Stability of the physics-dynamics in spectral models

Piet Termonia and Rafiq Hamdi

Royal Meteorological Institute of Belgium

Plans and rationale

- ARPEGE/ALADIN/AROME code is linked to IFS: code sharing, maintenance
- Both are spectral, but make distinctly opposite choices in the time-step organisation.
- Work is being carried out in the ALADIN community to make the time step more flexible to incorporate the IFS model structure as by Wedi (1999): M. Tudor and F. Vana.
- The Staniforth-Wood-Côté frame (2002) should be used to get insight and first guidance

Remarks

- We take the ARPEGE/ALADIN/AROME framework as the starting point, i.e. for the dynamics we base ourselves on Bénard (2003).
- Take the dynamics for granted: i.e. start with a “dynamical core” that is “adiabatically” stable.

Time step organisation: posibilities

- coupling of the physics parameterisation before or after the explicit part of the dynamics,
- coupling of the physics to the dynamics at different positions (in space and time) on the semi-Lagrangian trajectory,
- computing the physics parameterisation in a parallel or a fractional manner, and
- coupling the physics to the dynamics by updating the model state and using this for the dynamics, or computing the physics tendency and the dynamics tendencies separately and adding them to get the update, in other words treat the physics/dynamics in a fractional or a sequential manne

ALADIN/ARPEGE vs. SLAVEPP

	ARPEGE/ALADIN	SLAVEPP
phys. before/after dyn.	before	computed after and averag
on SL traj.	at t	at $t + \Delta t$
parallel / sequential physics calls	parallel	sequential
parallel /sequential phys.- dyn. coupling	sequential	parallel

SLAVEPP time step

	computation	1-D equivalent
1	inv. FFT, inv. Legendre transformation	
2	lin. terms	
3	compute departure point **D**	
4	interpolate to **D**	F_D^*
5	adiabatic explicit tendencies	F_A^{exp}
	and first-guess correction taken at the arrival point and time t	$\tilde{F}^+$
6	interpolate diab. tendencies of rad., conv. and cl. at t to **D**	T_D
7	tendencies of parameterised processes	
	computed in a fractional manner	G_M^{exp}
8	add tendencies of adiabatic and diabatic processes	F^{gp}
9	FFT, Legendre transformation	
10	Helmholtz, Horizontal diffusion	F_A^{dyn}

ARPEGE/ALADIN time step

	computation	1-D equivalent
1	inv. FFT, inv. Legendre transformation	
2	call physics	ϕ_α
3	update tendencies	F_A^*
4	compute departure point **D**	
5	interpolate to **D**	F_D^*
6	explicit part dynamics	F_A^{exp}
7	FFT, Legendre transformation	
8	Helmholtz, Horizontal diffusion	F_A^{dyn}

Methodology

Staniforth, Wood, Côté (2002)

$$\frac{\partial F}{\partial t} + U\frac{\partial F}{\partial x} + i\omega F = -\sum_{\alpha=1}^{N-p} \beta_\alpha F + \sum_{\alpha=p+1}^{N} R_\alpha \, e^{i[kx+\Omega_\alpha t]}$$

Generalisation

the framework proposed by Staniforth, Wood, Côté (2002) is extended to:

- take into account the spectral nature of the models and the difference between the real atmosphere and the background of the linearisation.

$$\frac{F_A^+ - F_D^0}{\Delta t} + \frac{i}{2}\omega^*(F_A^+ + F_D^0) + \frac{i}{2}(\omega - \omega^*)(F^{(0)} + F_D^0) = 0$$

$$F^{(0)} \equiv F_A^0 , \quad or \quad F^{(0)} = 2F_A^0 - F_A^-$$

“Dynamics” is unconditionally stable if: $\omega^* \geq \omega$

Simplified 1d ARPEGE/ALADIN

SP	Ders.	$\partial_x^p F_A = (ik)^p F$
	inv. S.T.	
	Physics $(lev.\, I)$	$\frac{G_\alpha - F_A^0}{\Delta t} = \xi_\alpha \phi_\alpha[F_A^0, G_\alpha]\,, \quad \alpha = 1, \ldots, M$
	Coupling	$F_A^* = F_A^0 + \Delta t \sum_{\alpha=1}^M \frac{G_\alpha - F_A^0}{\Delta t}$
	Interpolation	$F_D^* = e^{-ikU\Delta t} F_A^*$
GP	Expl. Dyn.	$F_A^{exp} = \left(1 - \frac{i\omega}{2}\Delta t\right) F_D^* - \frac{i}{2}(\omega - \omega^*)\Delta t F^{(0)}$
	Full TL first guess	$\tilde{F}^+ = F_A^{exp} - \frac{i}{2}\omega^* \Delta t\, F_A^*$
	Physics $(lev.\, II)$	$\frac{G_\alpha^{exp} - \tilde{F}^+}{\Delta t} = (1 - \xi_\alpha)(1 - \nu_\alpha)\phi[\tilde{F}^+; G_\alpha^{exp}]\,;$ $\alpha = 1, \ldots, M$
	Coupling	$G_A^{exp} = \tilde{F}^+ + \Delta t \sum_{\alpha=1}^M \frac{G_\alpha^{exp} - \tilde{F}^+}{\Delta t}$
	subtract first guess	$F^{gp} = G_A^{exp} + \frac{i}{2}\omega^* \Delta t\, F_A^*$
	S.T.	
SP	Impl. Dyn.	$F_A^{dyn} = \left[1 + \frac{i\omega^*}{2}\Delta t - \Delta t \sum_{\alpha=1}^M (1 - \xi_\alpha)\nu_\alpha \phi_\alpha^{imp}\cdot\right]^{-1} G_A^{exp}$

Simplified 1d version SLAVEPP

SP	Ders.	$\partial_x^p F_A = (ik)^p F$
	inv. S.T.	
	Physics $(lev.\,I)$	$\frac{G_\alpha - G_{\alpha-1}}{\Delta t} = \xi_\alpha \phi_\alpha [G_{\alpha-1}, G_\alpha]\,;\alpha = 1,\ldots,M\,, G_0 \equiv F_A^0$
	Interpolation	$F_D^* = e^{-ikU\Delta t} F_A^0\,, \quad T_D = e^{-ikU\Delta t} \frac{G_M - F_A^0}{\Delta t}$
GP	Expl. Dyn.	$F_A^{exp} = \left(1 - \frac{i\omega}{2}\Delta t\right) F_D^* - \frac{i}{2}(\omega - \omega^*)\Delta t\, F^{(0)}$
	Full TL first guess	$\tilde{F}^+ = F_A^{exp} - \frac{i}{2}\omega^* \Delta t\, F_A^0$
	Physics $(lev\,II)$	$\frac{G_\alpha^{exp} - G_{\alpha-1}^{exp}}{\Delta t} = (1-\xi_\alpha)(1-\nu_\alpha)\phi_\alpha [G_{\alpha-1}^{epx}, G_\alpha^{exp}]\,;$
		$\alpha = 1,\ldots,M\,, G_0^{exp} \equiv \tilde{F}^+$
	Coupling	$F^C = T_D \Delta t + G_M^{exp}$
	subtract first guess	$F^{gp} = F^C + \frac{i}{2}\omega^* \Delta t\, F_A^0$
	S.T.	
SP	Impl. Dyn.	$F_A^{dyn} = \left[1 + \frac{i\omega^*}{2}\Delta t - \Delta t \sum_{\alpha=1}^M (1-\xi_\alpha)\nu_\alpha \phi_\alpha^{imp}.\right]^{-1} F^{gp}$

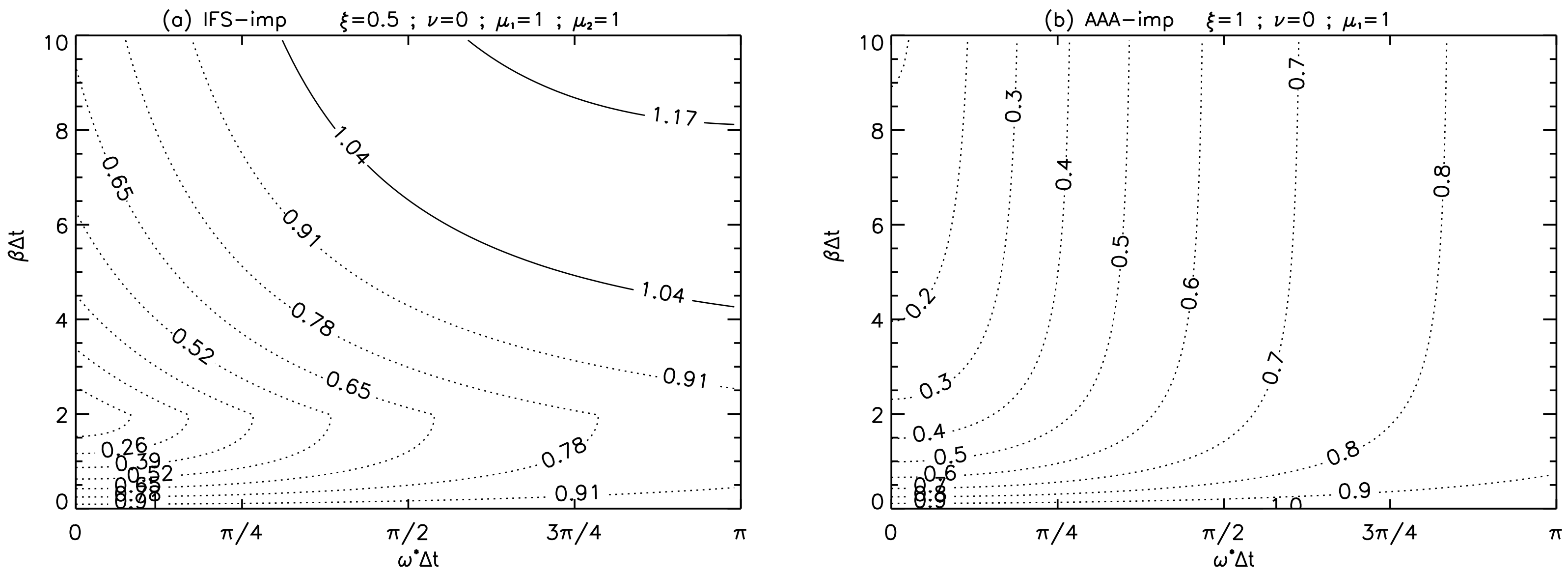
Generalisation

the framework proposed by Staniforth, Wood, Côté (2002) is extended to:

- take into account the spectral nature of the models and the difference between the real atmosphere and the background of the linearisation.
- allow a more numerical approach than analytic

Stability

- amplifi cation $|\mathcal{A}|$ given by $F_A^+ = \mathcal{A}\, F_A^0$ with $F_A^0 = 1$
- taking $|\mathcal{A}|_{\max} \equiv \max_{kU,\omega} |\mathcal{A}|$:



Accuracy

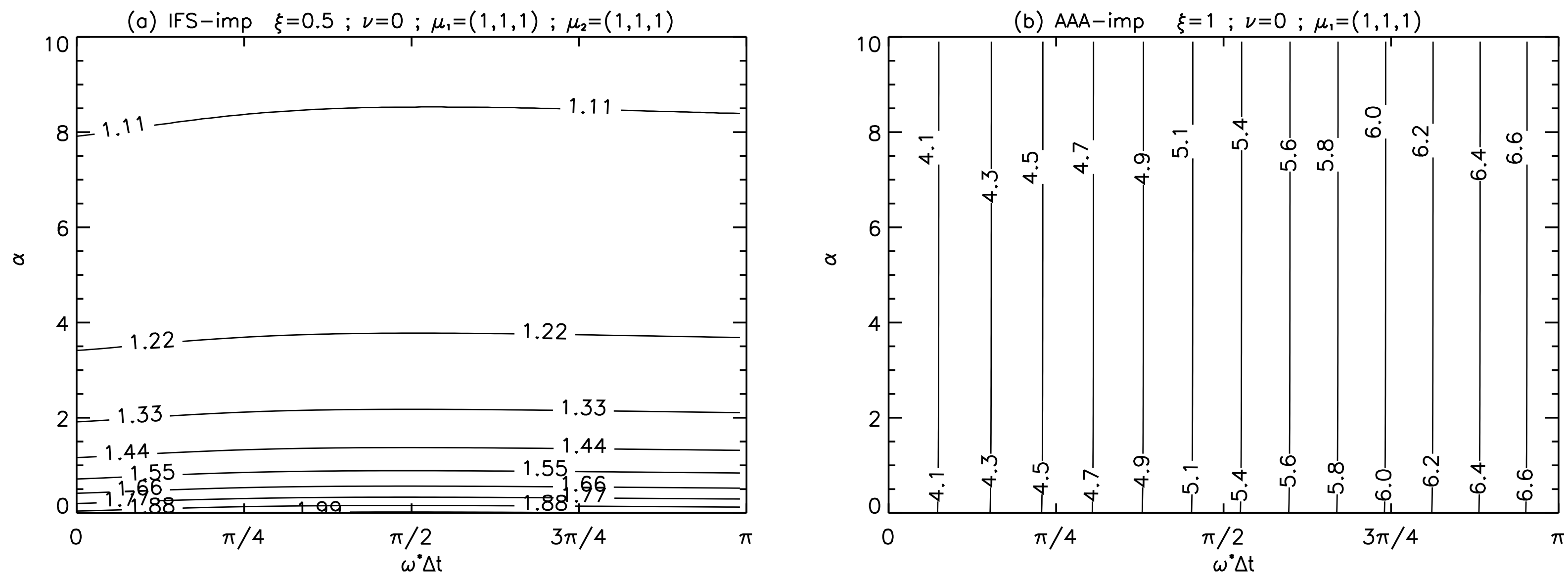
- The (idealized) SLAVEPP structure is second-order accurate: no surprise. AAA structure is first order accurate.
- The SLAVEPP structure represents the steady state exactly. AAA

$$(\phi^R + \phi^\beta)_{\text{par}} \mapsto \mathcal{D}^{exp} \mapsto \mathcal{D}^{imp}$$

does not: $(\beta^2 + \frac{\omega^2}{2})\Delta t + O(\Delta t^2)$ but improvements can be obtained by coupling after the explicit dynamics (maybe useful for ARPEGE climate?)

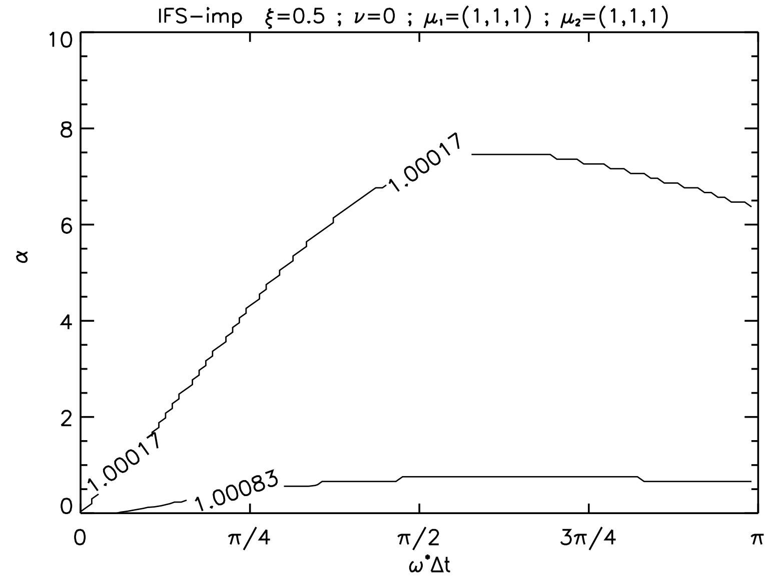
2 forcings, 1 diffusive

- IFS: $\mathcal{D}^{exp} \mapsto \frac{1}{2}\{(\phi^R \mapsto \phi^\beta \mapsto \phi^R)^0 + (\phi^R \mapsto \phi^\beta \mapsto \phi^R)^+\} \mapsto \mathcal{D}^{imp}$
- AAA: $(\phi^R + \phi^\beta + \phi^R) \mapsto \mathcal{D}^{exp} \mapsto \mathcal{D}^{imp}$



2 forcings, 1 diffusive

- IFS: $\mathcal{D}^{exp} \mapsto \frac{1}{2}\{(\phi^R \mapsto \phi^R \mapsto \phi^\beta)^0 + (\phi^R \mapsto \phi^R \mapsto \phi^\beta)^+\} \mapsto \mathcal{D}^{imp}$



Agreement with Dubal, Wood, Staniforth (2005)

Remark

- Conflicting conclusions between stability analysis and forcings?

Room for improvement?

SP	Ders.	$\partial_x^p F_A = (ik)^p F$
	inv. S.T.	
	Physics $(lev. I)$	$T^I[F_A^0]$
	Coupling $(lev. I)$	$F_A^* = F_A^0 + (1-\epsilon)(1-\kappa)\Delta t\, T_A^I$
	Interpolation	$F_D^* = e^{-ikU\Delta t} F_A^*;\, T_D^I = e^{-ikU\Delta t} T_A^I;\, F_D^0 = e^{-ikU\Delta t} F_A^0$
GP	Expl. Dyn.	$F_A^{exp} = \left(1 - \frac{i\omega}{2}\Delta t\right) F_D^* - \frac{i}{2}(\omega - \omega^*)\Delta t F^{(0)}$
	Full TL first guess	$\tilde{F}^+ = F_A^{exp} - \frac{i}{2}\omega^* \Delta t\, F_A^* + (1-\epsilon)\kappa\Delta t\, T_D^I$
	Physics $(lev. II)$	$T_A^{II}[\tilde{F}^+, F_D^0]$
	Coupling $(lev. II)$	$G_A^{exp} = \tilde{F}^+ + \epsilon(1-\delta)\Delta t\, T_D^I + (1-\zeta)\Delta t\, T_A^{II}$
	subtract first guess	$F^{gp} = G_A^{exp} + \frac{i}{2}\omega^* \Delta t\, F_A^*$
	S.T.	
SP	Impl. Dyn.	$F_A^{dyn} = \left[1 + \frac{i\omega^*}{2}\Delta t - \Delta t \sum_{\alpha=1}^{M}(1-\xi_\alpha)\nu_\alpha \phi_\alpha^{imp}\cdot\right]^{-1} F^{gp}$
	Coupling $(lev. III)$	$F_A^+ = F_A^{dyn} + \epsilon\delta\Delta t\, T_D^I + \zeta\Delta t\, T_A^{II}$

Room for improvement

$$\epsilon = \frac{1}{2}, \quad \zeta = 0, \quad T_A^{II}[F_D^0]$$

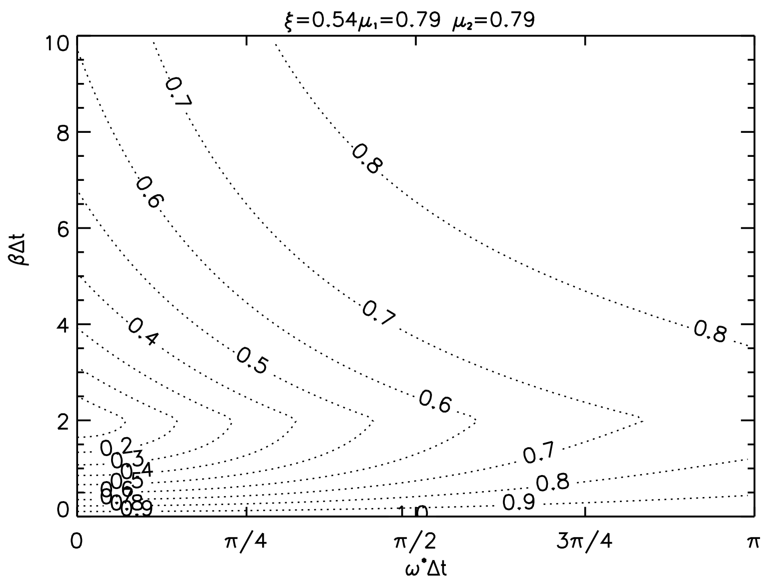
So concerning “where to couple” it is a mix between AAA and SLAVEPP We get second-order accuracy if

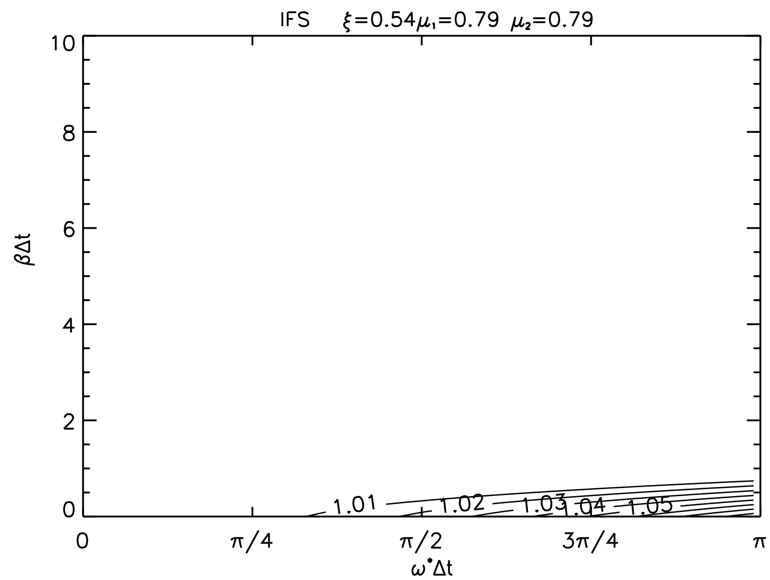
$$\mu = \tfrac{1}{16}(7 + \sqrt{33}) \quad \xi = \frac{2(1-2\mu)}{1-4\mu}$$

Room for improvement?

SP	Ders.	$\partial_x^p F_A = (ik)^p F$
	inv. S.T.	
	Physics $(lev.\,I)$	$T^I[F_A^0]$
	Coupling $(lev.\,I)$	$F_A^* = F_A^0 + \frac{1}{2}\Delta t\, T_A^I$
	Interpolation	$F_D^* = e^{-ikU\Delta t} F_A^*;\, T_D^I = e^{-ikU\Delta t} T_A^I;\, F_D^0 = e^{-ikU\Delta t} F_A^0$
GP	Expl. Dyn.	$F_A^{exp} = \left(1 - \frac{i\omega}{2}\Delta t\right) F_D^* - \frac{i}{2}(\omega - \omega^*)\Delta t F^{(0)}$
	Full TL first guess	$\tilde{F}^+ = F_A^{exp} - \frac{i}{2}\omega^* \Delta t\, F_A^*$
	Physics $(lev.\,II)$	$T_A^{II}[F_D^0]$
	Coupling $(lev.\,II)$	$G_A^{exp} = \tilde{F}^+ + \frac{1}{2}\Delta t\, T_D^I + \Delta t\, T_A^{II}$
	subtract first guess	$F^{gp} = G_A^{exp} + \frac{i}{2}\omega^* \Delta t\, F_A^*$
	S.T.	
SP	Impl. Dyn.	$F_A^{dyn} = \left[1 + \frac{i\omega^*}{2}\Delta t - \Delta t \sum_{\alpha=1}^{M} (1-\xi_\alpha)\nu_\alpha \phi_\alpha^{imp} \cdot\right]^{-1} F^{gp}$
	Coupling $(lev.\,III)$	$F_A^+ = F_A^{dyn}$

Room for improvement





Discussion

- Improvements: at first it was not clear that we can get improvements in both stability and accuracy, but from the last example is encouraging.
- We started from the AAA structure
- Exploring this space of possibilities for improvement