Demonstration of a cut-cell representation of 3D orography for flow over very steep hills

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With thanks to J. Klemp & W. Skamarock

WRF Development Testbed Center Visitor Program

Talk outline – Microscale Model

Current model features

including outline of cut-cell orographic representation

Details of computations for cut-cells

Results

- 2D benchmark case
- 3D equivalent case
- 2D very steep hill

Preliminary work

re-formulated advection terms & some early results

Future work

NCAS Microscale Model

Motivation:

Computer advances are exposing limitations of traditional terrain-following approaches for inclusion of orography

Current features:

3D, nonhydrostatic, fully compressible, Cartesian, limited area model

Advection-form equations: u, v, w, π', θ'

Time-splitting integration method (Klemp & Wilhelmson, 1978)

Fully explicit (microscale: $\Delta x \sim \Delta z$), 2nd-order schemes

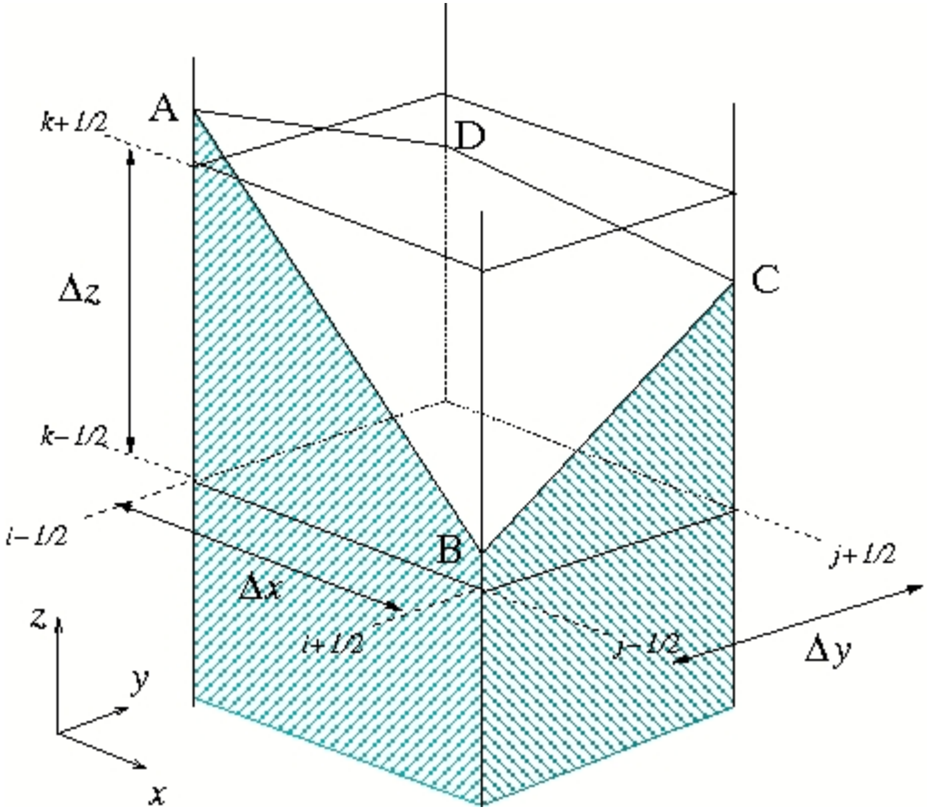
Fully parallelised (MPI)

Terrain-intersecting grid: cut-cell representation of orography

Finite-volume method for solving flows in cut-cells

NCAS Microscale Model

Cut-cell representation of orography (Steppeler et al., 2002):



Orographic surface:
Piecewise bilinear surfaces – continuous at grid-column boundaries

FV method:
Need fluxes across grid-box faces

Staggered grid:
u, v, w readily positioned for fluxes across faces that coincide with grid; zero flux across the orographic surface

Terrain-intersecting grid: cut-cell representation of orography

Finite-volume method for solving flows in cut-cells

Solving equations in the cut-cells:

Model equations:

$$\frac{\partial u}{\partial t} = -\mathbf{u}\cdot\nabla u - c_p\theta\frac{\partial \Pi'}{\partial x} + fv - Fw + D_u$$

$$\frac{\partial v}{\partial t} = -\mathbf{u}\cdot\nabla v - c_p\theta\frac{\partial \Pi'}{\partial y} - fu + D_v$$

$$\frac{\partial w}{\partial t} = -\mathbf{u}\cdot\nabla w - c_p\theta\frac{\partial \Pi'}{\partial z} + Fu + g\frac{\theta'}{\overline{\overline{\theta}}} + D_w$$

$$\frac{\partial \Pi'}{\partial t} = -\mathbf{u}\cdot\nabla \Pi' - \left(\frac{c_p}{c_v} - 1\right)\Pi\nabla\cdot\mathbf{u} + \frac{gw}{c_p\overline{\overline{\theta}}}$$

$$\frac{\partial \theta'}{\partial t} = -\mathbf{u}\cdot\nabla \theta' + w\frac{\partial \overline{\theta}}{\partial z} + D_\theta$$

Equations are in advection form for *u, v, w, π', θ'*

Following Steppeler et al. (2002), use an approximate FV method to compute:
1 – divergence term
2 – advection terms

What about the other terms?
3 – pressure gradient terms
4 – other terms

Solving equations in the cut-cells:

Model equations:

$$\frac{\partial u}{\partial t} = \boxed{-\mathbf{u}\cdot\nabla u} \boxed{-c_p\theta\frac{\partial \Pi'}{\partial x}} + fv - Fw + D_u$$

$$\frac{\partial v}{\partial t} = \boxed{-\mathbf{u}\cdot\nabla v} \boxed{-c_p\theta\frac{\partial \Pi'}{\partial y}} - fu + D_v$$

$$\frac{\partial w}{\partial t} = \boxed{-\mathbf{u}\cdot\nabla w} \boxed{-c_p\theta\frac{\partial \Pi'}{\partial z}} + Fu + g\frac{\theta'}{\overline{\theta}} + D_w$$

$$\frac{\partial \Pi'}{\partial t} = \boxed{-\mathbf{u}\cdot\nabla \Pi'} - \left(\frac{c_p}{c_v} - 1\right)\Pi\boxed{\nabla\cdot\mathbf{u}} + \frac{gw}{c_p\overline{\theta}}$$

$$\frac{\partial \theta'}{\partial t} = \boxed{-\mathbf{u}\cdot\nabla \theta'} + w\frac{\partial\overline{\theta}}{\partial z} + D_\theta$$

3

1

2

Equations are in advection form for *u, v, w, π', θ'*

Following Steppeler et al. (2002), use an approximate FV method to compute:

1 – divergence term

2 – advection terms

What about the other terms?

3 – pressure gradient terms

4 – other terms

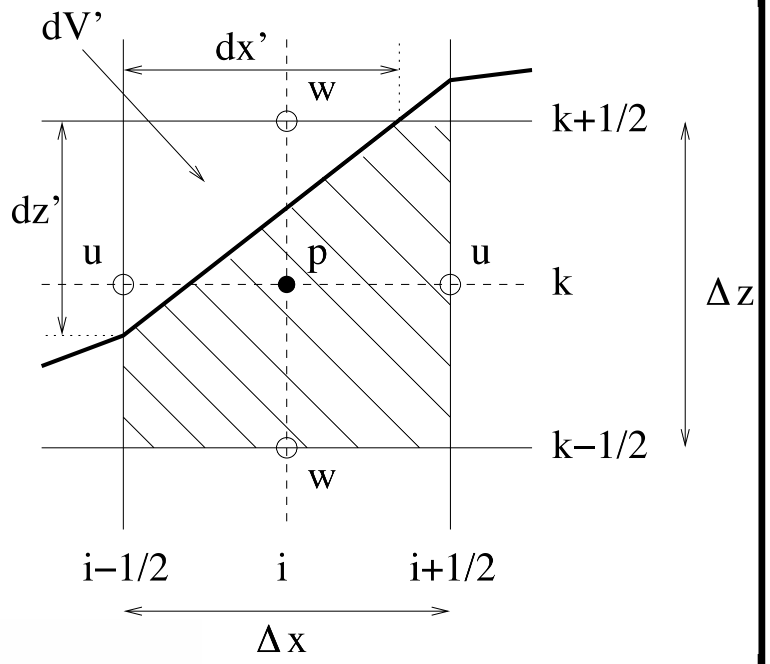
Solving equations in the cut-cells:

1 - Divergence term (pressure-point):

By Gauss's theorem:

$$\int_V \nabla \cdot \mathbf{F}\, dV = \oint_A \mathbf{F} \cdot \mathbf{d}A$$

express flux-divergence over volume *V* as a sum of fluxes across volume surface *A*.



In terms of Cartesian grid-cells:

$$\nabla \cdot \mathbf{u}_{i,k} = \frac{1}{dV'_{i,k}} \left(\{dz'u\}_{i+1/2,k} - \{dz'u\}_{i-1/2,k} + \{dx'w\}_{i,k+1/2} - \{dx'w\}_{i,k-1/2} \right)$$

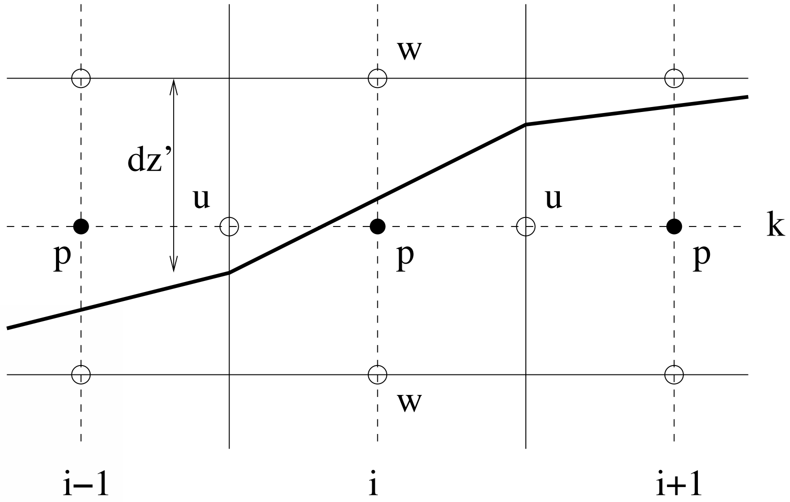
which reverts to FD computation for regular grid-cell,
i.e. for dx' = Δx, dz' = Δz, dV' = ΔxΔz

Solving equations in the cut-cells:

2 – Advection terms (pressure-point):

From Steppeler et al. (2002):

Advection terms are expressed in terms of flux-limiters (F) applied to the advective velocities, e.g. for a pressure point:



$$(\mathbf{u}\cdot\nabla\phi)_{p_{i,k}}\ :$$

$$u\frac{\partial\phi}{\partial x}_{p_{i,k}} = \frac{1}{2}\left\{(F_z u)_{i-1/2,k}\frac{(\phi_{i,k}-\phi_{i-1,k})}{\Delta x} + (F_z u)_{i+1/2,k}\frac{(\phi_{i+1,k}-\phi_{i,k})}{\Delta x}\right\}$$

$$\equiv \overline{F_z u \delta_x \phi}^{x}$$

where $F_z = dz' / \Delta z$

which reverts to FD computation for regular grid-cell, i.e. $F_z = 1$

Solving equations in the cut-cells:

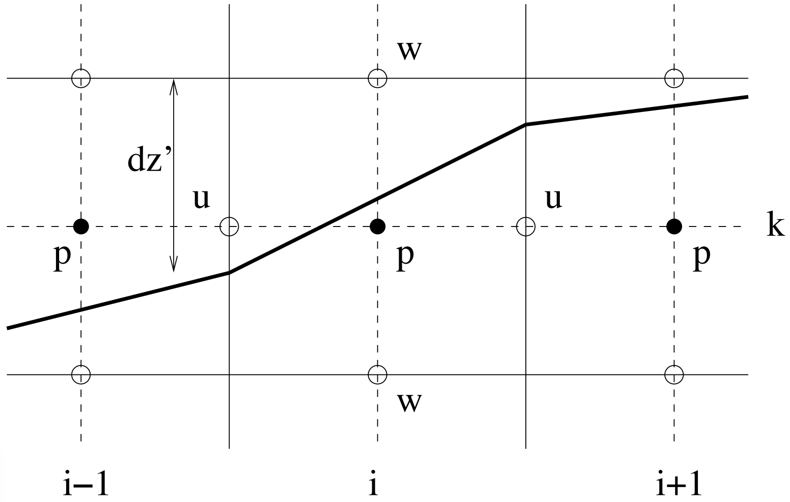
2b – Advection terms (e.g. u-point):

From Steppeler et al. (2002):

Advection terms are expressed in terms of flux-limiters (F) applied to the advective velocities.

For a u-point, requires averaging advective "fluxes":

$$(\mathbf{u}\cdot\nabla u)_{i-1/2,k} = \overline{\overline{F_z}^x\,\overline{u}^x\,\delta_x u}^x + \overline{\overline{F_x}^x\,\overline{w}^x\,\delta_z u}^z$$



where F_x = $dx' / \Delta x$,

F_z = $dz' / \Delta z$

which reverts to FD computation for regular grid-cell, i.e. F_x = F_z = 1

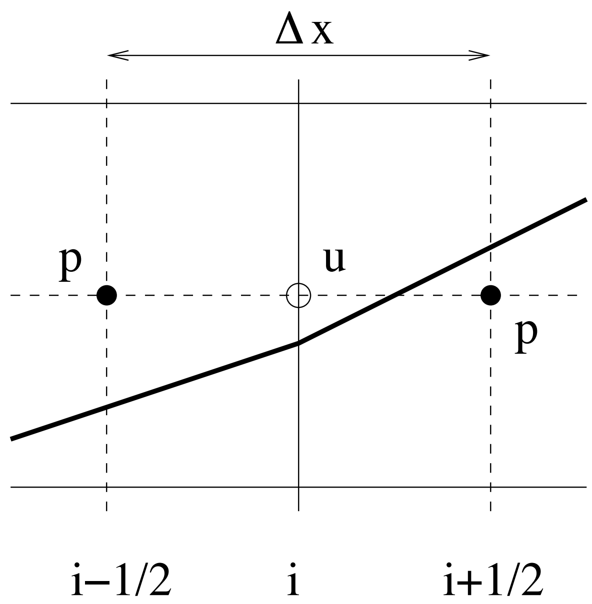
Solving equations in the cut-cells:

3 – Pressure-gradient terms:

Computations take no account of cut-cells

e.g. if a u solution exists on a cell-face, there must exist a pressure solution on either side

$$\frac{\partial p}{\partial x_i} = \frac{p_{i+1/2} - p_{i-1/2}}{\Delta x}$$



4 – other terms:

Same approach – no account is taken of cut-cells

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Comparison with benchmark case:

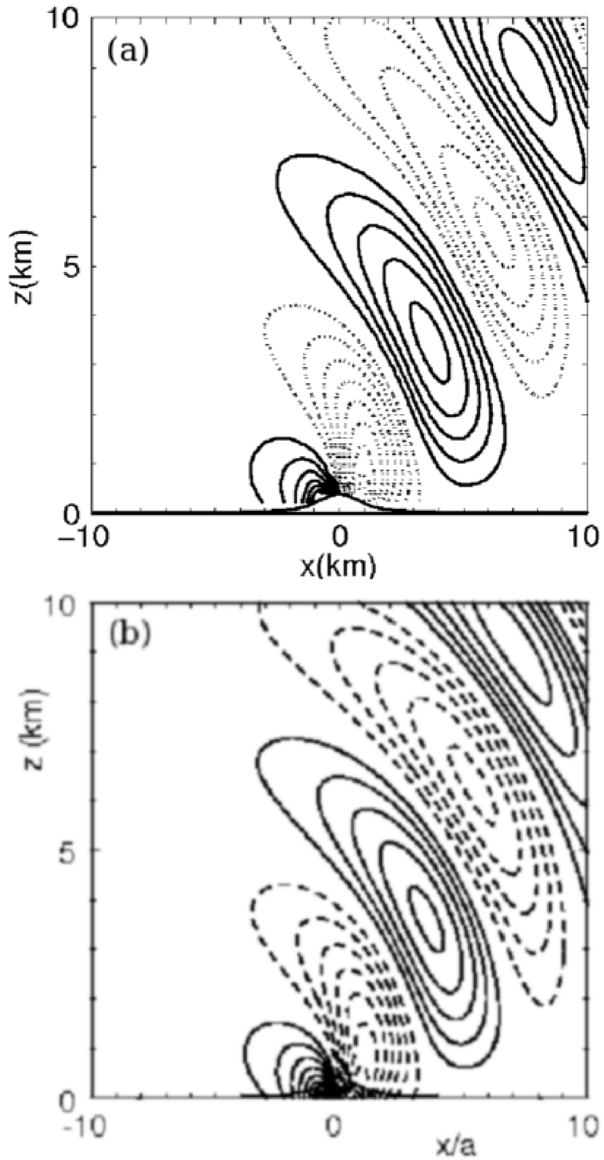
2D mountain waves:
Flow past bell-shaped hill:

$$h(x) = \frac{h_0}{(1 + x^2/a^2)}$$

Height h_0 = 400m, half-width a = 1 km.
Stratified background: $N = 0.01$ s^{-1}
Background wind: $U_0 = 10$ m s^{-1}

Model:
$\Delta x = 0.2a = 200$ m; $L_x = 300\ \Delta x$
$\Delta z = 200$ m; $L_z = 100\ \Delta z$
Absorbing layers in outer 50 grid-spacings

Results: *w* field
(a) model results: IT=50,000
(b) analytic solution (Gallus & Klemp 2000)



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3D bell-shaped hill:

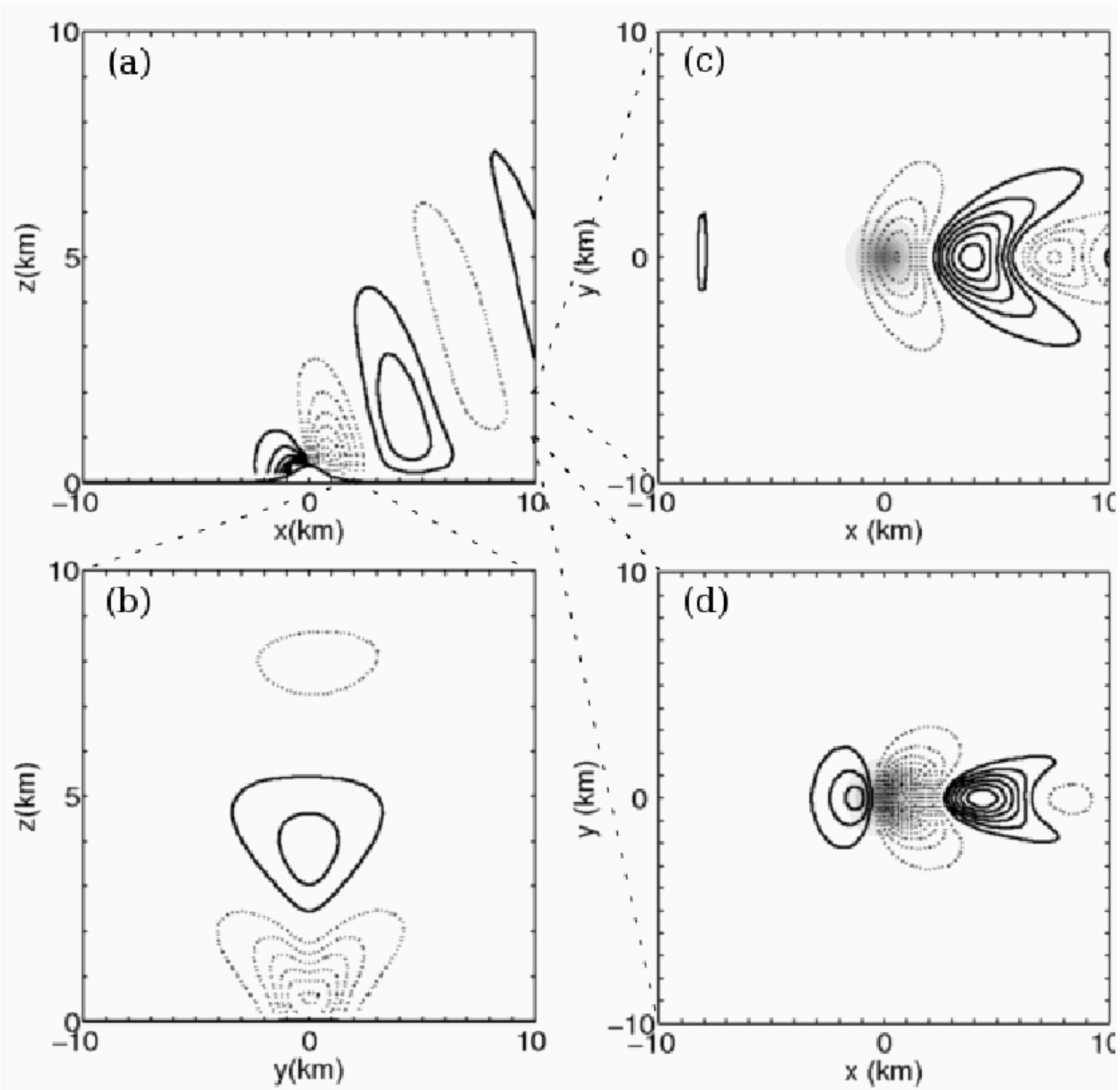
Equivalent set-up:
Flow past bell-shaped hill:

$$h(x,y) = \frac{h_0}{[(x^2+y^2)/a^2+1]^{3/2}}$$

h_0 = 400m, a = 1 km
N = 0.01 s^{-1}; U_0 = 10 m s^{-1}

Model:
Δx = 200 m; L_x = 300 Δx
Δy = 200 m; L_y = 200 Δy
Δz = 200 m; L_z = 100 Δz
Absorbing: outer 50Δ

Results: *w* field, IT=50,000
See "U-shaped" decaying oscillation (Smith, 1980)



Demonstration for very steep hill:

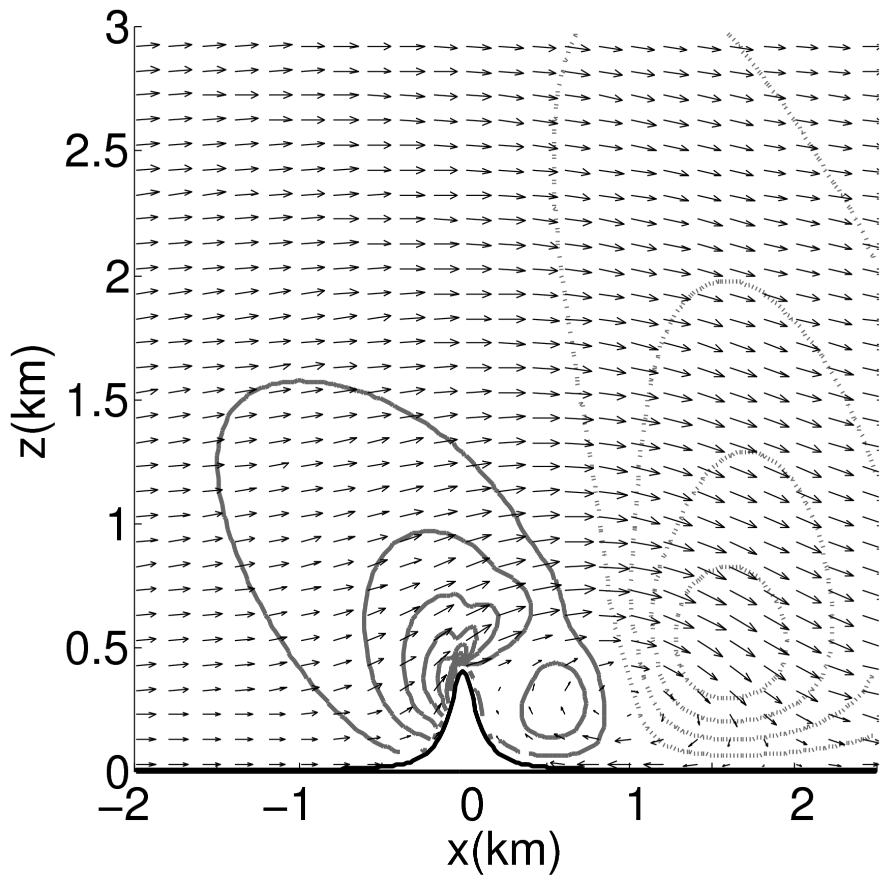
2D mountain waves:
Flow past bell-shaped hill:

$$h(x) = \frac{h_0}{(1 + x^2/a^2)}$$

h_0 = 400m, a = 100 m
N = 0.01 s^{-1}; U_0 = 10 m s^{-1}
Hill slopes:
aspect ratio, h_0/a = 4.0
max. gradient = 69°

Model:
Δx = 0.2a = 20 m; L_x = 400 Δx
Δz = 50 m; L_z = 300 Δz

Results (IT = 80,000):
Contoured w field with wind vectors



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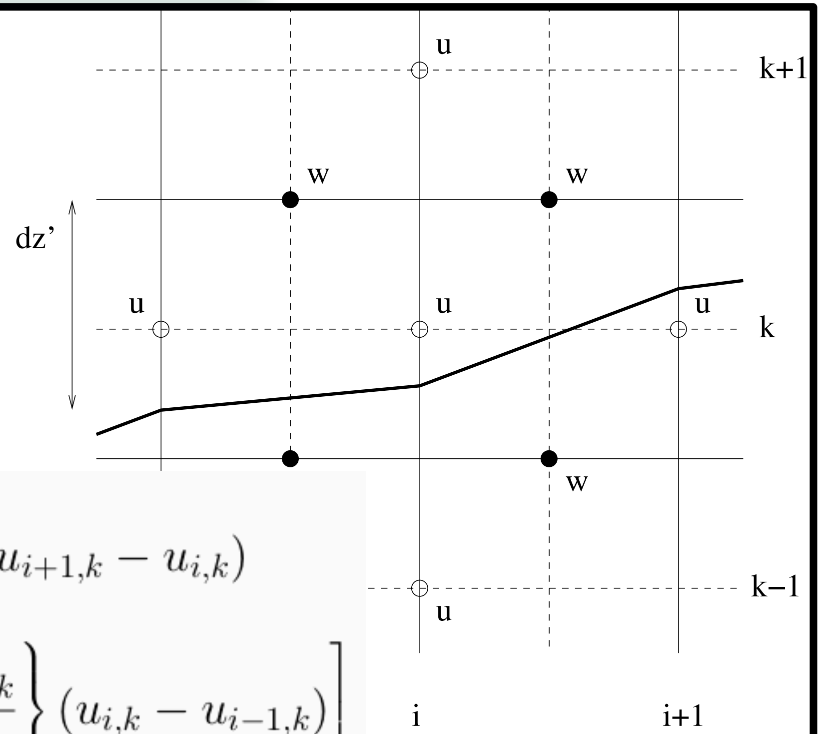
Re-formulating the cut-cell advection terms:

Advection terms (e.g. u-point):

Following recent NCAR visit (J. Klemp & W. Skamarock):



Considering re-formulation of advection terms, such that:

$$\mathbf{u}\cdot\nabla\phi = \nabla\cdot(\mathbf{u}\,\phi) - \phi\nabla\cdot\mathbf{u}$$

e.g. on a u-point:

$$\left(u\frac{\partial u}{\partial x}\right)_{i,k} = \frac{1}{2}\left[\frac{1}{2}\left\{\frac{(dz'u)_{i+1,k}}{dV'_{i+1/2,k}} + \frac{(dz'u)_{i,k}}{dV'_{i-1/2,k}}\right\}(u_{i+1,k} - u_{i,k}) + \frac{1}{2}\left\{\frac{(dz'u)_{i,k}}{dV'_{i+1/2,k}} + \frac{(dz'u)_{i-1,k}}{dV'_{i-1/2,k}}\right\}(u_{i,k} - u_{i-1,k})\right]$$

$$\equiv \overline{\frac{\overline{dz'u}^{x}}{dV'}\Delta_x u}^{x}$$

where $\Delta_x u = u_{i+1/2} - u_{i-1/2}$

Compared to earlier formulation: $\overline{\overline{F_z}^{x}\,\overline{u}^{x}\,\delta_x u}^{x}$

Comparison with earlier results:

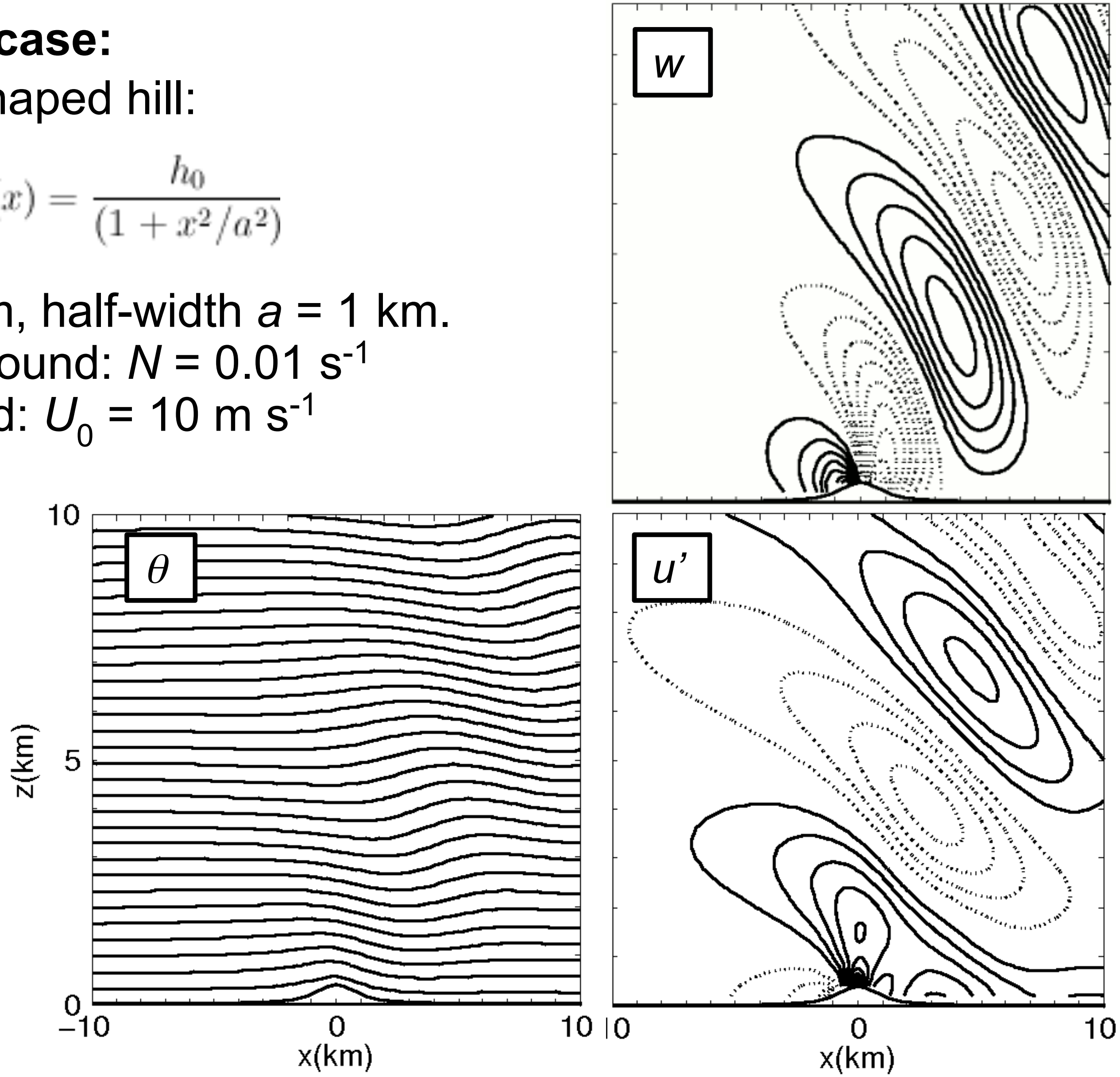
2D benchmark case:
Flow past bell-shaped hill:

$$h(x) = \frac{h_0}{(1 + x^2/a^2)}$$

Height h_0 = 400m, half-width a = 1 km.
Stratified background: $N = 0.01\ s^{-1}$
Background wind: $U_0 = 10\ m\ s^{-1}$

Result:

Only small difference from earlier results – most apparent in *u* field at lower boundary



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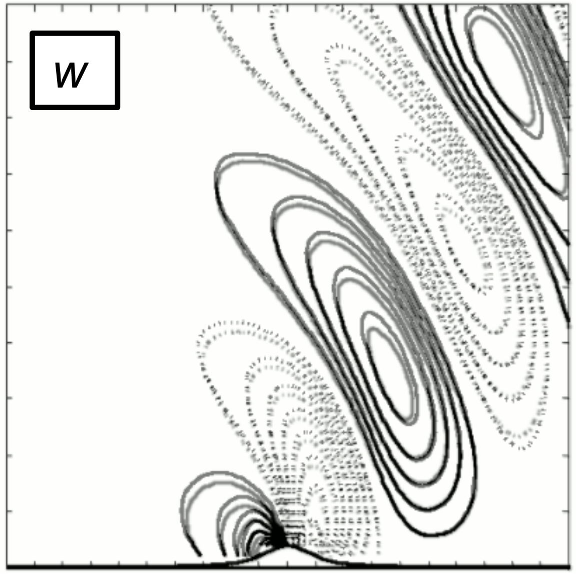

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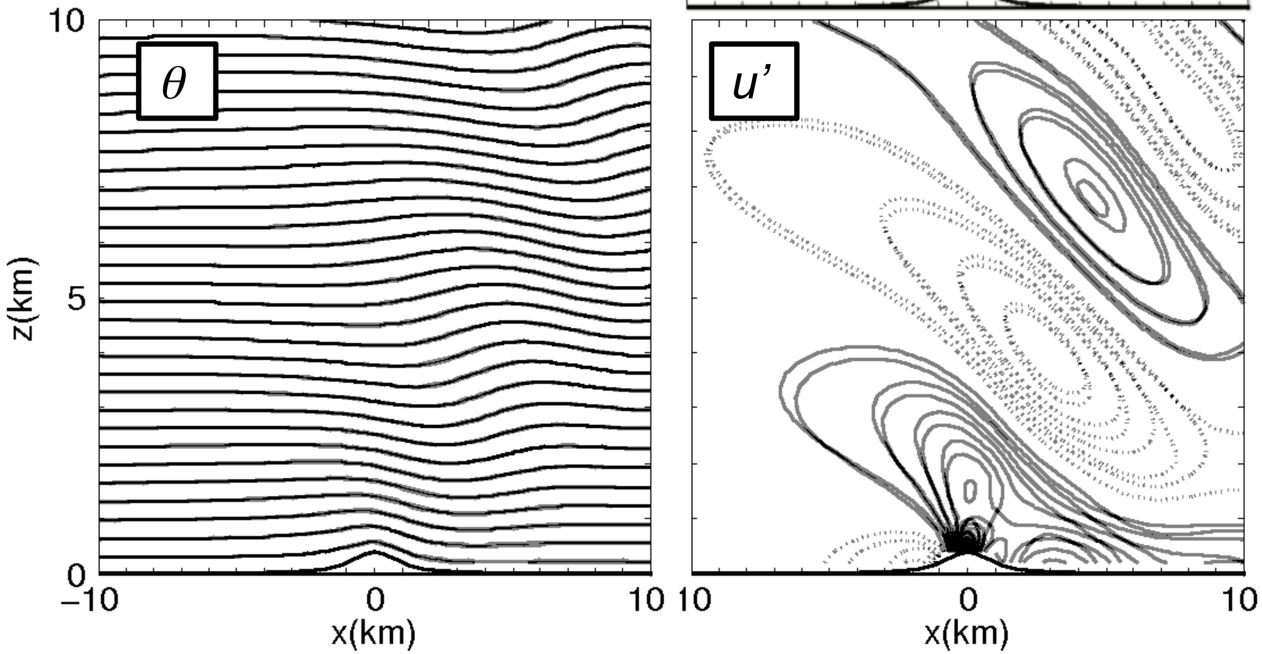

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New advection formulation – other results:

2D hydrostatic case:

Flow past hill:

$$h(x) = \frac{h_0}{(1 + x^2/a^2)}$$

Height h_0 = 400m,
half-width a = 10 km;
Background:
N = 0.01 s^{-1},
U_0 = 10 m s^{-1}

Model set-up:

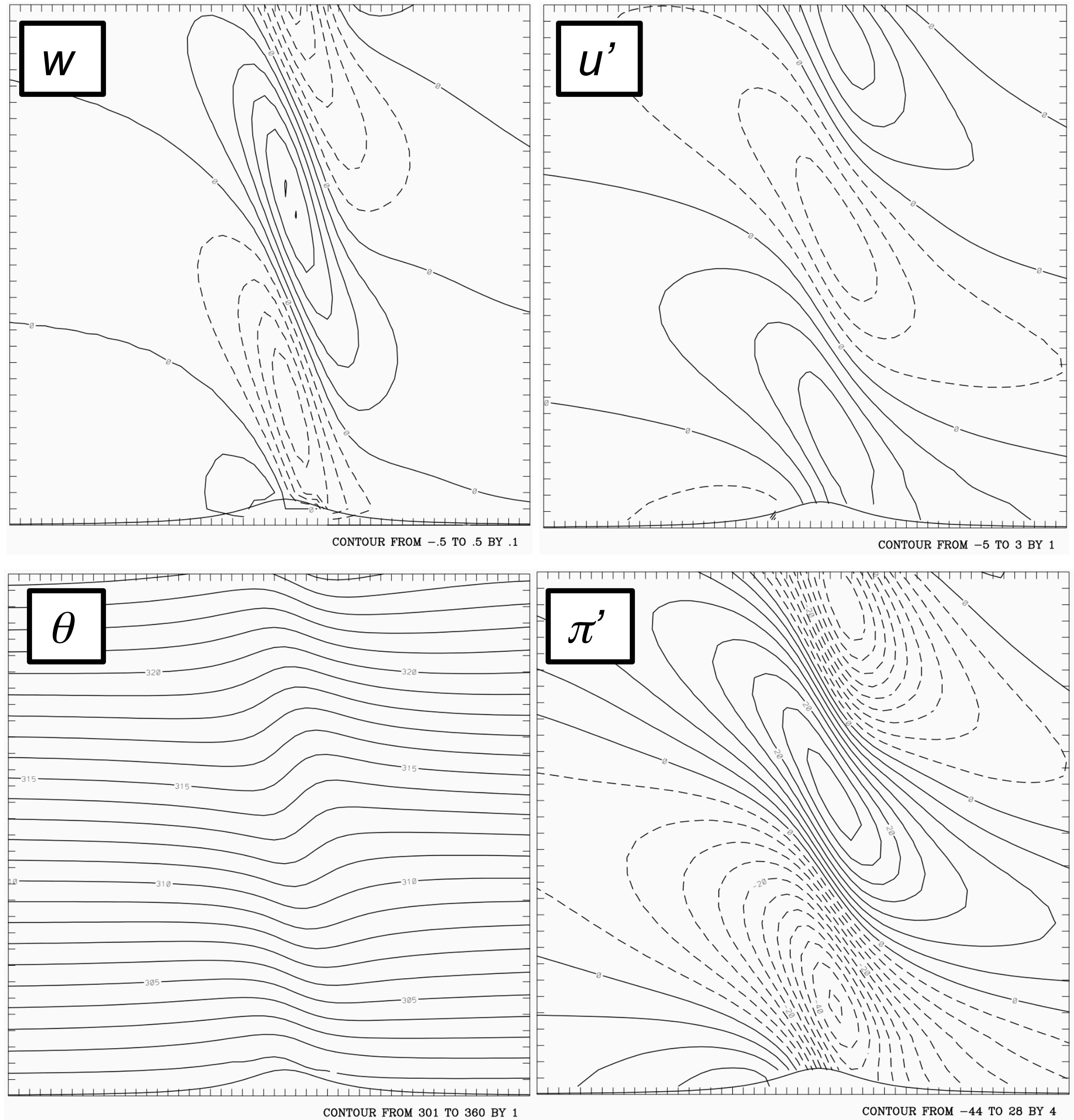
Minimal filtering, i.e.:

$\mu_2 \nabla^2 \phi$ ("del2")

diffusion term;
for very small μ_2

NB: Earlier results:

$\mu_4 \nabla^4 \phi$ ("del4")



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New advection formulation – other results:

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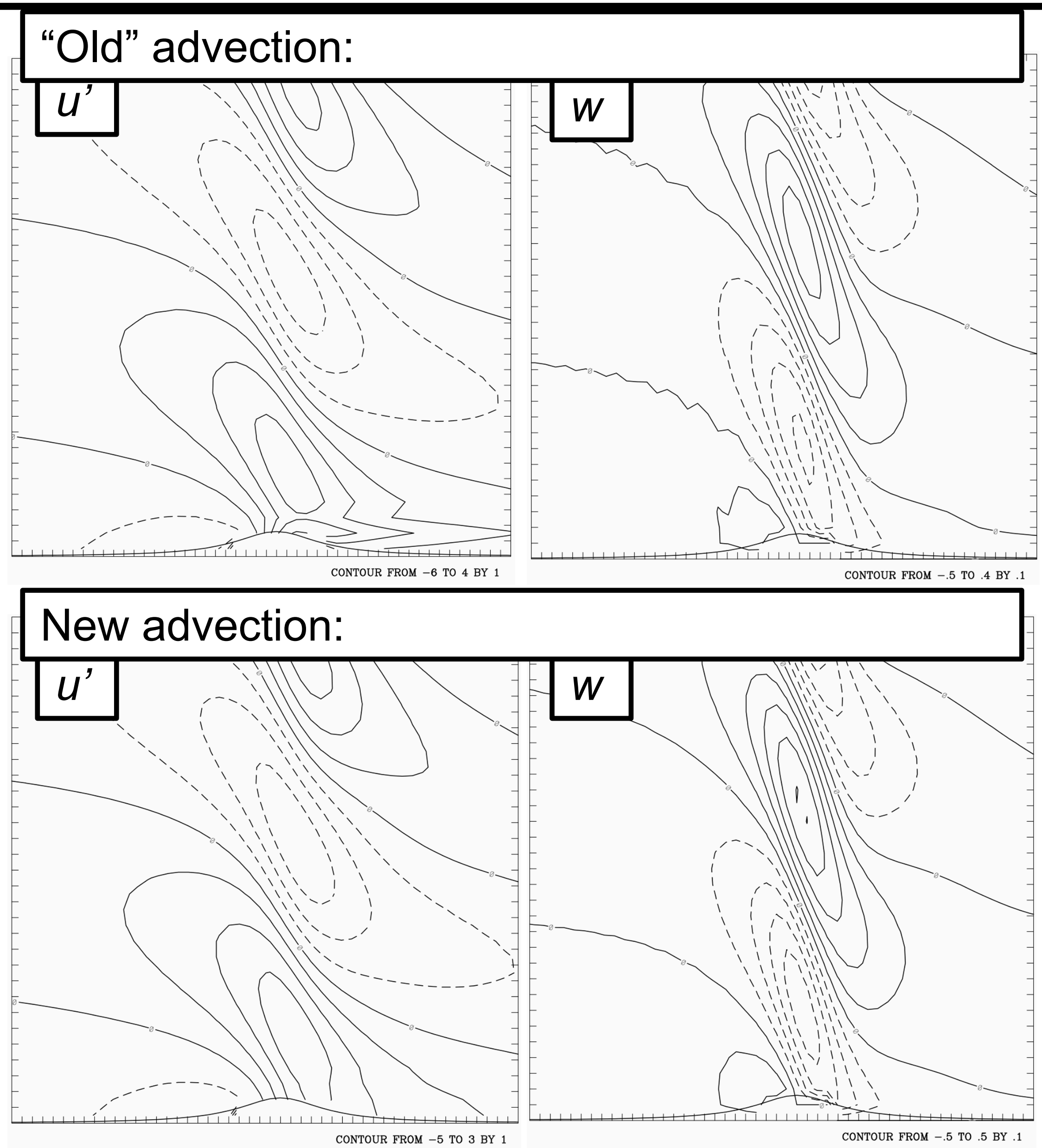
Height h_0 = 400m,
half-width a = 10 km;
Background:
N = 0.01 s^{-1},
U_0 = 10 m s^{-1}

Comparison:

“old” and new advection formulations (del2 diffusion)



New advection formulation – other results:

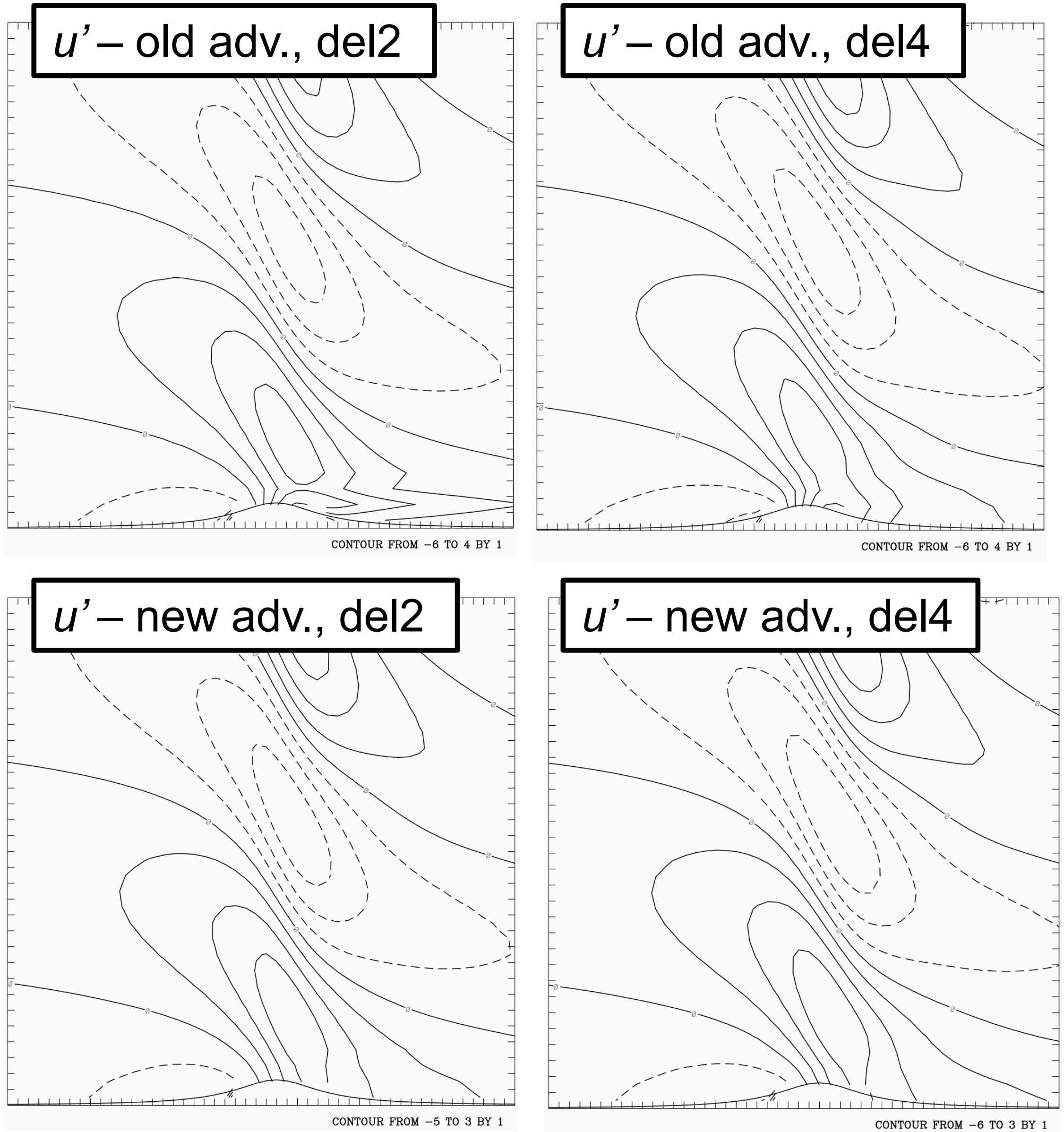
2D hydrostatic case:
Flow past hill:

$$h(x) = \frac{h_0}{(1 + x^2/a^2)}$$

Height h_0 = 400m,
half-width a = 10 km;
Background:
N = 0.01 s^{-1},
U_0 = 10 m s^{-1}

Comparison:

"old" and new advection formulations combined with del2 and del4 diffusion operators



Summary:

- Cut-cells implemented in 3D Microscale Model
- Results suggest good performance for medium steep hills & good potential for very steep hills
- Preliminary results from re-formulation of advection terms for cut-cells suggests improvement near lower boundary, particularly in horizontal winds (particularly evident with minimal numerical filtering)

Next steps:

- Further assessment of results for known cases & finding the "breaking point"
- Consider whether other terms (pressure-gradient / diffusion) could be handled better
- More consideration of new advection formulation & results

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