SINGULAR VECTORS IN A MOIST LIMITED–AREA MODEL

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Contents

1 Introduction and Questions 3
2 Singular Vectors (SVs) and Norms 4
3 The Mesoscale Adjoint Modeling System MAMS2 6
4 Moist SVs: Results and Relevance 8
5 SVs: Covariance Prediction and Spectra 9
6 Conclusions 15
1 Introduction and Questions

MOIST SVs

- What are the relative effects of initial perturbations (or errors) in the dynamical fields versus the moisture field?
- Are similar structures optimal for affecting both perturbation energy and precipitation?
- Is convective or non-convective precipitation generally more sensitive to initial perturbations?

ENSEMBLE PREDICTION AND SV SPECTRA

- obvious interest regarding data assimilation and predictability
- importance of SVs for covariance and ensemble prediction
- Hessian SVs and Spectra
2  Singular Vectors (SVs) and Norms

The problem:

Given a linearized model

\[ y = Mx \]

Find the \( x \) that maximizes

\[ L_2 = y^T L_2 y \]

Given

\[ L_1 = x^T L_1 x \]

The solution:

\[ x = L_1^{-\frac{1}{2}} z \]

where

\[ L_1^{-\frac{1}{2}T} M^T L_2 M L_1^{-\frac{1}{2}} z = \lambda z \]

Note: Different norms at initial and final times.
Norms Considered

Energy Norm:

\[ E = \frac{1}{N_w} \sum_{i,j,k} \Delta \sigma_k \left( u_{i,j,k}^2 + v_{i,j,k}^2 \right) + \frac{C_p}{T_r N_t} \sum_{i,j,k} \Delta \sigma_k T_{i,j,k}^2 + \frac{R T_r}{p_{sr}^2 N_t} \sum_{i,j} p_{i,j}^2 \]  \hspace{1cm} (1)

Moist Energy Norm (dry fields zero):

\[ E_m = \frac{L^2}{C_p T_r N_t} \sum_{i,j,k} \Delta \sigma_k q_{i,j,k}^2 \]  \hspace{1cm} (2)

Precipitation Rate Norm (used only as end–time norm; non–convective + convective precipitation):

\[ P = \frac{1}{N_t} \sum_{i,j} R_{i,j}^2 \]  \hspace{1cm} (3)

Dry and Moist Variance–weighted norm (penalize large \( q \) high up):

\[ V_d = \frac{1}{N_w} \sum_{i,j,k} \Delta \sigma_k \left( \frac{u_{i,j,k}^2}{V_{u,k}} + \frac{v_{i,j,k}^2}{V_{v,k}} \right) + \frac{1}{N_t} \sum_{i,j,k} \Delta \sigma_k T_{i,j,k}^2 \frac{T_{i,j,k}^2}{V_{T,k}} + \frac{1}{N_t} \sum_{i,j} p_{i,j}^2 \frac{p_{i,j}^2}{V_{p}} \]  \hspace{1cm} (4)

\[ V_m = \frac{1}{N_t} \sum_{i,j,k} q_{i,j,k}^2 \frac{q_{i,j,k}^2}{V_{q,k}} \Delta \sigma_k \]  \hspace{1cm} (5)
3 The Mesoscale Adjoint Modeling System MAMS2

The Mesoscale Adjoint Modeling System Version 2 (MAMS2)

- Primitive equations with water vapor
- Bulk PBL formulation (Deardorff)
- Stability–dependent vertical eddy diffusion (CCM3)
- RAS scheme (Moorthi and Suarez)
- Stable–layer precipitation
- $\Delta x = 80$ km
- 20–level model ($\Delta \sigma = 0.05$)
- $p_{\text{top}} = 100$ or 10 mb
- 12–hour forecasts for 4 synoptic cases
6 sets of SVs for each case

- $E \rightarrow E$
- $E_m \rightarrow E$
- $V_d \rightarrow E$
- $V_m \rightarrow E$
- $V_d \rightarrow P$
- $V_m \rightarrow P$

A larger value of $E$ can be produced with an initial constraint $V_m = 1$ compared with $V_d = 1$. 
4 Moist SVs: Results and Relevance

- Cases
- Amplifications
- Horizontal properties of SVs
- Vertical structures of SVs
- A peculiar SV
- Final–time SVs and growth mechanisms
- Nonlinear considerations and correlations
5 SVs: Covariance Prediction and Spectra
Uncertainty Prediction: Hessian SVs

The HSVs $Z_0$ solving the eigenvector problem:

$$M^T C^T C M Z_0 = (P^a)^{-1} Z_0 \Lambda \quad \text{s.t.} \quad Z_0^T (P^a)^{-1} Z_0 = I \quad (5.0.1)$$

are, when time–evolved, eigenvectors of $P^f$, because:

$$\left(C M P^a\right) M^T C^T C M Z_0 = \left(C M P^a\right) (P^a)^{-1} Z_0 \Lambda \quad \Rightarrow \quad \left(C P^f C^T\right) Z_t = Z_t \Lambda \quad (5.0.2)$$

The evolved HSVs $Z_t$ are the eigenvectors of $C P^f C^T$ – which is the forecast error covariance in the “final–time norm” $C$. Note the final–time orthogonality relationship:

$$Z_t^T Z_t = \left(C M Z_0\right)^T \left(C M Z_0\right) = Z_0^T M^T C^T C M Z_0 = Z_0^T (P^a)^{-1} Z_0 \Lambda = \Lambda \quad \Rightarrow \quad Z_t^T Z_t = \Lambda \quad (5.0.3)$$
Uncertainty Prediction: The SV–Decomposition of $P^a$

- Because the initial–time SVs satisfy (5.0.1), it is true that $P^a$ can be written as:

$$P^a = Z_0 Z_0^T$$  \hspace{1cm} (5.0.4)

This is a special square–root for $P^a$ (different from eigendecomposition and also not lower–triangular) \(\rightarrow\) the **SV–decomposition of $P^a$**

- Under linear dynamics this SV–decomposition becomes the eigendecomposition of the forecast error covariance matrix, because (5.0.5) is the same as (5.0.2) together with (5.0.3):

$$
\begin{align*}
\left(\begin{array}{c}
CM
\end{array}\right) P^a \left(\begin{array}{c}
CM
\end{array}\right)^T &= \left(\begin{array}{c}
CM
\end{array}\right) Z_0 Z_0^T \left(\begin{array}{c}
CM
\end{array}\right)^T
\quad \rightarrow \\
CP_f C^T &= Z_t Z_t^T
\end{align*}
$$

\hspace{1cm} (5.0.5)

- SV–decomposition implemented at ECMWF for generation of initial–time perturbations in the Ensemble–Prediction–System (only partly operational)
Multinormal Sampling Based on SV–Decomposition of $P^a$

- Transforming random variables

$$\mathbf{q} \sim \mathcal{N}(0, I) \quad \Rightarrow \quad \mathbf{x} = \mathbf{x}_0^c + \mathbf{V}^{1/2} \mathbf{q} \quad \rightarrow \quad \mathbf{x} \sim \mathcal{N}(\mathbf{x}_0^c, \mathbf{V}) \quad (5.0.6)$$

- Use **SV–decomposition of $P^a$** (possibly truncated to $N$ SVs) in (5.0.5) – to describe square–root of $P^a$ – in process of generating initial–time perturbed states $\mathbf{x}$:

$$\left(P^a\right)^{1/2} = Z_0^{(N)} \quad (5.0.7)$$

$$\mathbf{x}_i = \mathbf{x}_0^c + Z_0^{(N)} \mathbf{q}_i \quad i = 1, 2, \ldots, M \quad \Rightarrow \quad \mathbf{x} \sim \mathcal{N}\left(\mathbf{x}_0^c, (P^a)^{(N)}\right) \quad (5.0.8)$$

- Generating perturbations consistent with $P^a$ knowledge based on $N$ SVs

- Assumes normally distributed analysis errors; **non–eigendecomp. → eigendecomp.**

- Taking SV properties into nonlinear regime

- Strong similarity to operational *rotation* at ECMWF

- Free parameters: $N$ and $M$
exp. H1i1 QG model
left: initial pert.1
right: final pert.1

rms left: 11.4 / 7.2 / 4.4 m

rms right: 15.1 / 10.8 / 6.9 m

final = 2 days
Singular-Vector Spectra

- slope and shape of spectrum $\rightarrow$ relevant for truncations
- investigations with MAMS2
- dry truncated R-norm (14 Feb 1982): 169 growing SVs out of 4830 dimensions
- references:
  - Errico, Ehrendorfer, and Raeder 2001 ... Tellus
  - Ehrendorfer, Errico, and Raeder 1999 ... JAS
6 Conclusions

- The effect of “optimal” moisture perturbations can be as great as optimal perturbations of wind or temperature, even regarding their effects on wind and temperature.

- The geographic locations of leading SVs can be very dependent on the specific norms considered.

- The vertical structures of leading SVs can vary greatly from case to case, even when the same norms are considered.

- In some cases, leading SVs having only initial perturbations of wind and temperature produce nearly identical final–time SVs as those produced by the leading SVs having only moisture perturbations → inferred dependence on moist specific enthalpy.

- In most cases, different structures optimize the different final–time norms.

- An example of an initial–time SV dominated by wind divergence above the tropopause was obtained.
• Sensitivities to non–convective precipitation have not been shown to always dominate sensitivities to convective precipitation.

• TLM results of precipitation processes over 12 hours match NLM results quite well, even for perturbations as large as 2 g kg$^{-1}$.

• The effects of possible moisture errors and of appropriately linearized moist physics should not be neglected or treated as second–order.

• Hessian SVs

• Multinormal Sampling

• Spectra and number of growing SVs
References


