SINGULAR VECTORS IN A MOIST

LIMITED–AREA MODEL

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Contents

1 Introduction and Questions

MOIST SVs

- What are the relative effects of initial perturbations (or errors) in the **dynamical** fields versus the **moisture** field?
- Are similar structures optimal for affecting **both** perturbation energy and precipitation?
- Is **convective** or **non–convective** precipitation generally more sensitive to initial perturbations?

ENSEMBLE PREDICTION AND SV SPECTRA

- obvious interest regarding data assimilation and predictability
- *importance of SVs for covariance and ensemble prediction*
- Hessian SVs and Spectra

2 Singular Vectors (SVs) and Norms

The problem:

Given a linearized model

$$\mathbf{y} = \mathbf{M}\mathbf{x}$$

Find the $\mathbf{x}$ that maximizes

$$L_2 = \mathbf{y}^{\mathrm{T}}\mathbf{L}_2\mathbf{y}$$

Given

$$L_1 = \mathbf{x}^{\mathrm{T}}\mathbf{L}_1\mathbf{x}$$

The solution:

$$\mathbf{x} = \mathbf{L}_1^{-\frac{1}{2}}\mathbf{z}$$

where

$$\mathbf{L}_1^{-\frac{1}{2}\mathrm{T}}\mathbf{M}^{\mathrm{T}}\mathbf{L}_2\mathbf{M}\mathbf{L}_1^{-\frac{1}{2}}\mathbf{z} = \lambda\mathbf{z}$$

Note: *Different* norms at initial and final times.

Norms Considered

Energy Norm:

$$E = \frac{1}{N_w} \sum_{i,j,k} \Delta\sigma_k \left(u'^2_{i,j,k} + v'^2_{i,j,k}\right) + \frac{C_p}{T_r N_t} \sum_{i,j,k} \Delta\sigma_k T'^2_{i,j,k} + \frac{R T_r}{p_{sr}^2 N_t} \sum_{i,j} p'^2_{s\ i,j} \tag{1}$$

Moist Energy Norm (dry fields zero):

$$E_m = \frac{L^2}{C_p T_r N_t} \sum_{i,j,k} \Delta\sigma_k q'^2_{i,j,k} \tag{2}$$

Precipitation Rate Norm (used only as end–time norm; non–convective + convective precipitation):

$$P = \frac{1}{N_t} \sum_{i,j} R'^2_{t\ i,j} \tag{3}$$

Dry and Moist Variance–weighted norm (penalize large q high up):

$$V_d = \frac{1}{N_w} \sum_{i,j,k} \Delta\sigma_k \left(\frac{u'^2_{i,j,k}}{V_{u\ k}} + \frac{v'^2_{i,j,k}}{V_{v\ k}} \right) + \frac{1}{N_t} \sum_{i,j,k} \Delta\sigma_k \frac{T'^2_{i,j,k}}{V_{T\ k}} + \frac{1}{N_t} \sum_{i,j} \frac{p'^2_{s\ i,j}}{V_p} \tag{4}$$

$$V_m = \frac{1}{N_t} \sum_{i,j,k} \frac{q'^2_{i,j,k}}{V_{q\ k}} \Delta\sigma_k \tag{5}$$

3 The Mesoscale Adjoint Modeling System MAMS2

The Mesoscale Adjoint Modeling System Version 2 (MAMS2)

- Primitive equations with water vapor
- Bulk PBL formulation (Deardorff)
- Stability–dependent vertical eddy diffusion (CCM3)
- RAS scheme (Moorthi and Suarez)
- Stable–layer precipitation
- $\Delta x = 80$km
- 20–level model ($\Delta\sigma = 0.05$)
- $p_{\rm top} = 100$ or 10 mb
- 12–hour forecasts for 4 synoptic cases

6 sets of SVs for each case

- $E \rightarrow E$
- $E_m \rightarrow E$
- $V_d \rightarrow E$
- $V_m \rightarrow E$
- $V_d \rightarrow P$
- $V_m \rightarrow P$

A larger value of E can be produced with an initial constraint $V_m = 1$ compared with $V_d = 1$.

4 Moist SVs: Results and Relevance

- Cases
- Amplifications
- Horizontal properties of SVs
- Vertical structures of SVs
- A peculiar SV
- Final–time SVs and growth mechanisms
- Nonlinear considerations and correlations

5 SVs: Covariance Prediction and Spectra

Uncertainty Prediction: Hessian SVs

The HSVs Z_0 solving the eigenvector problem:

$$\boxed{\mathsf{M}^{\mathrm{T}}\mathsf{C}^{\mathrm{T}}\mathsf{C}\mathsf{M}\mathsf{Z}_0 = (\mathsf{P}^{\mathsf{a}})^{-1}\mathsf{Z}_0\Lambda \qquad \text{s.t.} \quad \mathsf{Z}_0^{\mathrm{T}}(\mathsf{P}^{\mathsf{a}})^{-1}\mathsf{Z}_0 = \mathsf{I}} \tag{5.0.1}$$

are, when time–evolved, eigenvectors of P^{f}, because:

$$\Big(\mathsf{C}\underbrace{\mathsf{M}\mathsf{P}^{\mathsf{a}}\Big)\mathsf{M}^{\mathrm{T}}}_{\equiv \mathsf{P}^{\mathsf{f}}}\mathsf{C}^{\mathrm{T}}\underbrace{\mathsf{C}\mathsf{M}\mathsf{Z}_0}_{\equiv \mathsf{Z}_{\mathsf{t}}} = \Big(\mathsf{C}\mathsf{M}\mathsf{P}^{\mathsf{a}}\Big)(\mathsf{P}^{\mathsf{a}})^{-1}\mathsf{Z}_0\Lambda \qquad \longrightarrow \qquad \boxed{\Big(\mathsf{C}\mathsf{P}^{\mathsf{f}}\mathsf{C}^{\mathrm{T}}\Big)\mathsf{Z}_{\mathsf{t}} = \mathsf{Z}_{\mathsf{t}}\Lambda} \tag{5.0.2}$$

The evolved HSVs Z_{t} *are the eigenvectors of* $\mathsf{C}\mathsf{P}^{\mathsf{f}}\mathsf{C}^{\mathrm{T}}$ *– which is the forecast error covariance in the "final–time norm"* C. Note the final–time orthogonality relationship:

$$\mathsf{Z}_{\mathsf{t}}^{\mathrm{T}}\mathsf{Z}_{\mathsf{t}} = \Big(\mathsf{C}\mathsf{M}\mathsf{Z}_0\Big)^{\mathrm{T}}\Big(\mathsf{C}\mathsf{M}\mathsf{Z}_0\Big) = \mathsf{Z}_0^{\mathrm{T}}\underbrace{\mathsf{M}^{\mathrm{T}}\mathsf{C}^{\mathrm{T}}\mathsf{C}\mathsf{M}\mathsf{Z}_0}_{} = \overbrace{\mathsf{Z}_0^{\mathrm{T}}(\mathsf{P}^{\mathsf{a}})^{-1}\mathsf{Z}_0}^{}\Lambda = \Lambda \qquad \longrightarrow \qquad \boxed{\mathsf{Z}_{\mathsf{t}}^{\mathrm{T}}\mathsf{Z}_{\mathsf{t}} = \Lambda} \tag{5.0.3}$$

Uncertainty Prediction: The SV–Decomposition of P^a

- Because the initial–time SVs satisfy (5.0.1), it is true that P^a can be written as:

$$\mathsf{P}^\mathsf{a} = \mathsf{Z}_0 \mathsf{Z}_0^{\mathrm{T}} \tag{5.0.4}$$

  This is a special square–root for P^a (different from eigendecomposition and also not lower–triangular) $\rightarrow$ the **SV–decomposition of** P^a

- Under linear dynamics this SV–decomposition becomes the eigendecomposition of the forecast error covariance matrix, because (5.0.5) is the same as (5.0.2) together with (5.0.3):

$$\Big(\mathsf{CM}\Big)\mathsf{P}^\mathsf{a}\Big(\mathsf{CM}\Big)^{\mathrm{T}} = \Big(\mathsf{CM}\Big)\mathsf{Z}_0\mathsf{Z}_0^{\mathrm{T}}\Big(\mathsf{CM}\Big)^{\mathrm{T}} \quad \rightarrow \quad \boxed{\mathsf{CP}^\mathsf{f}\mathsf{C}^{\mathrm{T}} = \mathsf{Z}_\mathsf{t}\mathsf{Z}_\mathsf{t}^{\mathrm{T}}} \tag{5.0.5}$$

- SV–decomposition implemented at ECMWF for generation of initial–time perturbations in the Ensemble–Prediction–System (only partly operational)

Multinormal Sampling Based on SV–Decomposition of P^{a}

- Transforming random variables

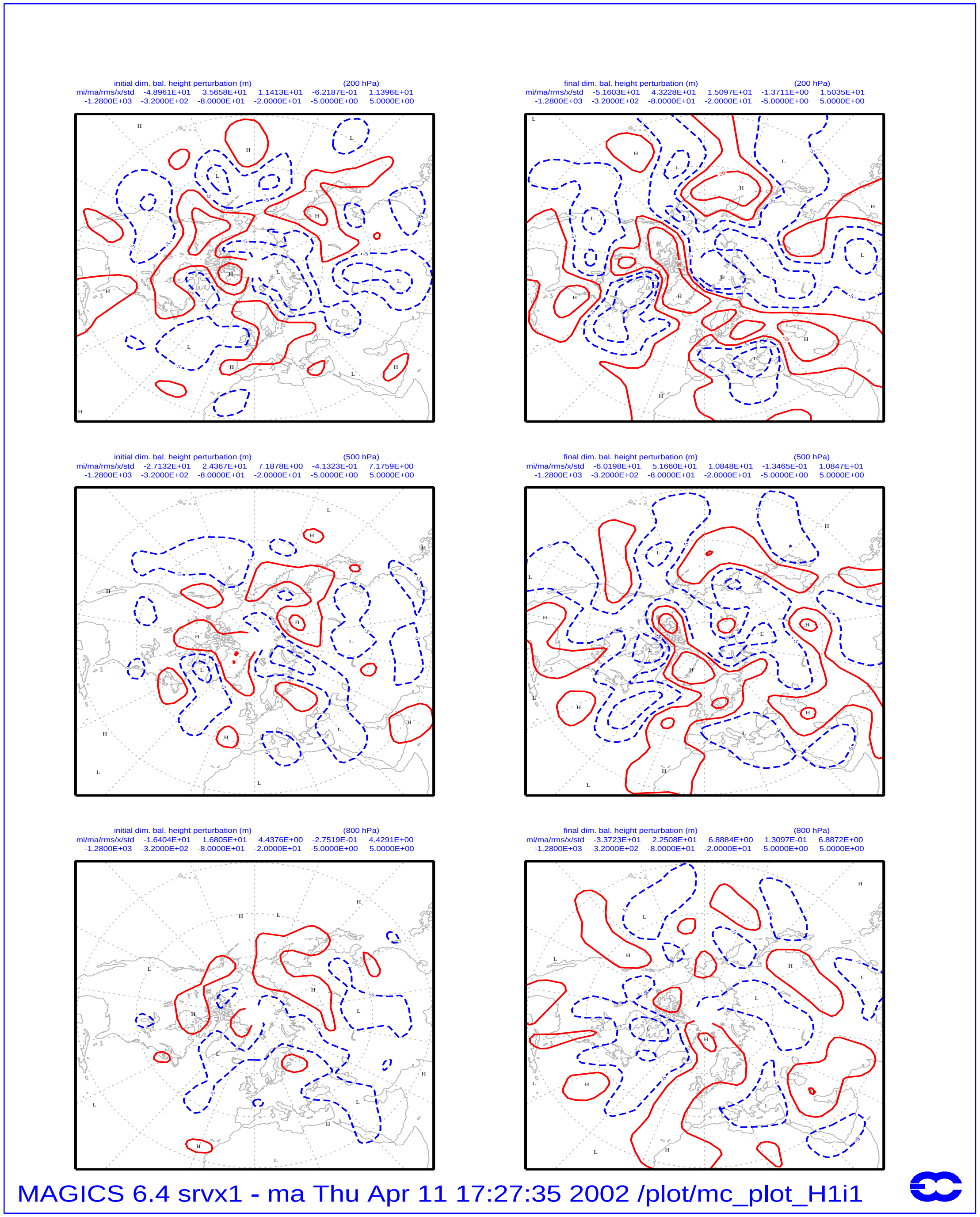
$$\mathbf{q} \sim \mathcal{N}(0, \mathsf{I}) \qquad \Rightarrow \qquad \mathbf{x} = \mathbf{x}_0^c + \mathsf{V}^{1/2}\mathbf{q} \qquad \rightarrow \qquad \mathbf{x} \sim \mathcal{N}(\mathbf{x}_0^c, \mathsf{V}) \tag{5.0.6}$$

- Use **SV–decomposition of** P^a (possibly truncated to N SVs) in (5.0.5) – to describe square–root of P^{a} – in process of generating initial–time perturbed states $\mathbf{x}$:

$$(\mathsf{P}^a)^{1/2} = \mathsf{Z}_0{}^{(N)} \tag{5.0.7}$$

$$\mathbf{x}_i = \mathbf{x}_0^c + \mathsf{Z}_0^{(N)}\mathbf{q}_i \qquad i = 1, 2, ..., M \quad \Rightarrow \quad \boxed{\mathbf{x} \sim \mathcal{N}\Big(\mathbf{x}_0^c, (\mathsf{P}^{\mathsf{a}})^{(N)}\Big)} \tag{5.0.8}$$

- Generating perturbations consistent with P^{a} knowledge based on N SVs
- Assumes normally distributed analysis errors; • non–eigendecomp. $\rightarrow$ eigendecomp.
- Taking SV properties into nonlinear regime
- Strong similarity to operational *rotation* at ECMWF
- free parameters: N and M



exp. H1i1 QG model

left: initial pert.1

right: final pert.1

rms left: 11.4 / 7.2 / 4.4 m

rms right: 15.1 / 10.8 / 6.9 m

final = 2 days

Singular–Vector Spectra

- slope and shape of spectrum → relevant for truncations
- investigations with MAMS2
- dry truncated R-norm (14 Feb 1982): **169** growing SVs out of 4830 dimensions
- references:

  Errico, Ehrendorfer, and Raeder 2001 ... Tellus

  Ehrendorfer, Errico, and Raeder 1999 ... JAS

6 Conclusions

- The effect of “optimal” moisture perturbations can be as great as optimal perturbations of wind or temperature, even regarding their effects on wind and temperature.
- The geographic locations of leading SVs can be very dependent on the specific norms considered.
- The vertical structures of leading SVs can vary greatly from case to case, even when the same norms are considered.
- In some cases, leading SVs having only initial perturbations of wind and temperature produce nearly identical final–time SVs as those produced by the leading SVs having only moisture perturbations $\rightarrow$ inferred dependence on moist specific enthalpy.
- In most cases, different structures optimize the different final–time norms.
- An example of an initial–time SV dominated by wind divergence above the tropopause was obtained.

- Sensitivities to non–convective precipitation have not been shown to always dominate sensitivities to convective precipitation.
- TLM results of precipitation processes over 12 hours match NLM results quite well, even for perturbations as large as 2 g kg^{-1}.
- The effects of possible moisture errors and of appropriately linearized moist physics should not be neglected or treated as second–order.

- Hessian SVs
- Multinormal Sampling
- Spectra and number of growing SVs

References

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