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Probabilistic forecasts of precipitation in terms of quantiles

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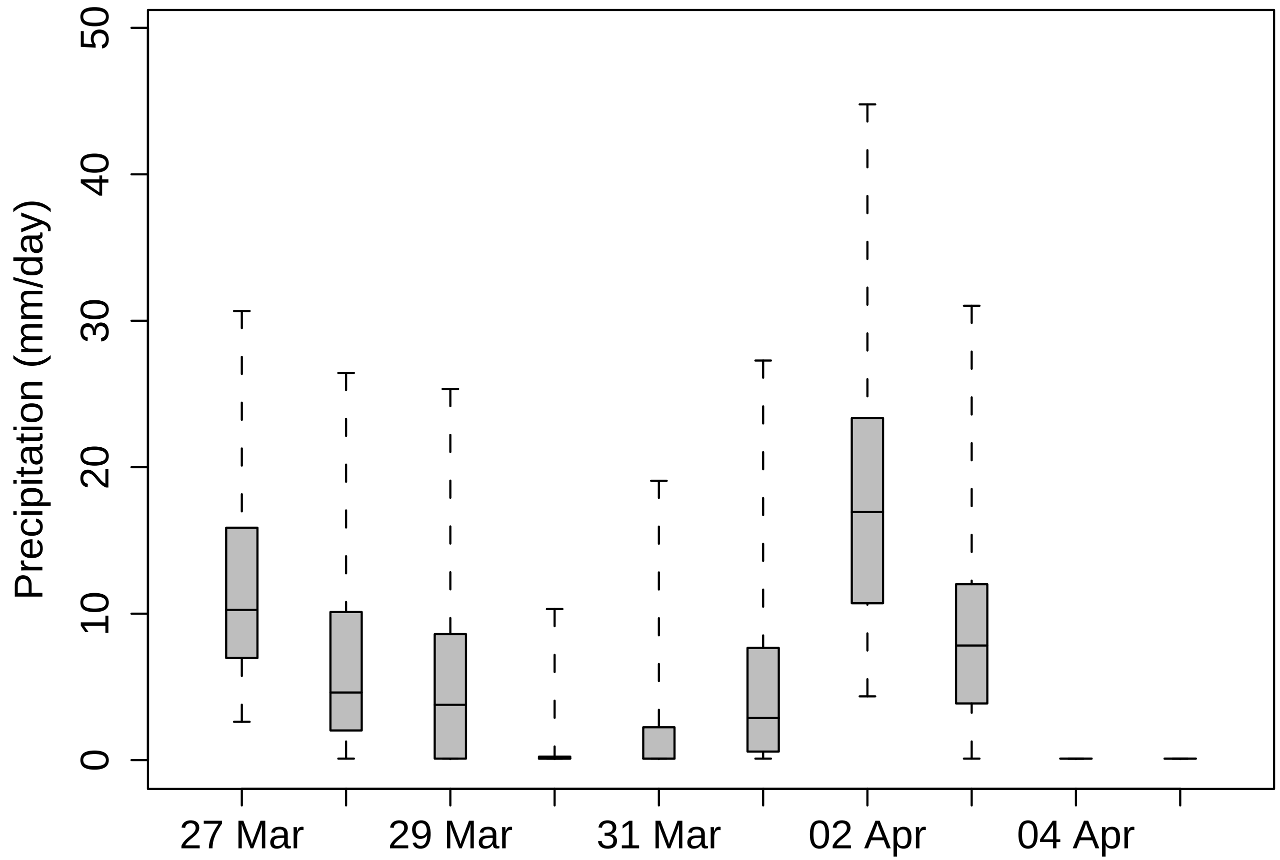
Example of quantile forecasts



Methods

Quantile regression:

estimate of the θ-th quantile by

$$\arg\min_{\alpha} \sum_i \rho_\theta \left(r_i - \alpha_0 - \sum\nolimits_k \alpha_k x_{ik} \right)$$

where

$$\rho_\theta(u) = \begin{cases} u\theta & u \geq 0 \\ u(1-\theta) & \text{else} \end{cases}$$

r_i observations

x_i predictors (NWP model output)

α_i regression parameters

NOTE: minimization must be repeated for each θ

Local quantile regression:

estimate of the θ-th quantile at predictor value x by

$$\arg\min_{\alpha} \sum_i \rho_\theta\left(r_i - \alpha_0 - \sum\nolimits_k \alpha_k x_{ik}\right) w\left(\left\|x - x_i\right\| ; \lambda\right)$$

where

- $w()$ weight function, defined such that weather situations similar to x are given largest weight and, hence, greatest impact on the fit
- λ smoothing parameter. Fraction of data to be used

NOTE: minimization must be repeated for each x (and θ)

Problem:

Precipitation is a discrete/continuous variable

Solution:

Estimation in two steps

i. probability of precipitation (discrete)

   Discriminant analysis, logistic regression (GLM), probit regression (GLM), neural networks, classification trees, …

ii. precip. amounts given occurrence of precip. (continuous)

   (Local) quantile regression using data with observed precipitation only

Forecasting quantiles

Assume the p-th quantile, q_p, is of interest

- Estimate probability of precipitation, π, at step (i)

- Decide which quantile at step (ii) to estimate?

$$P\left(R \leq q_p \mid R > 0\right) = \frac{P\left(0 < R \leq q_p\right)}{P(R > 0)} = \frac{1 - P(R = 0) - P(R > q_p)}{P(R > 0)} = 1 - \frac{1 - p}{\pi}$$

- At step (ii) estimate this quantile

Example:

- Assume the 5, 25, 50, 75, and 95 percentiles are wanted
- probability of precipitation estimated to 0.65
  - → only the 50, 75, and 95 percentiles must be estimated
- At step (ii):
  estimate the 23.1, 61.5, and 92.3 conditional percentiles

Software for quantile regression

- Koenker & D'Orey

  J. R. Statist. Soc., Ser. C, 1987, 36, 383-393

  J. R. Statist. Soc., Ser. C, 1993, 43, 410-414

  http://lib.stat.cmu.edu/apstat/229 (Fortran 77)

- R: package "quantreg" (Koenker)

  http://cran.r-project.org/

Examples: Daily precipitation

Location: Brekke i Sogn (north of Bergen, Norway)

Data: ECMWF (12+66 UTC) and daily observations (525 days)

Experiments

EC output from high-resolution model

RR, MSLP, $RH_{925-500}$, $RH_{925-700}$, $Q_{925-500}$, $Q_{925-700}$, $DZ_{925-500}$, $DZ_{925-700}$

W, S, F, Vo (basic variables in a Lamb classification algorithm)

EPS ALL methods applied to each member, then averaging

RR

EPS STATS statistics of ensemble as predictors

MIN, 5, 25, 50, 75, 95 percentiles, MAX,

probability of more than 0.1, 1, 5 mm/day

Selection of predictors/smoothing

- Cross-validation (5 parts)
  → Selection based on quality of forecasts
- Separately for PoP and amounts given precipitation

Verification measures

- Probability of precipitation
  - Brier scores and reliability diagrams
- Amounts (conditional quantiles)
  - Reliability: chi-square test
  - Sharpness: distribution of prediction intervals (50% and 90%)

EC

PoP (probit regression)

Log(RR+0.1), W, log(RR+0.1)*W, $RH_{925\text{-}500}$*Vo

Amounts (local quantile regression)

RR, W, and S

Smoothing: 0.7

EPS ALL

PoP (probit regression)

Log(RR+0.1)

Amounts (quantile regression)

RR and RR^2

EPS STATS

PoP (probit regression)

Log(MIN+0.1), log(MEDIAN+0.1), log(MAX+0.1)

Amounts (local quantile regression)

25 and 75 percentiles of ensemble

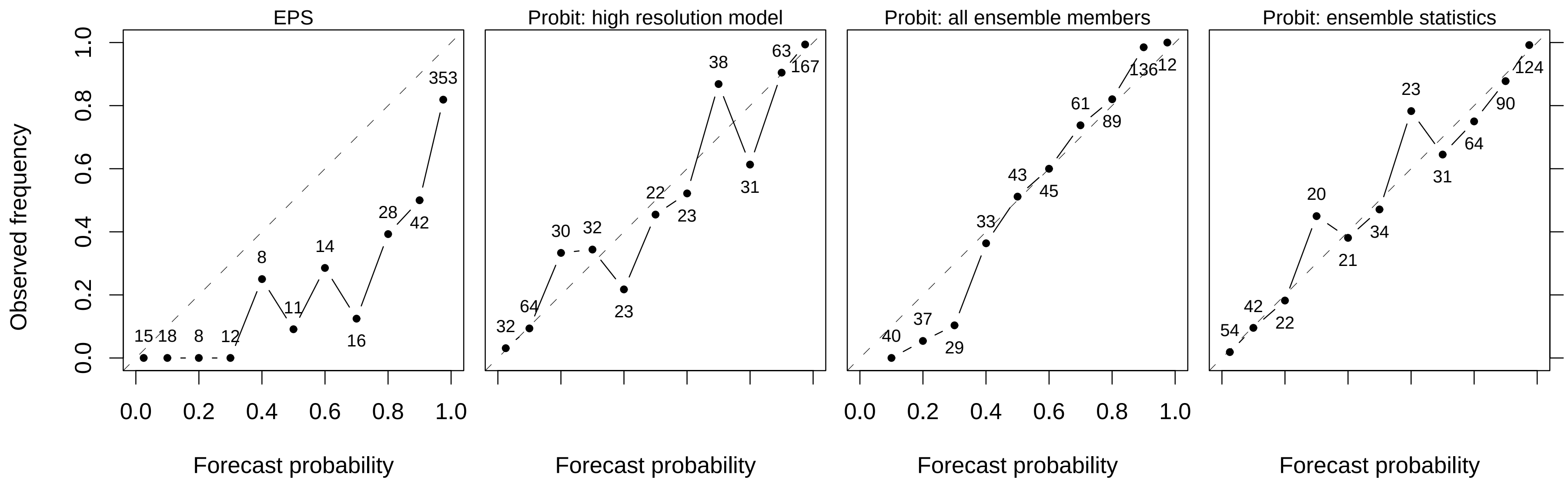
Smoothing: 0.6

Evaluation and comparison of final forecasts

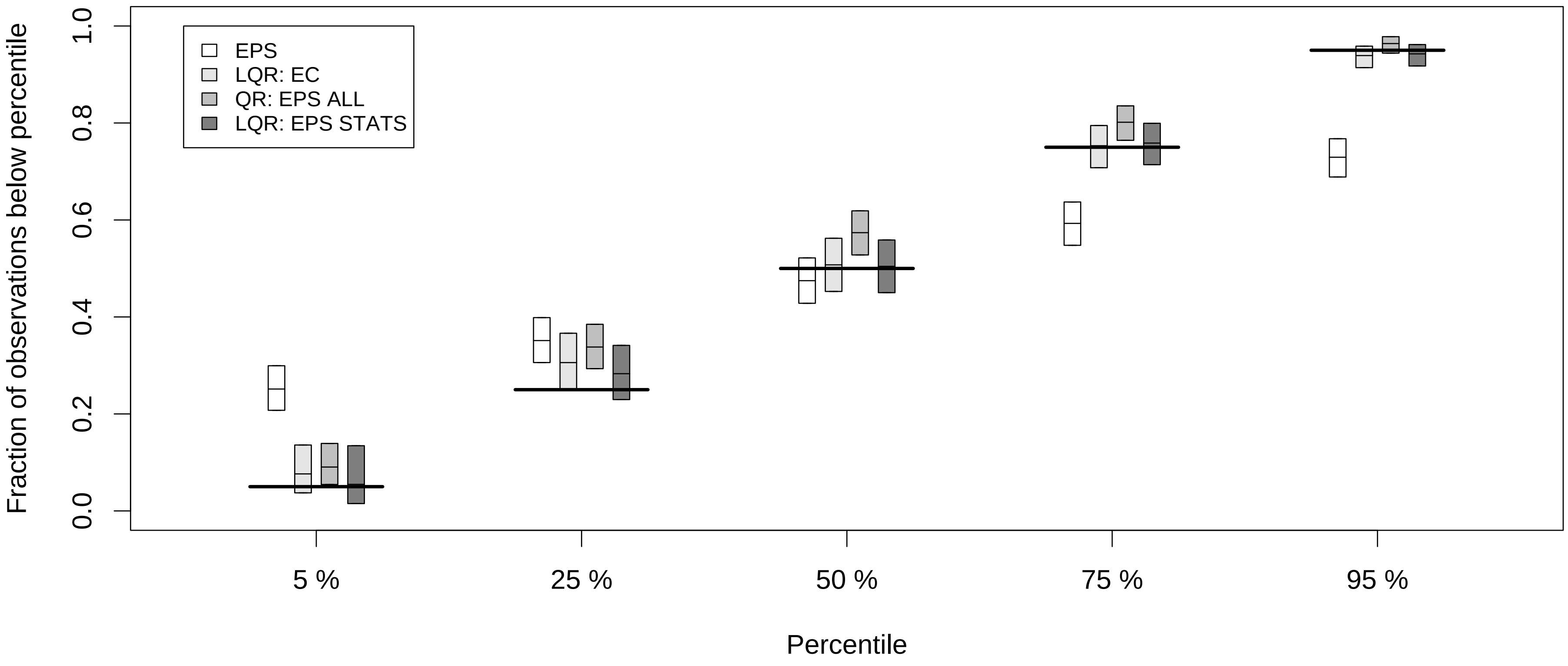
- New cross-validation

- Verification as for selecting predictors, but
  - Quantiles are not conditioned on occurrence of precipitation
  - Reliability tests (confidence intervals) separately for each quantile
  - Only cases where quantiles exist are used

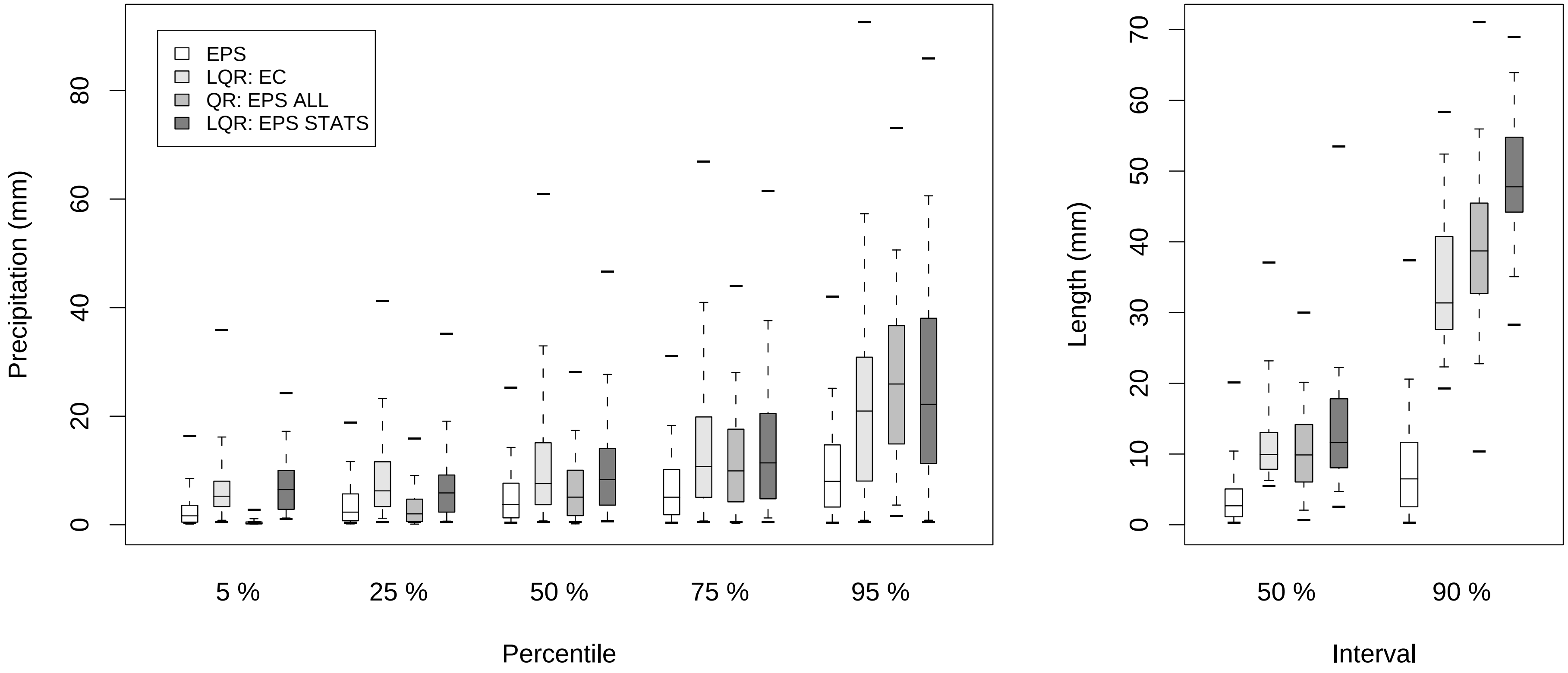
Reliability diagrams for probability of precipitation forecasts



Reliability of forecasted percentiles



Distributions of forecasted percentiles and intervals



Summary of experiments

- Raw EPS not reliable (as point forecast)
- Best reliable forecasts obtained by using output from the high-resolution model
- Forecasts based on ensembles would improve for longer lead times and more variables available
- Applying methods to each ensemble member and then averaging, not recommended

Why use quantile regression ?

- Produces well-calibrated forecasts
- No strong assumptions needed
- Any information can be included as predictors
- Dealing with ensembles easier
- Quantile forecasts ideal for graphical presentations in time

Future work and possibilities

- Verification scores for quantile forecasts
  - → Automatic and efficient predictor selection
- Local quantile regression
  - Different predictors for different quantiles
  - Weighting
  - Smoothing dependent on quantile
- Use of ensembles
- Properties of quantile forecasts for extreme events
  - how to “control” extrapolations
- Quantile forecasts for other variables, e.g. wind speed and temperature