis: numerical stability

$\dot{F} = \lambda F$



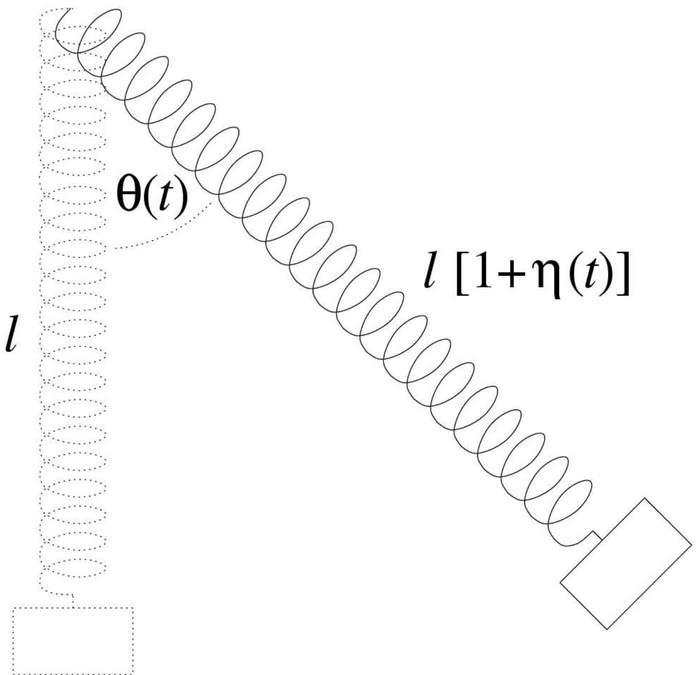
Im(λ)Δt

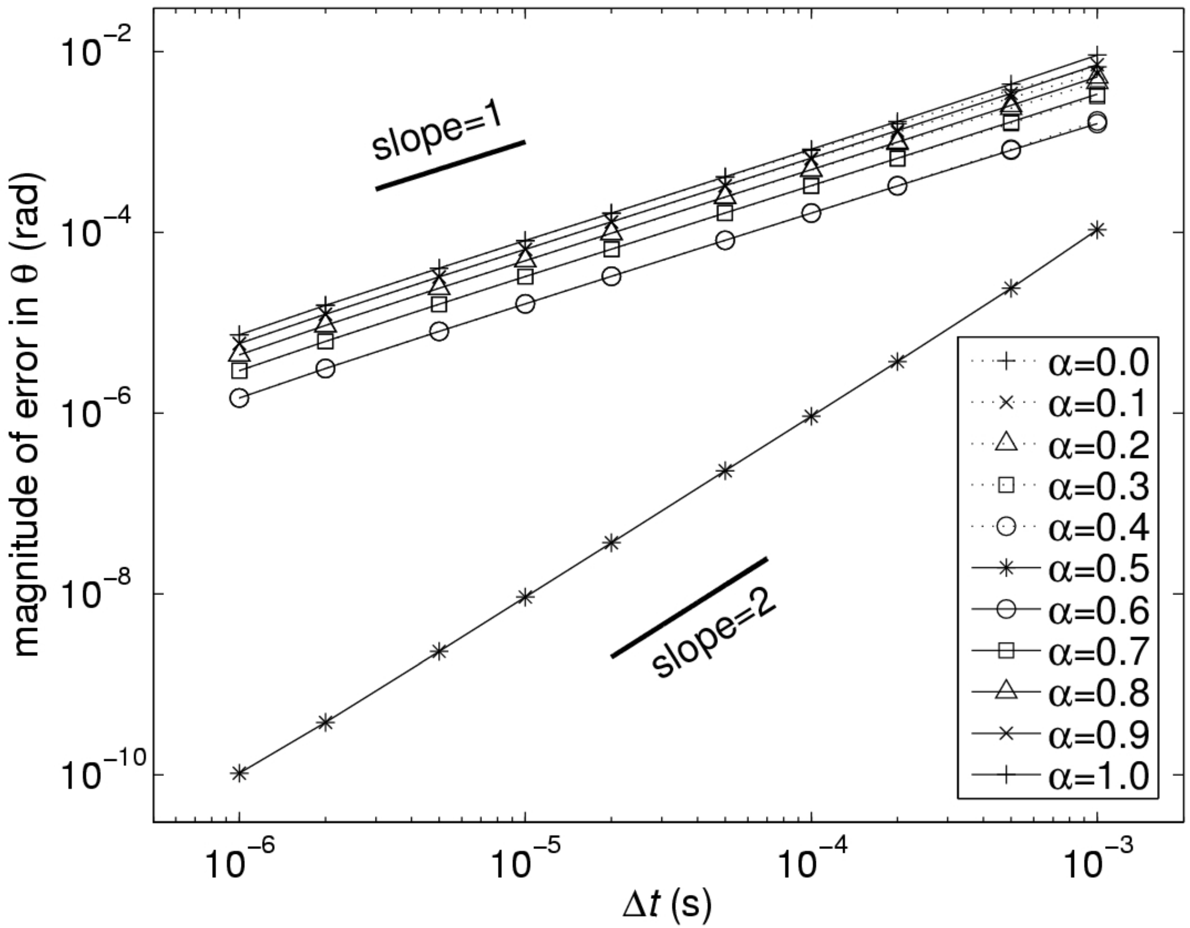
Re(λ)Δt

(Thanks to Yu-heng Tseng)

Analysis: numerical convergence

Semi-implicit integrations of the elastic pendulum (or "swinging spring")





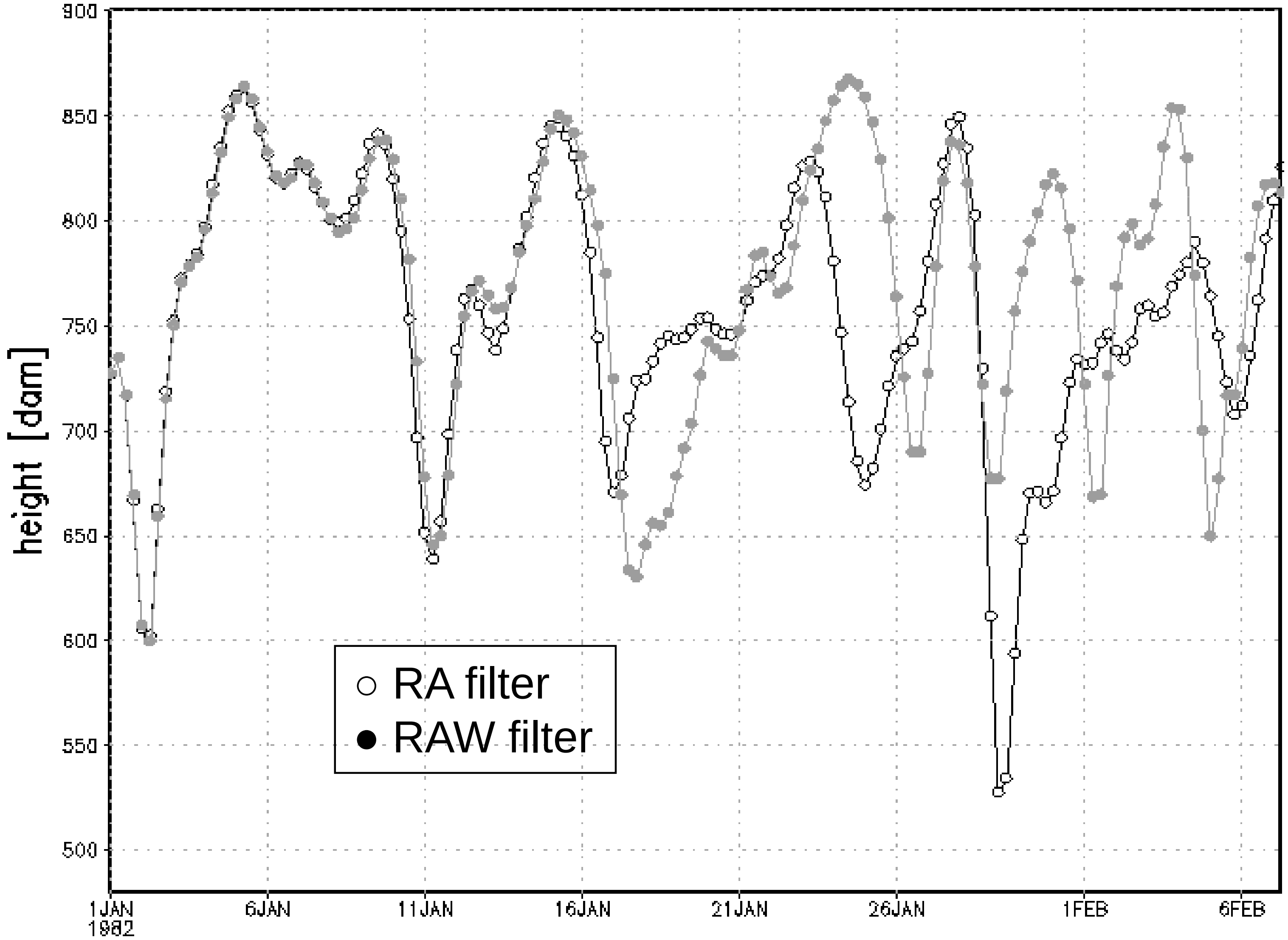
(Williams 2011)

Implementation in existing code

```
! Compute tendency at this time step
tendency = […]

! Leapfrog step
x_next = x_last + tendency*2*delta_t

! Compute filter displacement
d = nu*(x_last - 2*x_this + x_next)/2

! Apply filter
x_this = x_this + d
```

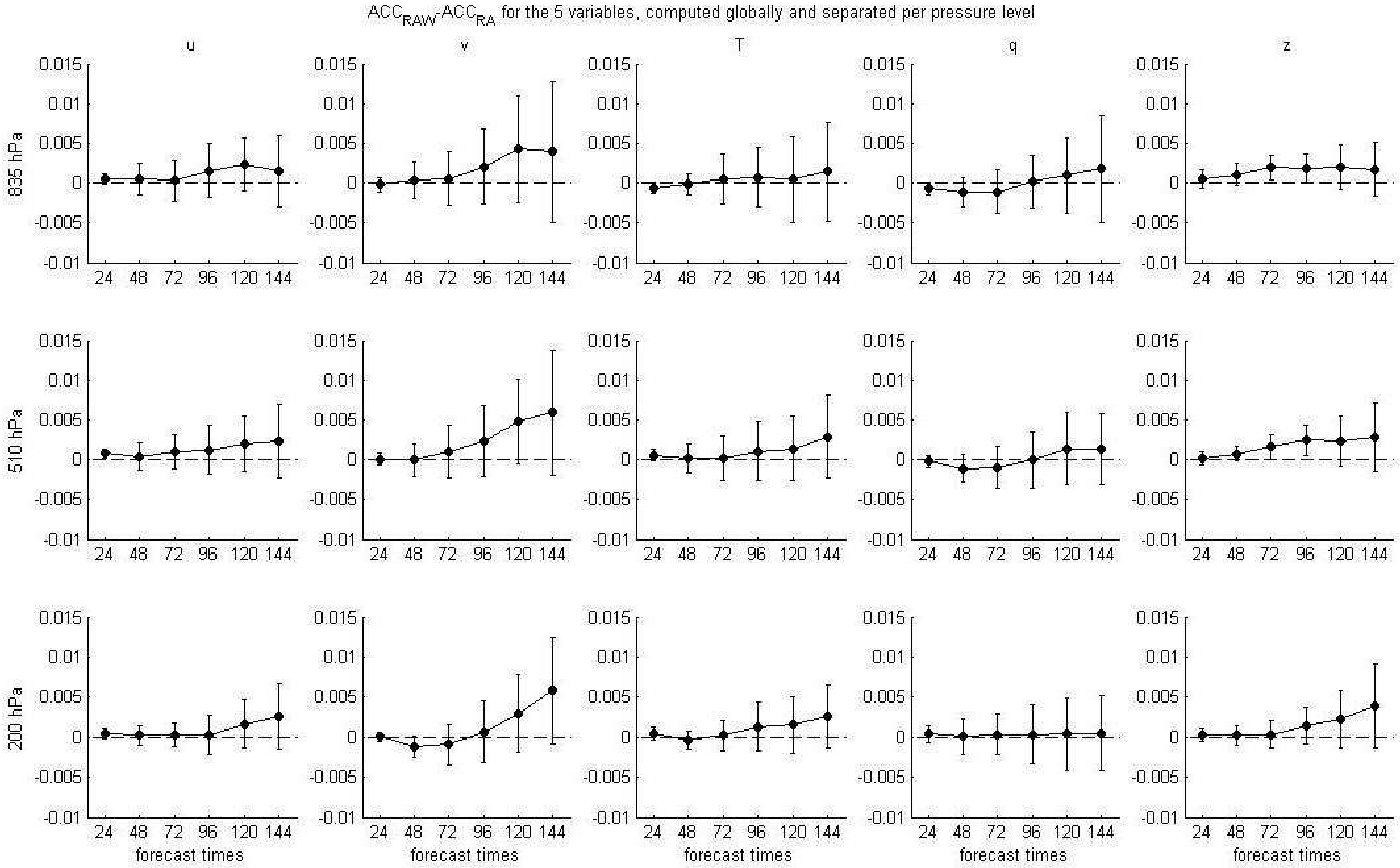
Implementation in SPEEDY

500 hPa geopotential height in Maryland



(Amezcua, Kalnay & Williams 2011)

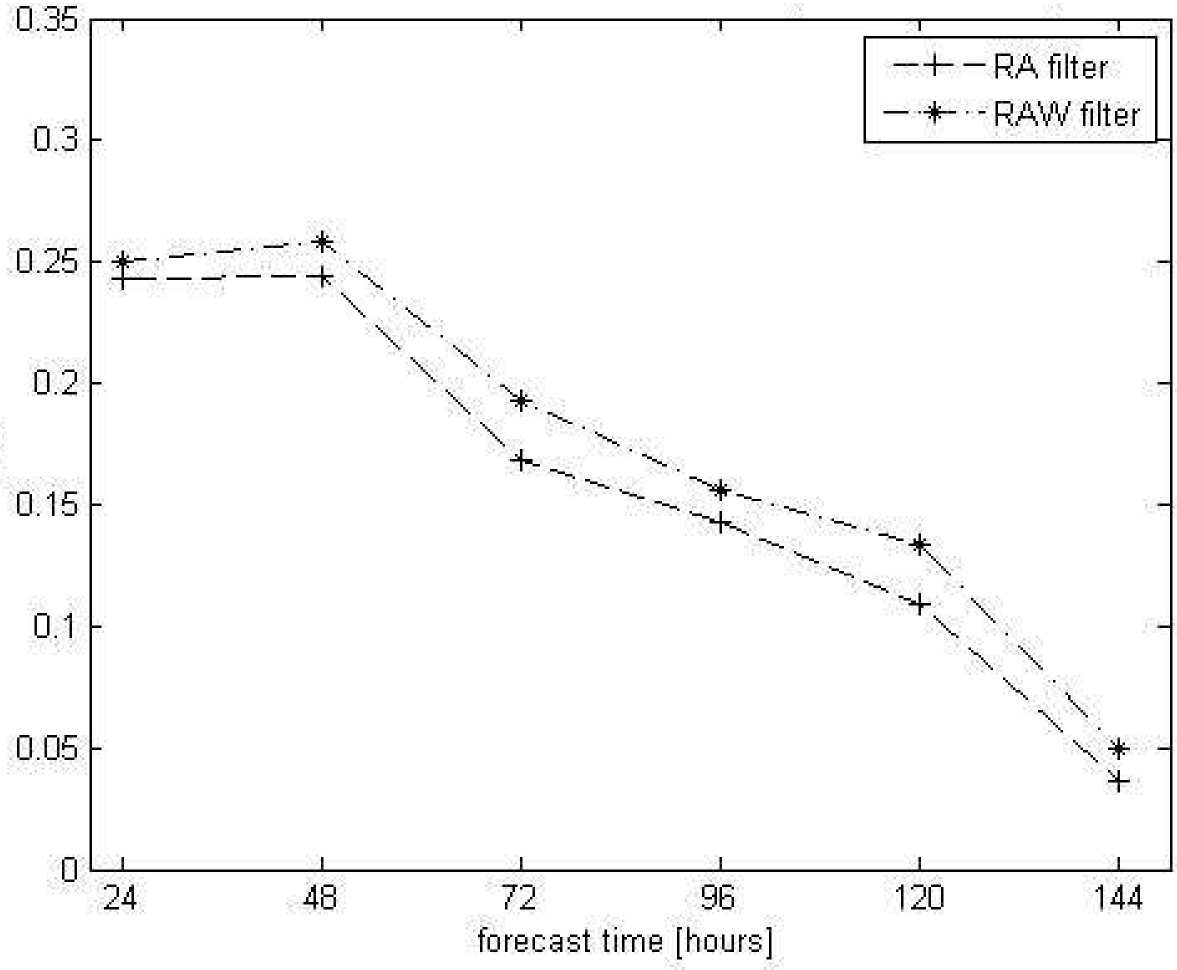
Implementation in SPEEDY



(Amezcua, Kalnay & Williams 2011)

Implementation in SPEEDY

ACC for surface pressure in the tropics (25°S-25°N)



5-day forecasts made using the RAW filter have approximately the same skill as 4-day forecasts made using the RA filter

(Amezcua, Kalnay & Williams 2011)

Summary

- Time stepping is an important contributor to model error
- The Robert-Asselin filter is widely used but is dissipative and reduces accuracy
- The RAW filter has approximately the same stability but much greater accuracy
- Implementation in an existing code is trivial and there is virtually no extra computational cost

The Sunday Times, 21 Nov 2010

Feature Young scientists

WELCOME TO THE BOFFIN BANK

The brain drain has finally been plugged. Young scientists like these are staying in Britain, fired up with breathtaking new ideas and inspired by a Nobel prize. By Amy Turner. Portraits: Loubie-Lou

The Sunday Times, 21 Nov 2010

Feature

'THE CLIMATE CHANGE DEBATE IS ESPECIALLY VOCIFEROUS. **I HAVE FOLDERS MARKED "HATE MAIL".** I KEEP A RECORD OF ALL THE CLIMATE SCEPTICS' THREATS'

PAUL WILLIAMS, 33, WEATHER AND CLIMATE, READING UNIVERSITY

My inbox and filing cabinet have folders marked "hate mail". I keep a record of all the climate sceptics' threats, partly to provide a list of suspects if I ever disappear, but mainly because it's funny to read how many times someone can call you a "caulkhead". It's a strange insult: it means someone for whom three generations of parents have lived on the Isle of Wight.

The climate-change debate is especially vociferous because everyone feels they know the weather. Climate scientists are the first to admit that the prediction models we use aren't perfect, which makes the sceptics jump up and down with delight.

My greatest idea for improving climate modelling came to me when I was walking along a beach in California. A model is millions of lines of computer code containing the laws of physics applied to the atmosphere, ocean and ice. In real life, as we know, time flows continuously, but in computer models, time has to be divided into discreet chunks. The model makes predictions by "leapfrogging" rhythmically from one chunk to the next, a process that's inherently unstable, but I've found a way to stabilise the leapfrogging which is being tested around the world.

Within 10 years, I think we'll see a model that predicts the weather and climate change exactly — it's the only way to resolve the debate.

You get a bit desensitised to your own gloomy predictions. It's true the oceans and the atmosphere are getting hotter. If we do nothing and it gets more than two degrees warmer than before the industrial revolution, ice will melt and we'll be in big trouble. We're basically conducting a massive experiment with our planet. But humanity is enormously impressive. We're quite capable of averting disaster. It's a question of whether we choose to.

What really bothers me is flying to conferences. I've been told you can't micromanage these things, so I fly all over the world for my career, despite the irony of it.

Further information

Williams PD (2009) A proposed modification to the Robert-Asselin time filter. *Monthly Weather Review* **137**(8), pp 2538-2546.

Williams PD (2011) The RAW filter: An improvement to the Robert-Asselin filter in semi-implicit integrations. *Monthly Weather Review*, in press.

Amezcua J, Kalnay E, and Williams PD (2011) The effects of the RAW filter on the climatology and forecast skill of the SPEEDY model. *Monthly Weather Review* **139(2**), pp 608-619.

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