L-Galerkin Operators on Polygonal Serendipidity Grids for spherical spectral Element Discretiations

J. Steppeler

CSC , Hamburg

Offenbach 2013

Plan of Lecture:

- Numerical developments now ?
- Finite Elements and spectral elements
- Polygonal grids
- Serendipity
- Different L-Galerkin operators: Quadrature, CG/DG
- Cut cells as an (very irregular) case
- Some analysis of different L-Galerkin operators
- Initial results

The spectral elements

- SE: Standard spectral elements (Quadrature G-Lobatto points)
- SES schemes are local and use a basis function interpolation
- Pre-regularisation (simple o3)
- Mass conserving interpolation

Reprinted from JOURNAL OF COMPUTATIONAL PHYSICS Vol. 19, No. 4, December 1975

On a High Accuracy Finite Difference Method

J. STEPPELER

Deutscher Wetterdienst, Zentralamt, Offenbach, West Germany

Received November 18, 1974; revised May 5, 1975

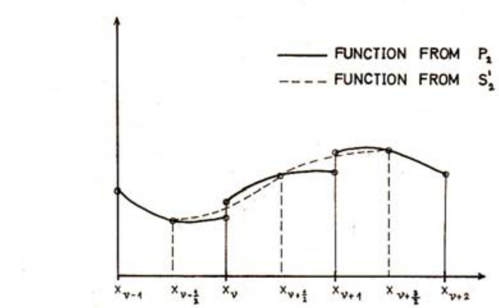
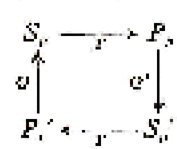


FIG. 2. Approximating functions from P_2 by functions from S_2.

Numerical Procedure

A time step is just a time translation mapping T fol... shiftings Q or Q'. For example, one obtains in this wa... from a function in S_ν. In the next time step one agai...



L-Galerkin procedure

The Galerkin/L-Galerkin Zoo

- Operators used: Galerkin G; $\frac{\partial}{\partial x};\frac{\partial}{\partial y}$; Regularization R ($divRF \in C; for F \in C$); Collocation K
- Simple 3rd order L-Galerkin: $\frac{\partial \rho}{\partial t} = divRF; F \in C; \rho \in C$

  Standard Galerkin: $\frac{\partial \rho}{\partial t} = GdivF; F \in C; \rho \in C$

  Pre regularization: $\frac{\partial \rho}{\partial t} = div\rho v \Rightarrow \frac{\partial \rho}{\partial t} = divR\rho Gv$
- From the basis-function representation follow 1st order conservation and other minemic properties.
- **This flexibility gives CG the properties normally associated with DG**
- **Pre-regularization make schemes like Baumgardner or standard o4 conserving**

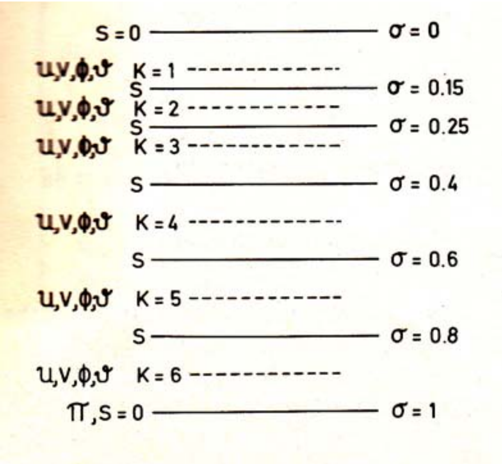
Numerical properties of different Serendipity Schemes

Table 1:

CFL numbers for different schemes

Scheme	CFL	conservation	suitable for irregular resolution
O1O1	2.8	yes	no
Classic O4:	2.1	not known	in 1-d
O2 Standard:	1.9	yes	yes
O2 with O4 diff:	4.	Yes	limited
O2 with O2/O4 diff:	1.4	yes	yes
O3 with O4 diff:	3.8	yes	limited
O3 standard	1.6	yes	yes

An early d-3 example of o3 spectral elements



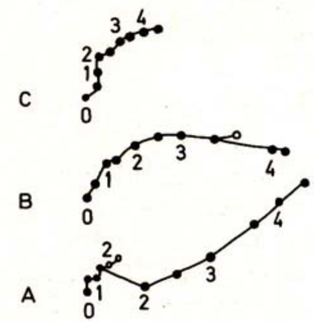
A Baroclinic Model Using a High Accuracy Horizontal Discretization

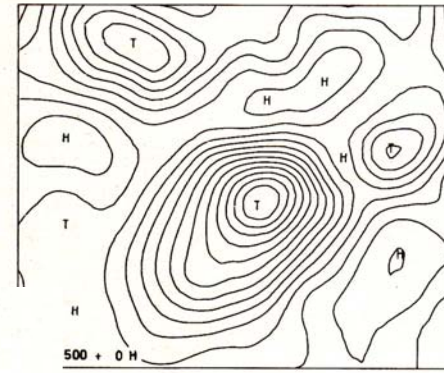
Ein baroklines Modell mit sehr genauer horizontaler Diskretisierung

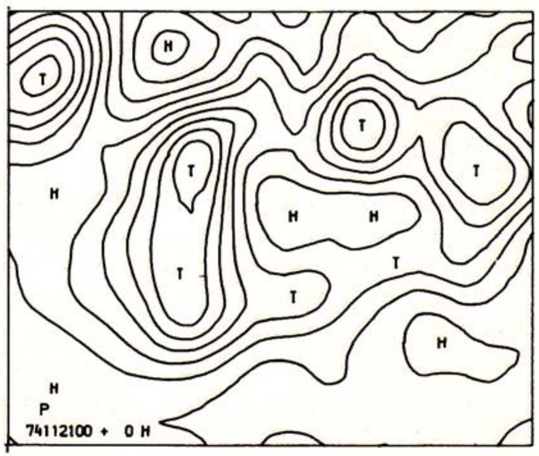
J. Steppeler

Deutscher Wetterdienst, 605 Offenbach

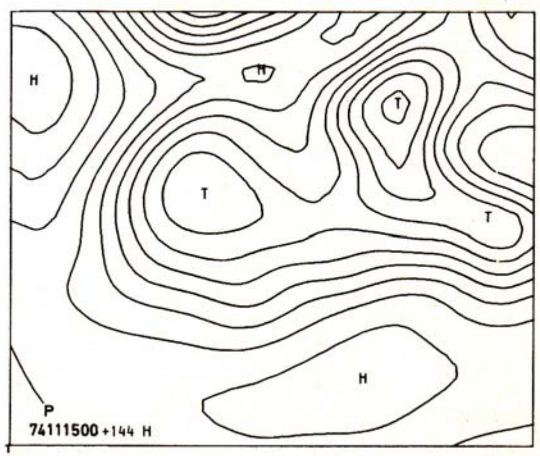
(Manuscript received 17.5.1976, in revised form 30.9.1976)



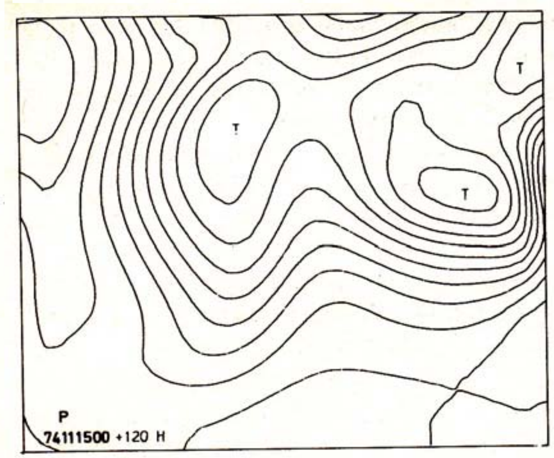




Verifying analysis

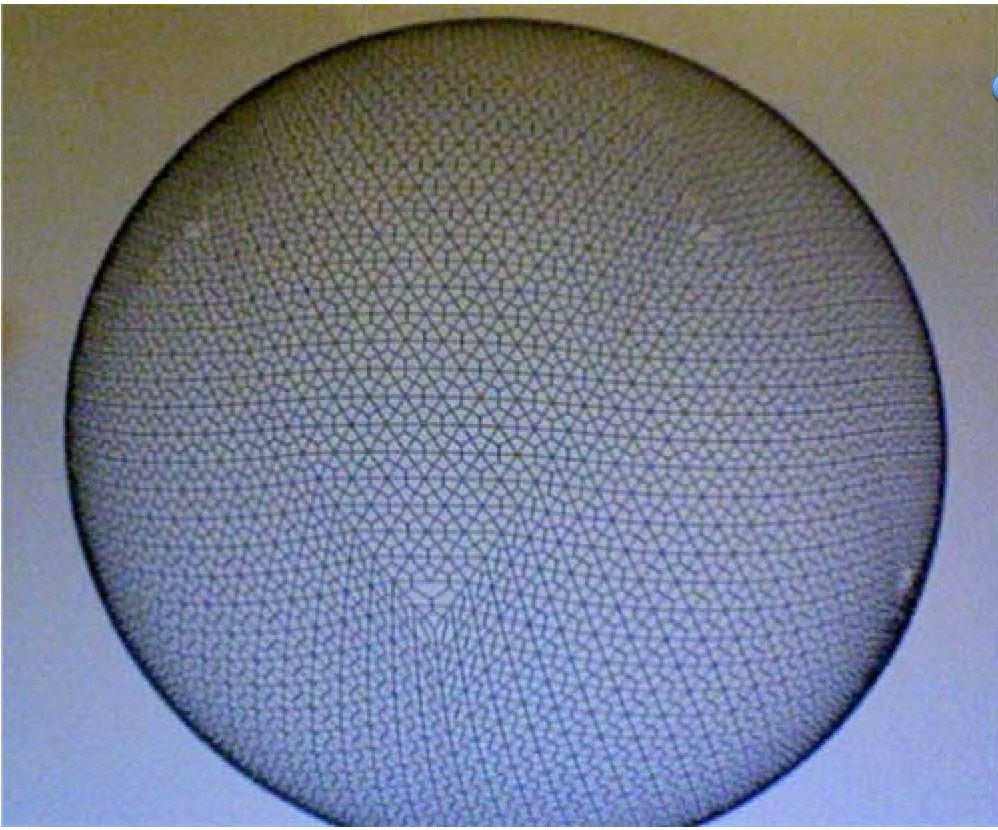


BKF + 6 days

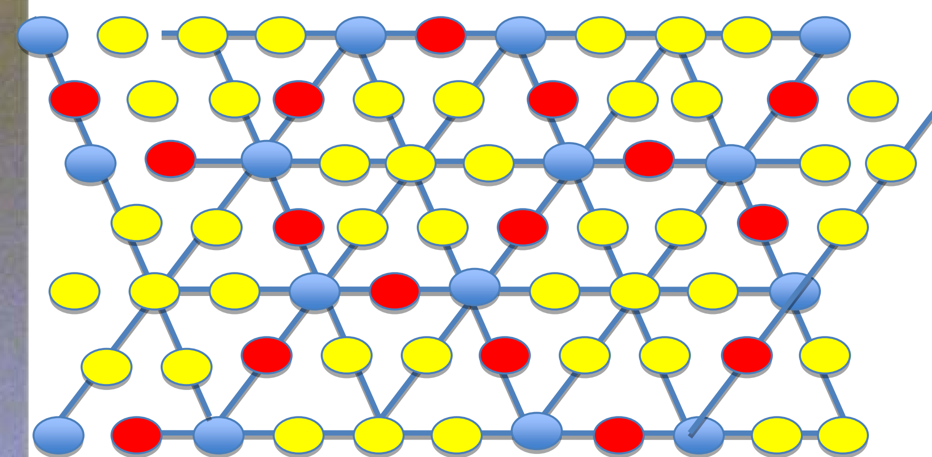


SE + 6 days

Serendipity Hexagonal Sparse Grids: o2



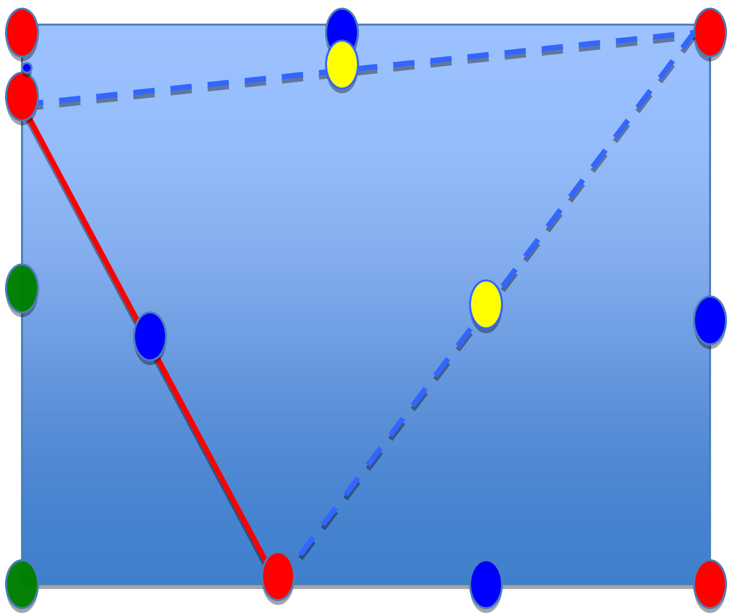
O2: full grid (yellow) **Main points** (blue)
O2 points (red)

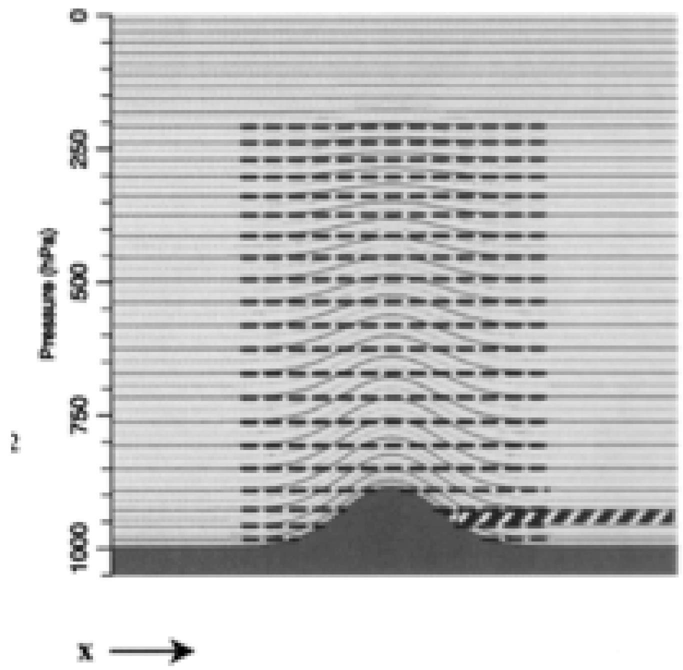


3-d, o2:
2*3+3 : (7+2+2)*3=

9 : 36

Cut Cells, Triangulation (irregular), high order points, phantom points, diagnostic points

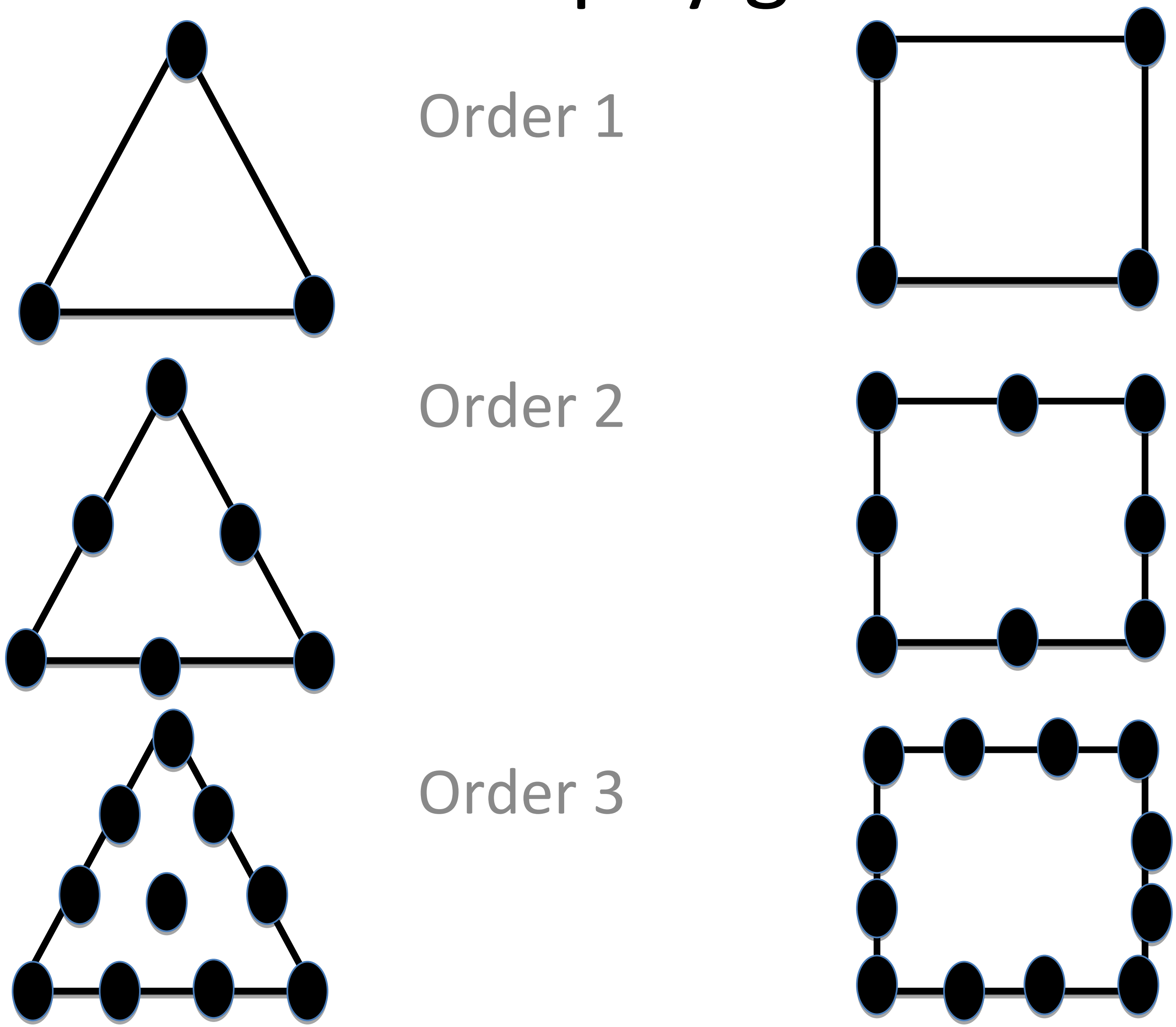




Numeric developments making models wonderful

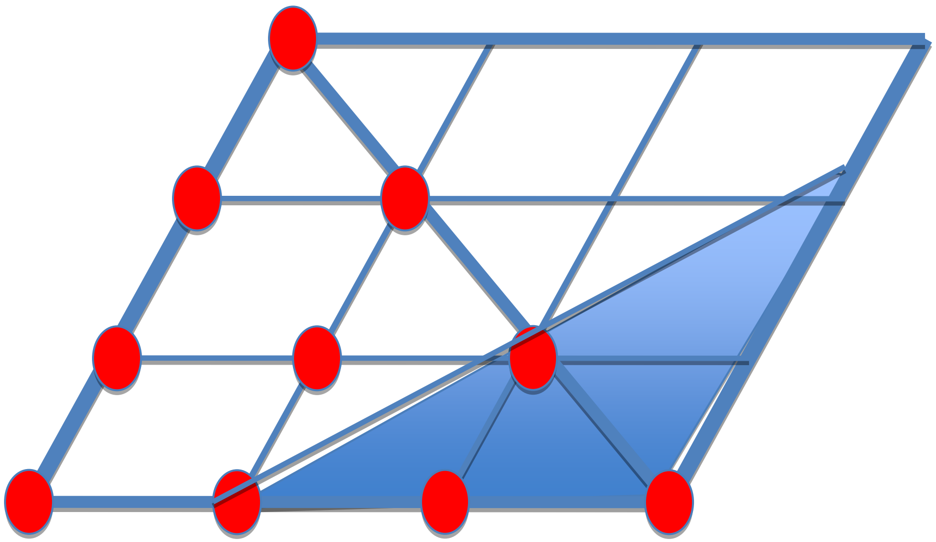
- Polygonal Grids, Hexagons, Icosahedron
- RK3/RK4, nh, hexagons
- Spectral elements: uniform high order, no grid smoothing, easy 1:2 refinement, scalability, conservation/minemic properties
- Cut cells
- Real life models with these properties are rare
- None of the models has **<u>all</u>** these properties

Serendipity grids



The task managing of serendipity 2nd or 3rd order is the same as for rectangular grids

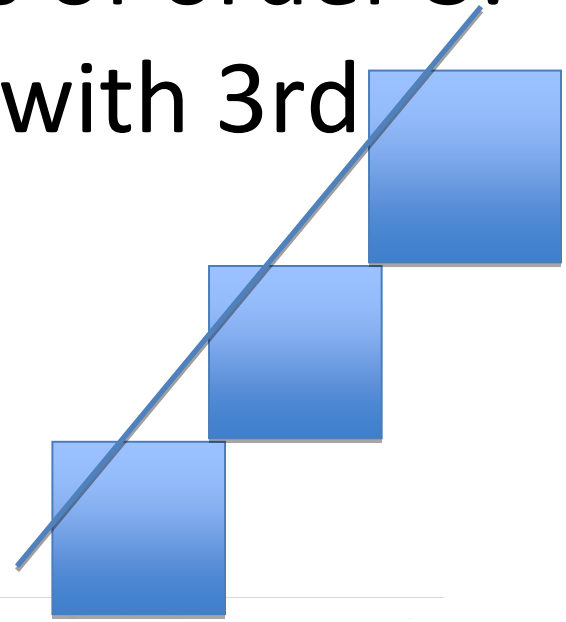
- On the icosaheron triangular serendipity is equivalent to a full grid on Rhomboids

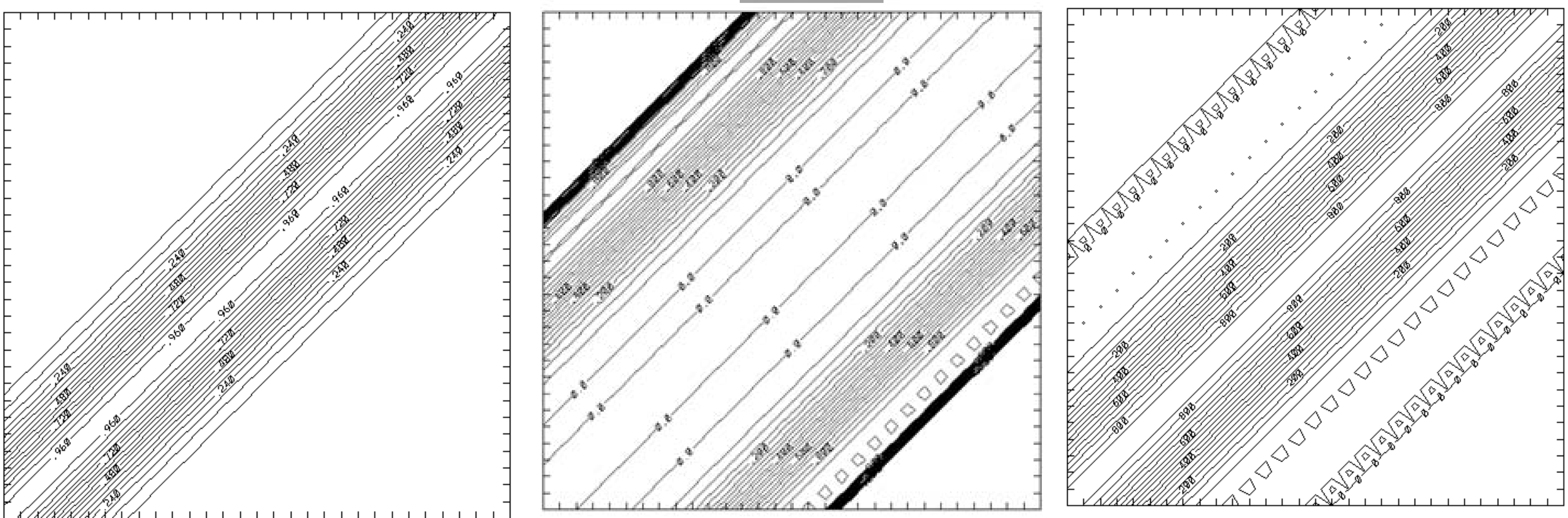


3rd spectral elements with cut cells

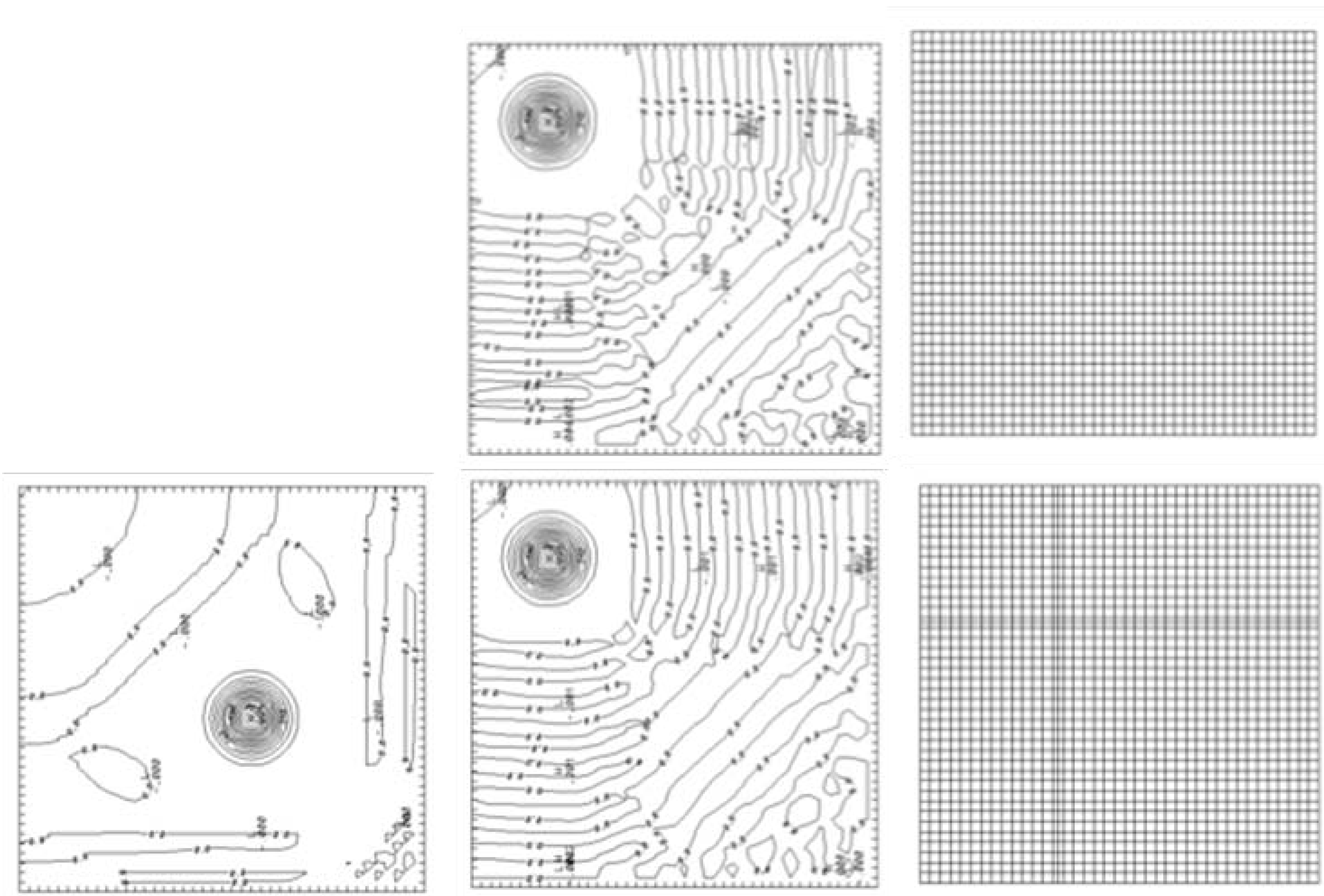
spectral elements of order 3:

=> cut cells work with 3rd order Spectral-elements



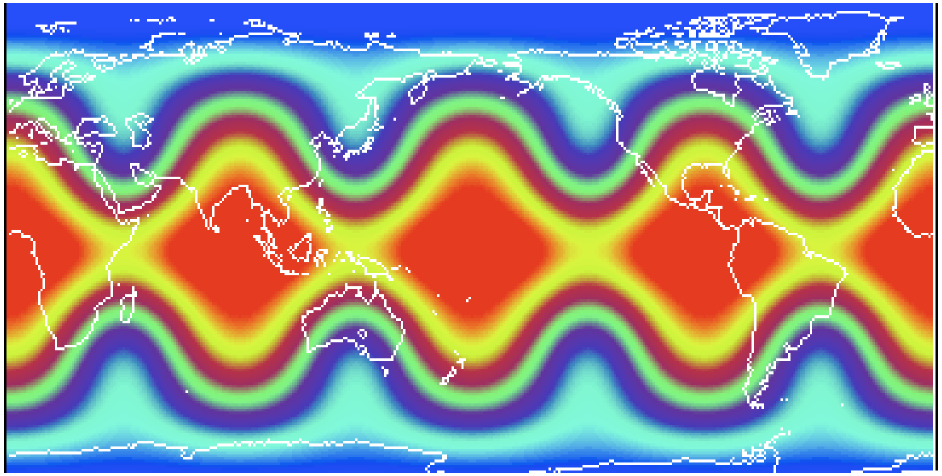


Standard O3 L-Galerkin is suitable for irregular resolution

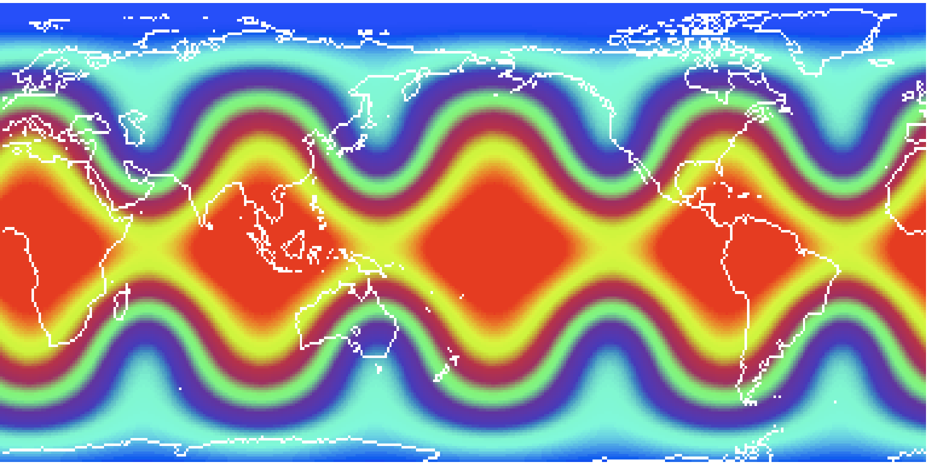


Sparse rhomboidal/conserving serendipity elements Test case 6

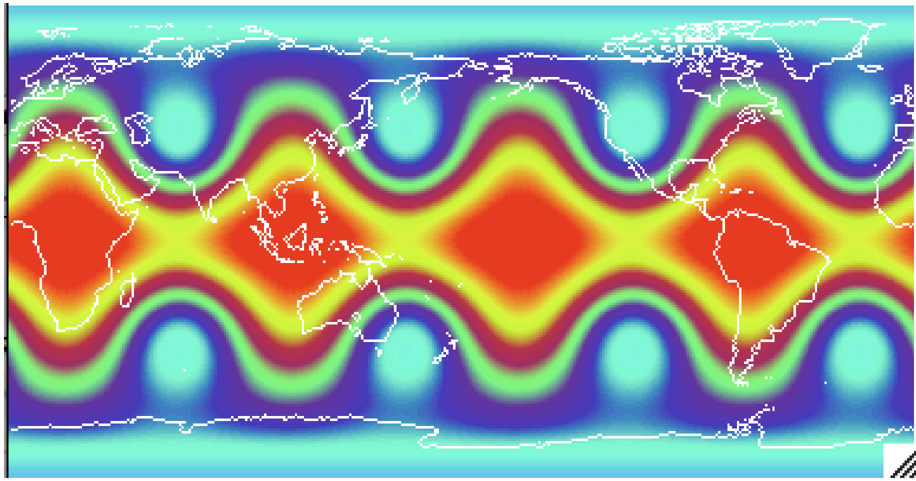
Initial



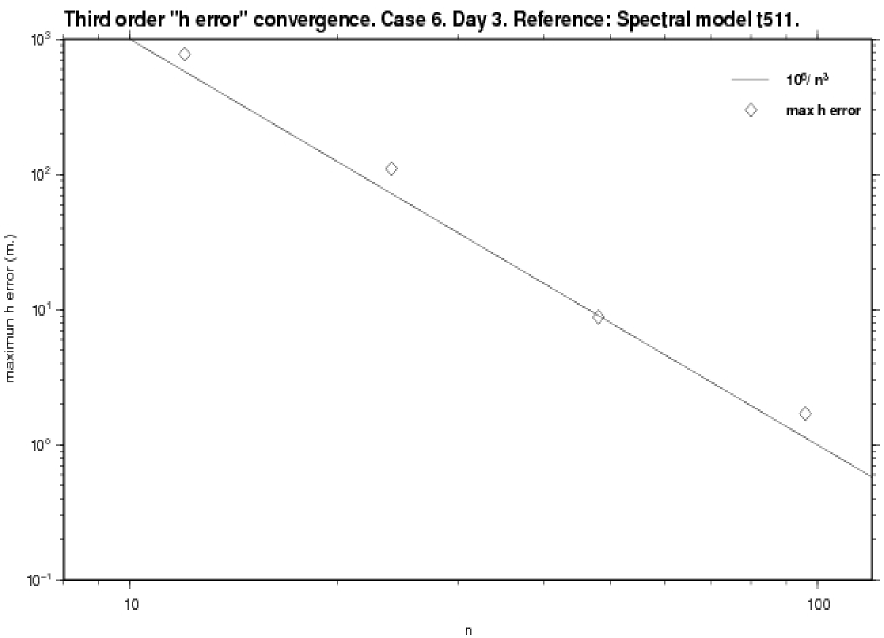
Day1

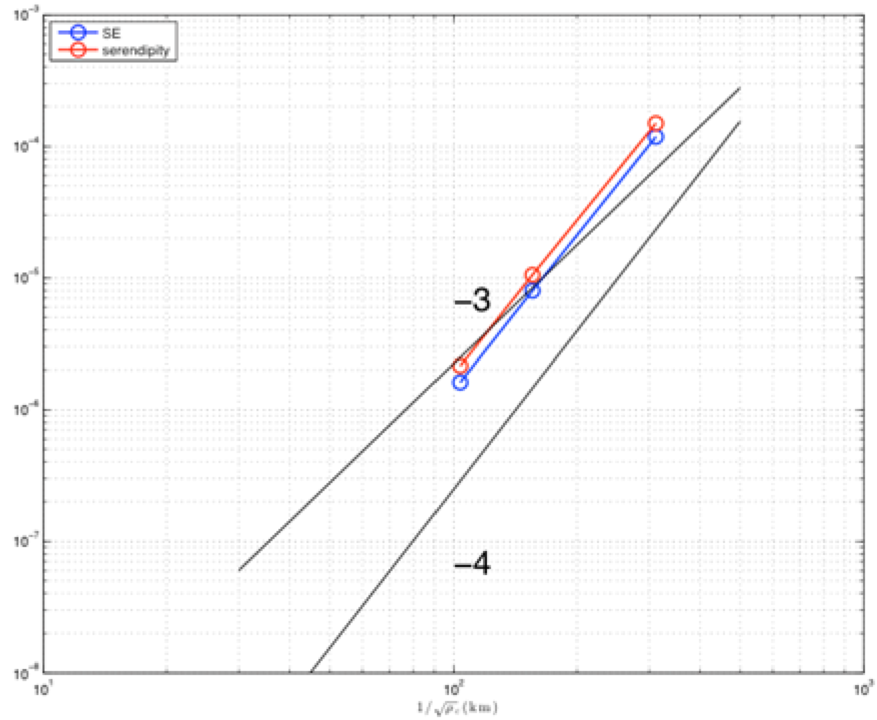


Day 10

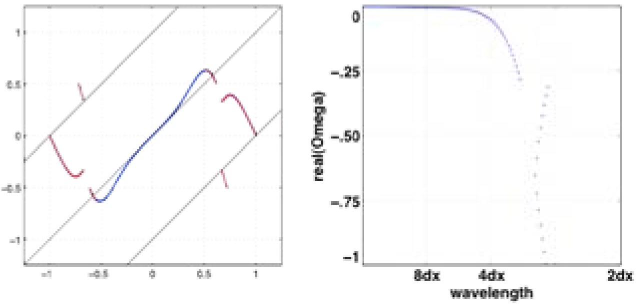


Third Order Convergence of Shallow Water Model at Day 3

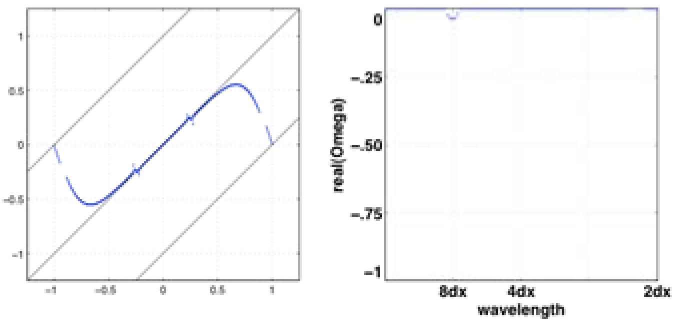




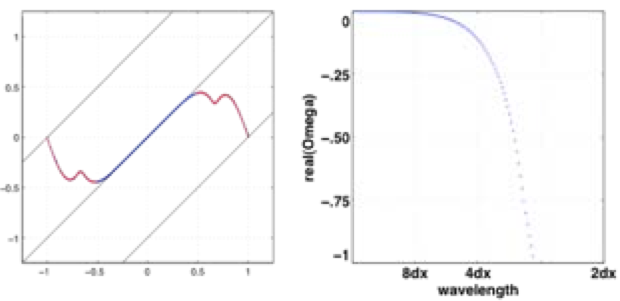
Simple O3: linear analysis, L-Galerkin



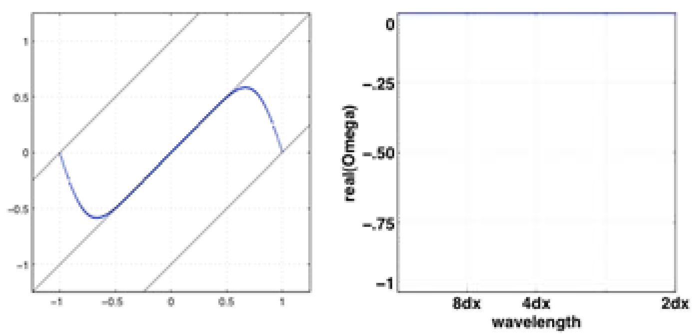
Eq spaced L-Gal, Diffusion o4



Eq spaced SE, no diffusion



Eq spaced SE, Diffusion o4



GL spaced SE no diffusion

Pre-Galerkin

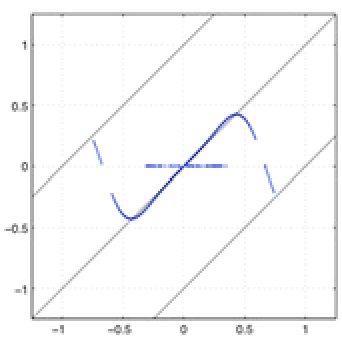
equally spaced method stable, CFL with RK4: 3.9 => LA=1.4

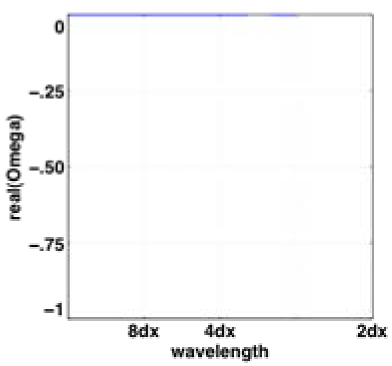
Comparison:

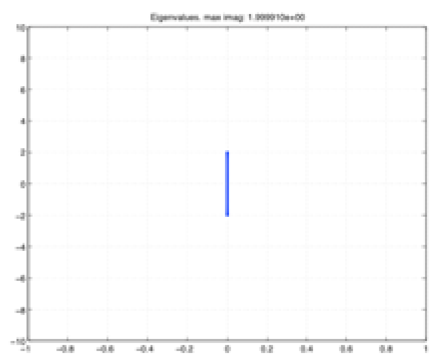
Standard GL/SE: LA=2.7;normal Pre-Galerkin, O3 based: LA=2.

conservative eq spaced:LA=2.97;

eq. spaced/quadrature: unstable



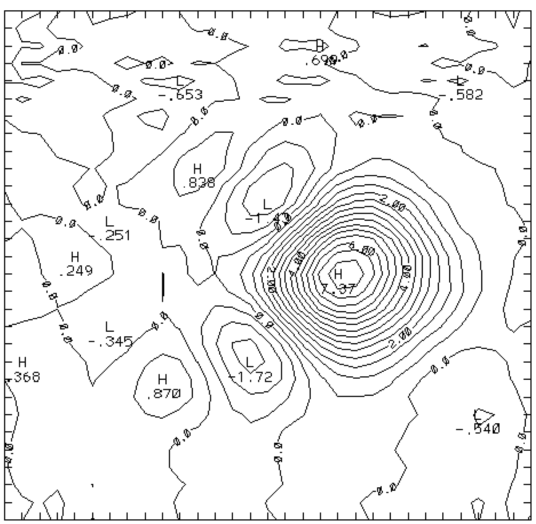


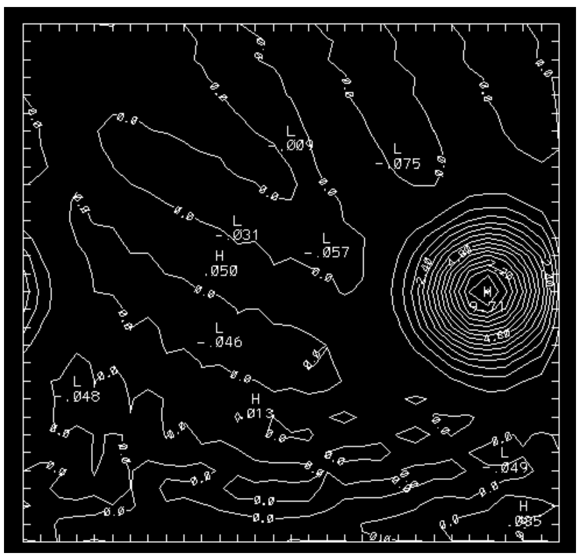


2nd order elements: 3rd order by super-convergence

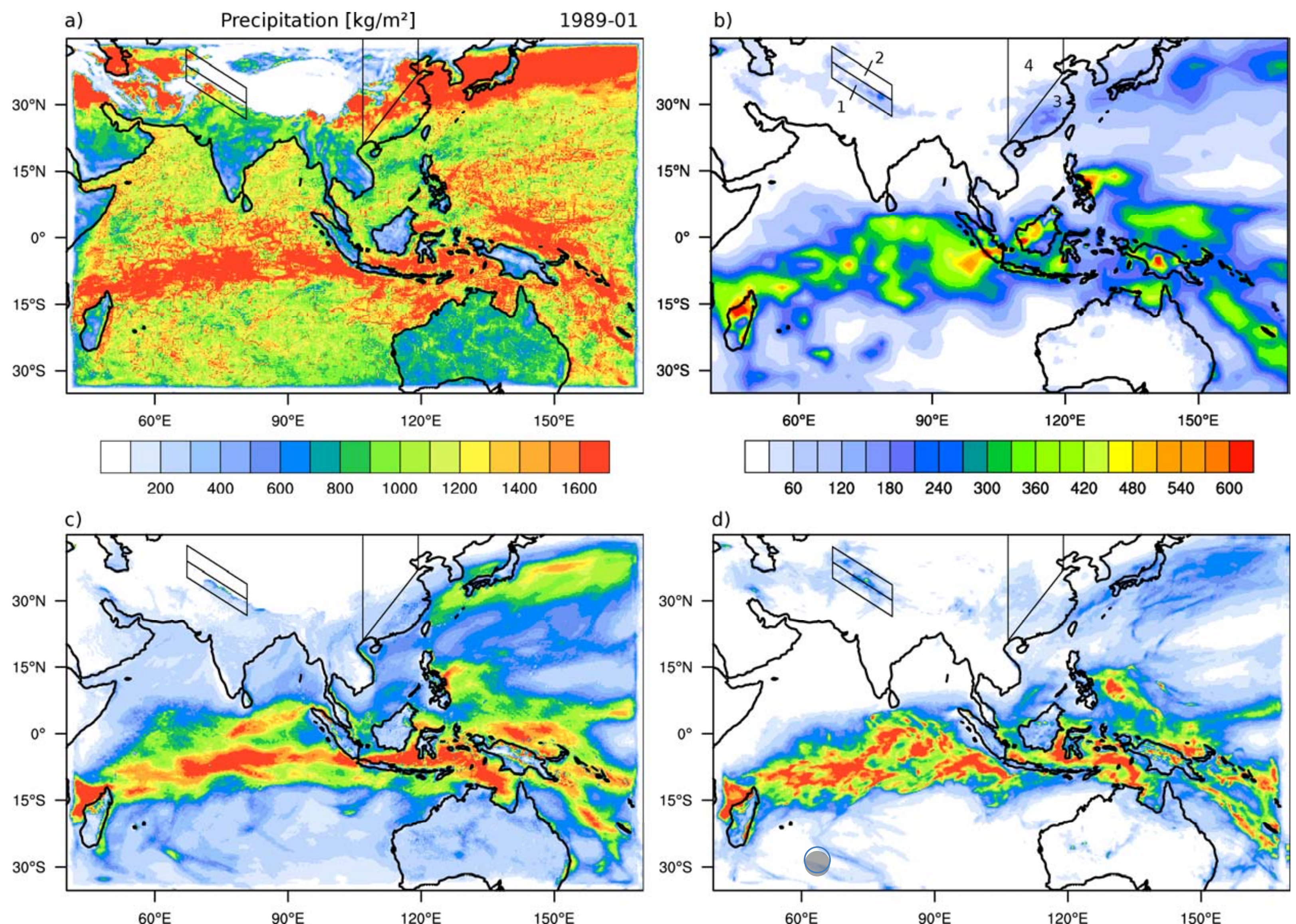
CFL(1-d)=4.

10 Rotations 1.3 Rotations



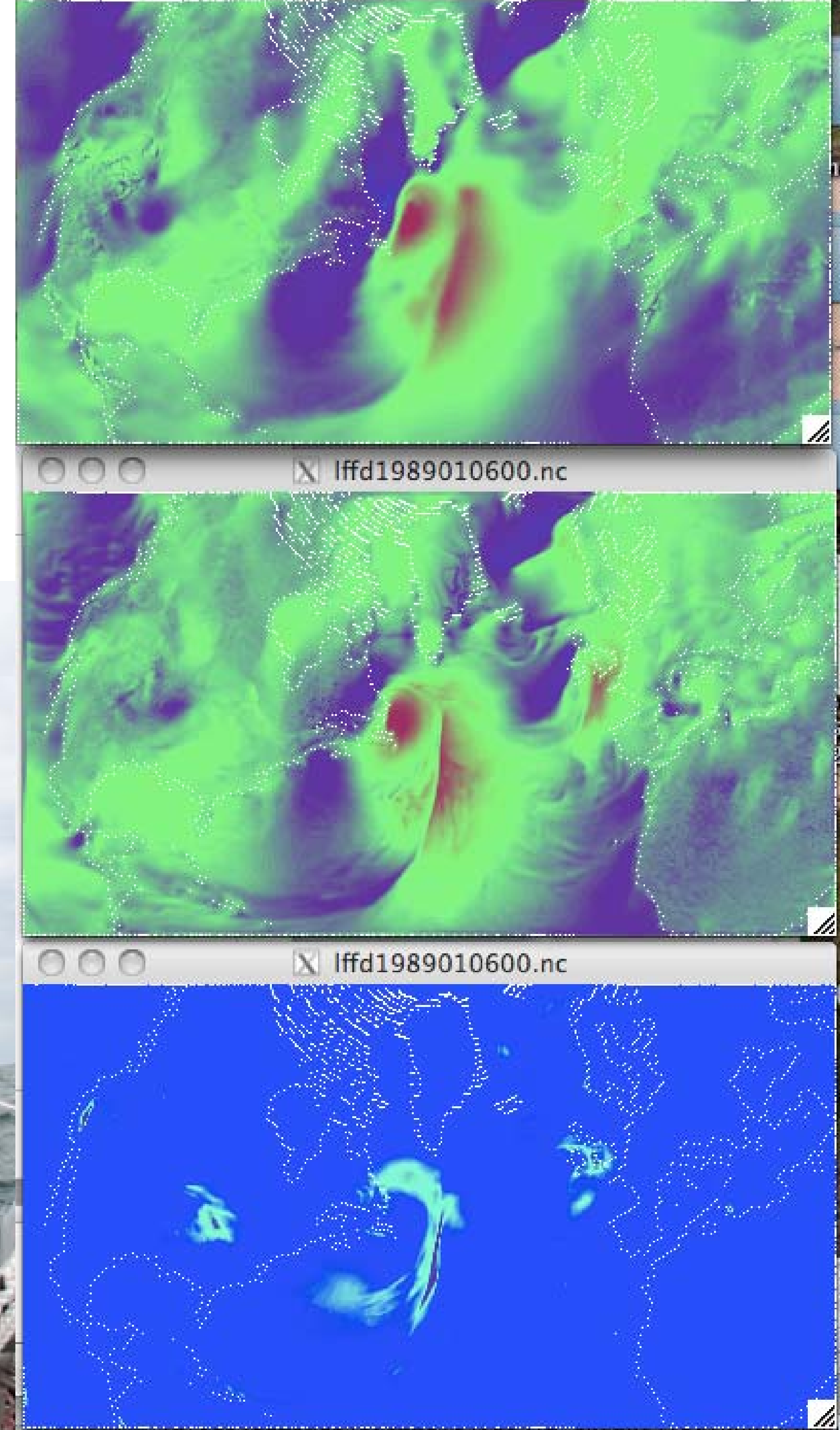


Pecip: a, control, b, obs, c, cut cell, d, CLM

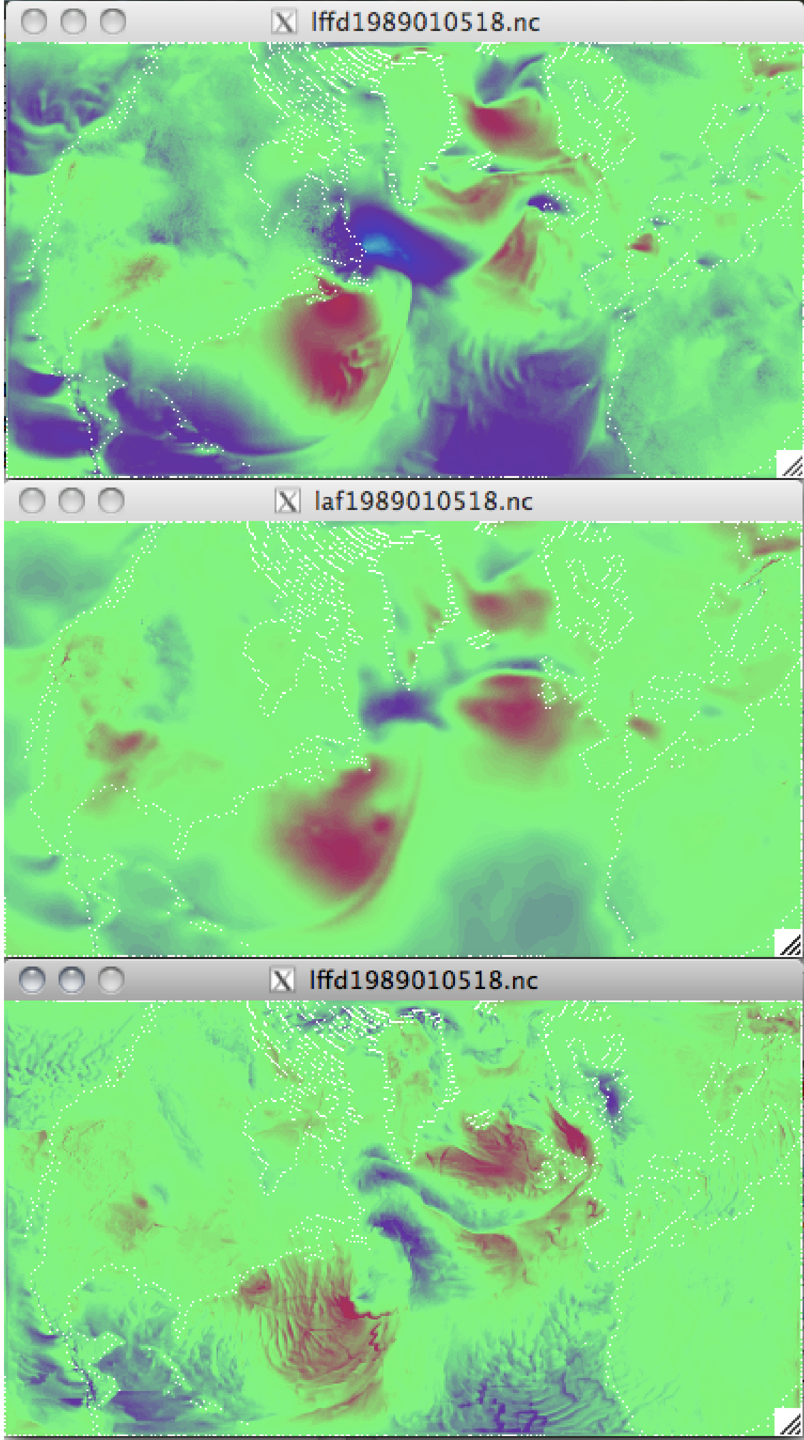


Cut-cell 5 days: V

Questions?



U 4.75 days



4.75 days V