Exploring an iterative approach to the semi-implicit, semi-Lagrangian time-stepping in the Met Office non-hydrostatic model

SRNWP workshop on Numerical Techniques
December 2006, Zagreb, Croatia

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Talk outline

1. Brief description of the semi-implicit semi-Lagrangian (SISL) predictor-corrector scheme of the non-hydrostatic Unified Model
2. An iterative version of the Unified Model SISL scheme
3. Results from a high resolution mesoscale case study
4. Global model results

Main features of the Unified Model

Met Office Unified Model (UM) Davies et al (2005, QJRMS):

- A single code library used for operational weather forecasts (global, regional, mesoscale), climate predictions and as a research tool:
  ▷ It operates efficiently on a wide spectrum of horizontal resolutions, from 200 km down to 1 km
- Non-hydrostatic, deep atmosphere, semi-Lagrangian model
- Finite difference gridpoint model with terain following height based vertical coodinate: C-grid in the horizontal, stretched quadratic in the vertical with Charney-Phillips staggering
- Two time-level semi-implicit predictor-corrector time integration
- 3D Helmholtz equation

The UM semi-implicit semi-Lagrangian scheme

Consider the prognostic equation

$$\frac{D\mathbf{X}}{Dt} = \mathbf{L}(\mathbf{x}, t, \mathbf{X}) + \mathbf{N}(\mathbf{x}, t, \mathbf{X}) + \mathbf{S}(\mathbf{x}, \mathbf{t}, \mathbf{X}) + \mathbf{F}(\mathbf{x}, \mathbf{t}, \mathbf{X})$$

where, $\mathbf{x}$, $\mathbf{L}$, $\mathbf{N}$ denote position, linear and nonlinear dynamical terms and $\mathbf{S}$, $\mathbf{F}$ slow and fast physics forcing. Semi-implicit semi-Lagrangian (SISL) target discretization (Staniforth & Côté, MWR 1991):

$$\frac{\mathbf{X}^{n+1} - \mathbf{X}_d^n}{\Delta t} = (1-\alpha)(\mathbf{L}+\mathbf{N}+\mathbf{S}+\mathbf{F})_d^n + \alpha(\mathbf{L}+\mathbf{N}+\mathbf{S}+\mathbf{F})^{n+1}, \quad 1/2 \leq \alpha \leq 1.$$

For CPU cost efficient solution the implicit nonlinear coupling should be reduced: *A predictor-corrector approach is used*.

UM predictor-corrector time scheme

Compute $\mathbf{X^{n+1}}$ via a predictor-corrector approach:

$$\begin{aligned} \mathbf{X}^{(1)} &= \mathbf{X}_d^n + (1-\alpha)\Delta t(\mathbf{L}+\mathbf{N})_d^n + \Delta t\,(\mathbf{S})_d^n + \alpha\Delta t(\mathbf{L}+\mathbf{N})^n \\ \mathbf{X}^{(2)} &= \mathbf{X}^{(1)} + \Delta t\mathbf{F}(\mathbf{X}^n, \mathbf{X}^{(1)}, \mathbf{X}^{(2)}) \\ \mathbf{X}^{(3)} - \alpha\Delta t\mathbf{L}^{(3)} &= \mathbf{X}^{(2)} + \alpha\Delta t(\mathbf{N}^* - \mathbf{N}^n - \mathbf{L}^n) \end{aligned}$$

where,

$\mathbf{X^{(1)}}$: first predicted value, $\mathbf{X^{(2)}}$: predicted value after fast physics, $\mathbf{X^{(3)}} \equiv \mathbf{X^{n+1}}$ the final estimate and $\mathbf{N}^* \approx \mathbf{N}^{n+1}$.

Eliminating the intermediate stages:

$$\mathbf{X}^{n+1} = \mathbf{X}_d^n + (1-\alpha)\Delta t(\mathbf{L}+\mathbf{N}+\mathbf{S})_d^n + \alpha\Delta t\left[\mathbf{L}^{n+1} + \mathbf{N}^* + \mathbf{F}(\mathbf{X}^n, \mathbf{X}^{(1)}, \mathbf{X}^{(2)})\right].$$

Departure point calculation

Approximate

$$\mathbf{x}_a - \mathbf{x}_d = \int_{t^n}^{t^n+\Delta t} \mathbf{U}\left[\mathbf{x}\left(t\right), t\right] dt$$

as,

$$\mathbf{x}_a - \mathbf{x}_d \approx \Delta t \cdot \mathbf{U}\left[(\mathbf{x}_a + \mathbf{x}_d)/2, t^{n+\frac{1}{2}}\right].$$

Compute $\mathbf{x}_d$ iterating

$$\mathbf{x}_d^{[l+1]} = \mathbf{x}_\alpha - \Delta t \mathbf{U}_*^{[l]}, \qquad l = 0, 1$$

where,

$$\mathbf{U}_* \equiv \widetilde{\mathbf{U}}\left(\frac{\mathbf{x}_a + \mathbf{x}_d}{2}, t^{n+1/2}\right), \quad \widetilde{\mathbf{U}}(\mathbf{x}, t^{n+1/2}) \equiv \frac{3}{2}\mathbf{U}\left(\mathbf{x}, t^n\right) - \frac{1}{2}\mathbf{U}\left(\mathbf{x}, t^n - \Delta t\right).$$

Extrapolation introduces a weak instability Cordero et al (QJRMS, 2005).

Iterative SISL scheme for the UM

Stability of the SI scheme can be enhanced using an iterative fixed point algorithm: Côté et al (MWR, 1998), Cullen (QJRMS, 2001), Bénard (MWR, 2003), Cordero et al (QJRMS, 2005).

Recently developed iterative version of the UM scheme:

$$\begin{aligned}
\mathbf{X}^{(1)[\ell]} &= \mathbf{X}^n_{d_\ell} + (1-\alpha)\Delta t\,(\mathbf{L}+\mathbf{N})^n_{d_\ell} + \Delta t\,(\mathbf{S})^n_{d_\ell} + \alpha\Delta t\,(\mathbf{L}+\mathbf{N})^{(3)[\ell-1]} \\
\mathbf{X}^{(2)[\ell]} &= \mathbf{X}^{(1)[\ell]} + \Delta\mathbf{F}(\mathbf{X}^n, \mathbf{X}^{(1)[\ell]}, \mathbf{X}^{(2)[\ell]}) \\
\mathbf{X}^{(3)[\ell]} - \alpha\Delta t\mathbf{L}^{(3)[\ell]} &= \mathbf{X}^{(2)[\ell]} + \alpha\Delta t\left(\mathbf{N}^* - \mathbf{N}^{(3)[\ell-1]} - \mathbf{L}^{(3)[\ell-1]}\right)
\end{aligned}$$

where, $\ell = 1, 2, \ldots$ and

$$\mathbf{L}^{(3)[\ell]} \equiv \mathbf{L}\left(\mathbf{X}^{(3)[\ell]}\right), \quad \mathbf{N}^{(3)[\ell]} \equiv \mathbf{N}(\mathbf{X}^{(3)[\ell]}), \quad \mathbf{L}^{(3)[0]} \equiv \mathbf{L}\left(\mathbf{X}^n\right), \quad \mathbf{N}^{(3)[0]} \equiv \mathbf{N}(\mathbf{X}^n)$$

- For $\ell = 1$ the current non-iterated scheme is obtained
- For $\ell > 1$ more stable and accurate scheme

Departure point calculation in the iterative scheme

Likewise, compute $\mathbf{x}_d$ iterating

$$\mathbf{x}_{d_\ell}^{[l+1]} = \mathbf{x}_\alpha - \Delta t \mathbf{U}_*^{[l]}, \qquad l = 0, 1$$

where,

$$\mathbf{U}_* \equiv \widetilde{\mathbf{U}} \left(\frac{\mathbf{x}_a + \mathbf{x}_{d_\ell}}{2}, t^{n+1/2} \right),$$

$$\widetilde{\mathbf{U}}(\mathbf{x}, t^{n+1/2}) \equiv \begin{cases} (1-\gamma)\mathbf{U}\left(\mathbf{x}, t^n - \Delta t\right) + \gamma \mathbf{U}\left(\mathbf{x}, t^n\right), & \ell = 1 \\ \frac{1}{2}\left(\mathbf{U}^{(3)[\ell-1]}(\mathbf{x}) + \mathbf{U}^n(\mathbf{x})\right), & \ell > 1 \end{cases}$$

where:

- $\ell = 1$:
  - $\triangleright\, \gamma = 3/2$: 2nd order extrapolating scheme
  - $\triangleright\, \gamma = 1$: 1st order non-extrapolating scheme
- $\ell > 1$: 2nd order interpolating scheme: stable + more accurate

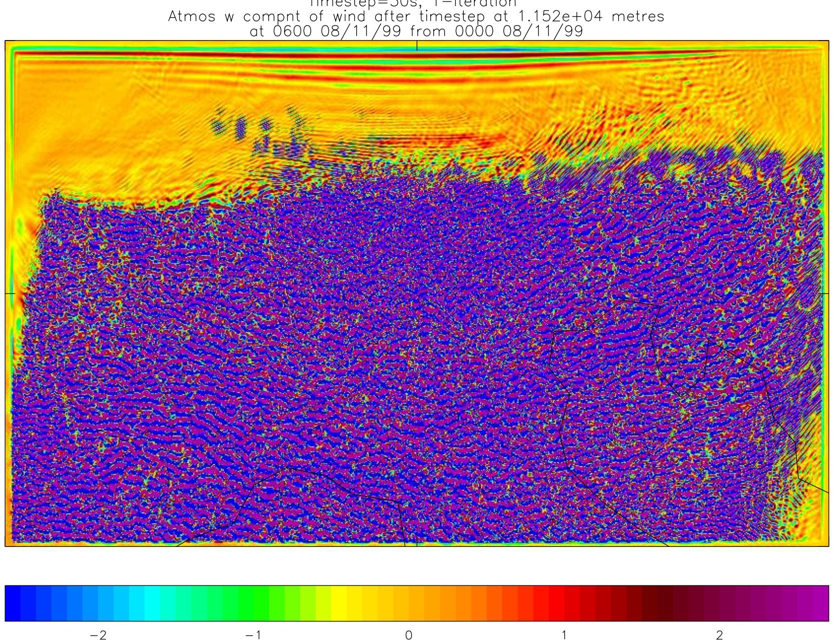
Resulting improvements in the UM

- More stable and accurate departure point calculation
- Improved handling of the deep atmosphere Coriolis terms in momentum equations:
  ▷ Non-iterated scheme: explicit handling
  ▷ Iterated scheme: semi-implicit handling
- Improved semi-implicit handling of the nonlinear vertical pressure gradient term $\mathbf{N}_{\mathbf{vpg}} = c_p \theta_v \nabla \Pi$ in momentum equations:
  ▷ Non-iterated scheme: $\mathbf{N}^*_{\mathbf{vpg}} \equiv c_p \theta_v^{n+1} \nabla \Pi^{n+1} \approx \theta_v^{(2)} \nabla \Pi^{n+1}$, i.e. a partially updated θ_v
  ▷ Iterated scheme: $\mathbf{N}^*_{\mathbf{vpg}} \equiv c_p \theta_v^{n+1} \nabla \Pi^{n+1} \approx c_p \theta_v^{(3)[\ell-1]} \nabla \Pi^{n+1}$, i.e. a fully updated θ_v
- Improved physics-dynamics coupling

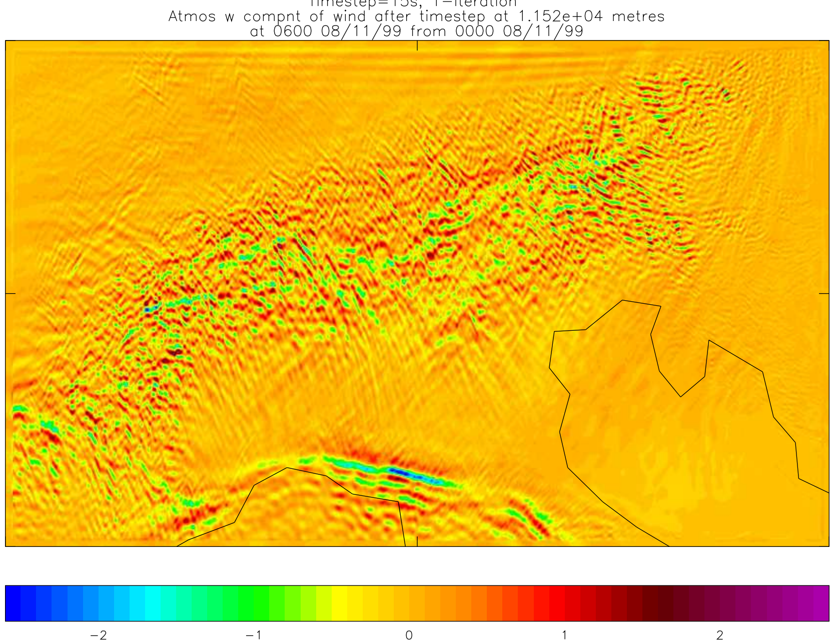
Details of discretization submitted for publication in QJRMS.

Mesoscale Alpine Programme (MAP) case study

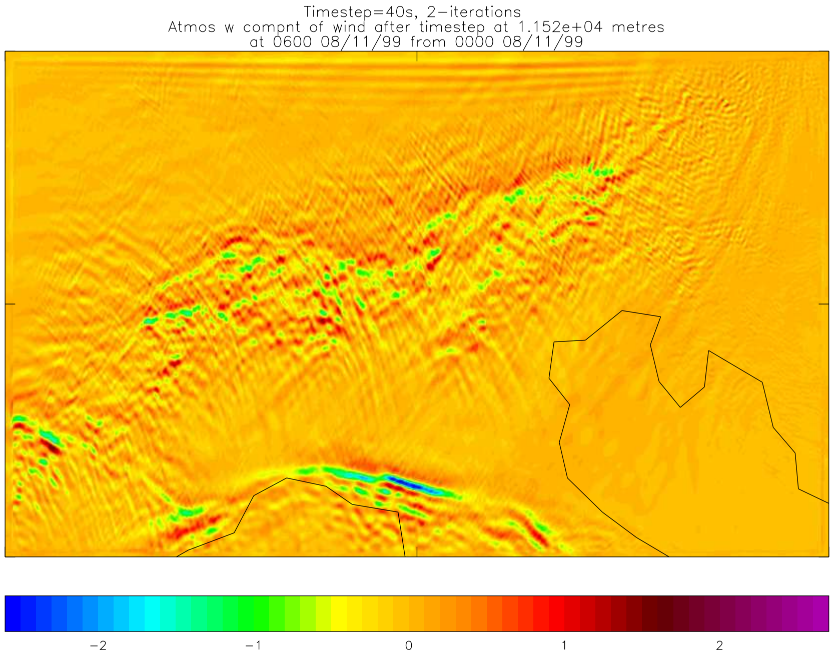
- Mesoscale case study over the Alps (Smith, QJRMS 2003)
- High horizontal resolution (1km). Deep, narrow, low-lying valleys are well resolved
- Monotone, fully-interpolating SISL for θ is used here. In operations, a non-interpolating in the vertical SL scheme for θ is used
  - ▷ With a fully interpolating scheme, a more realistic simulation is obtained in this case study
  - ▷ However, stability is weakened and a shorter timestep is required
- Stability improves when the 2-iteration scheme is used.



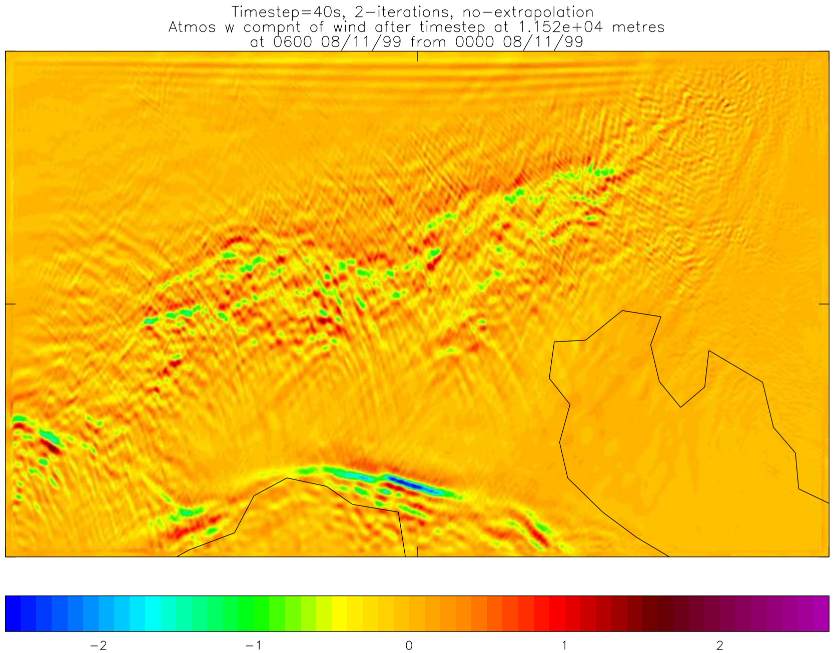
(a) **W at 12km: Timestep=30s, 1-iteration**



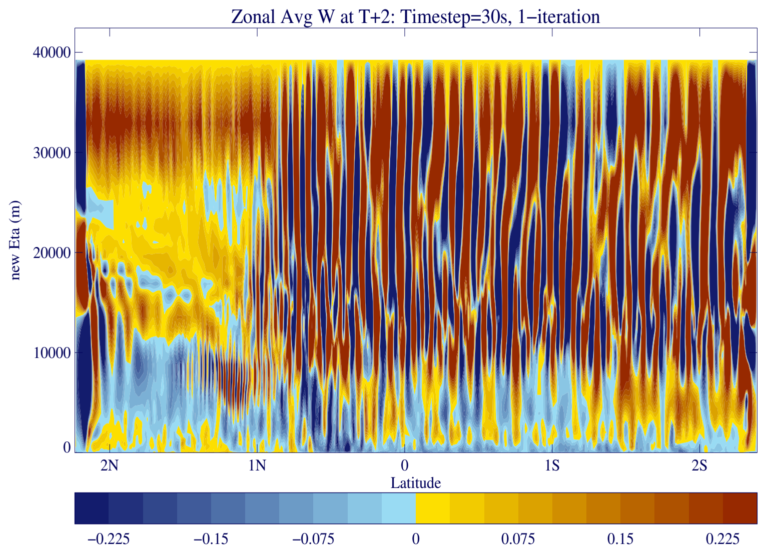
(b) **W at 12km, Timestep=15s, 1-iteration**



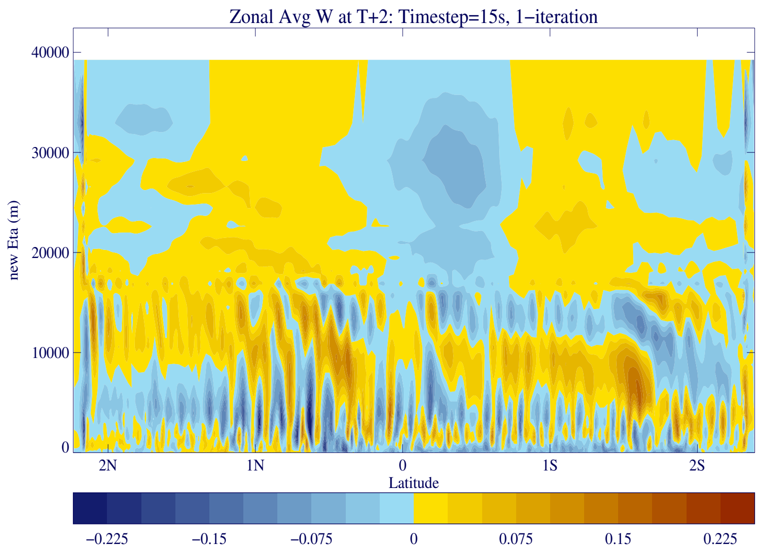
(c) **W at 12km, Timestep=40s, 2-iterations**



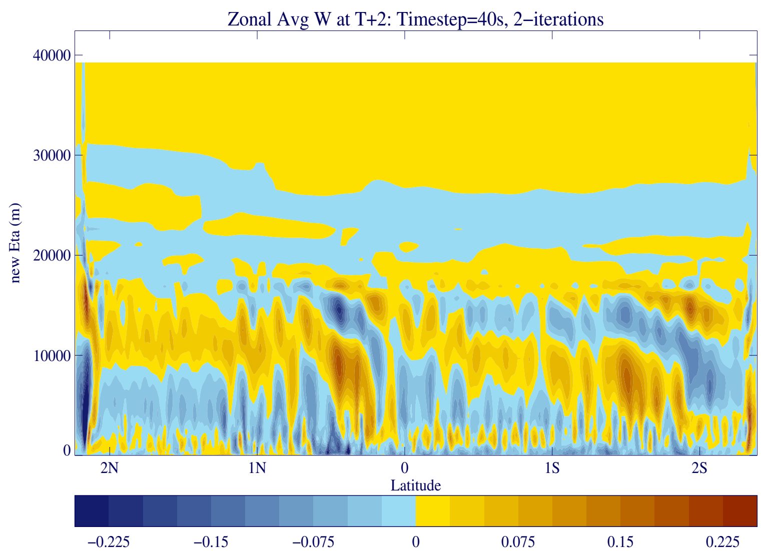
(d) **W at 12km, Timestep=40s, 2-iterations, non-extrapolating**



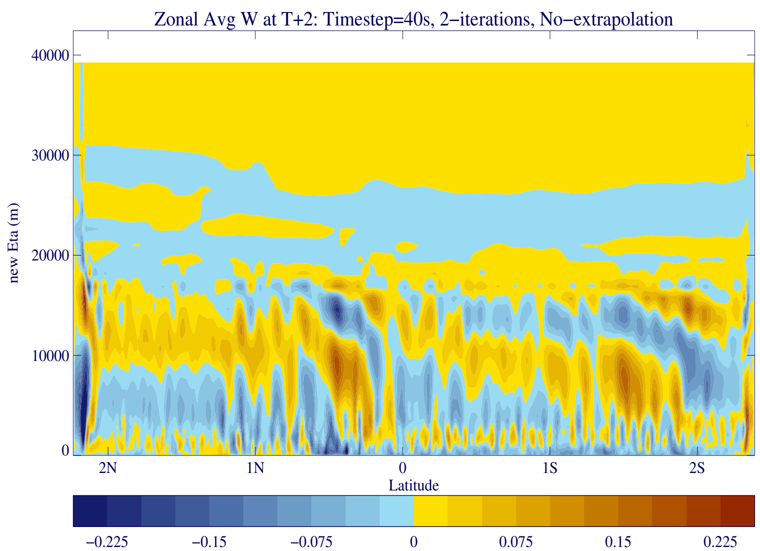
(e) **zonally averaged W at T+2: Timestep=30s, 1-iteration**



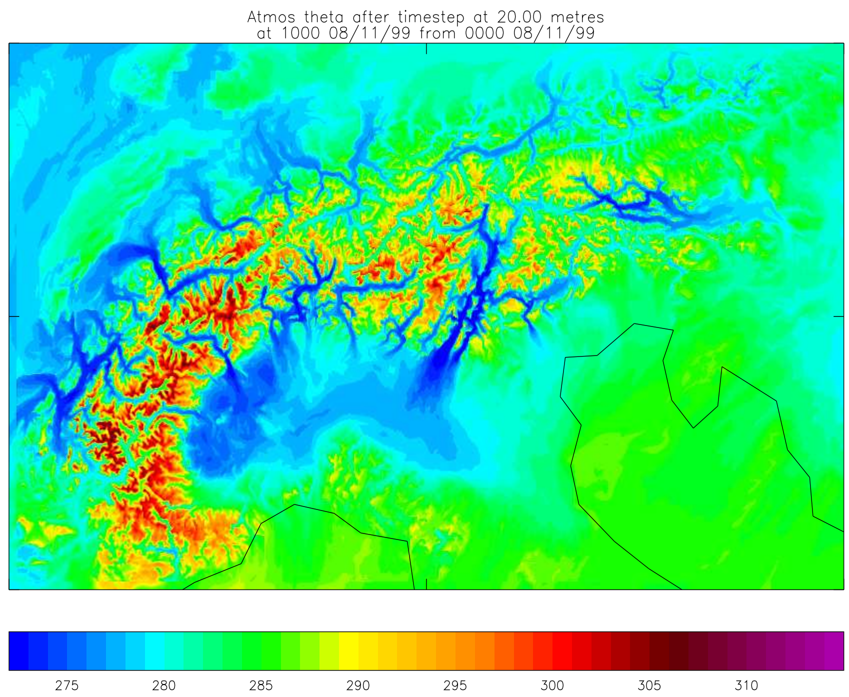
(f) **Zonally averaged W at T+2, Timestep=15s, 1-iteration**



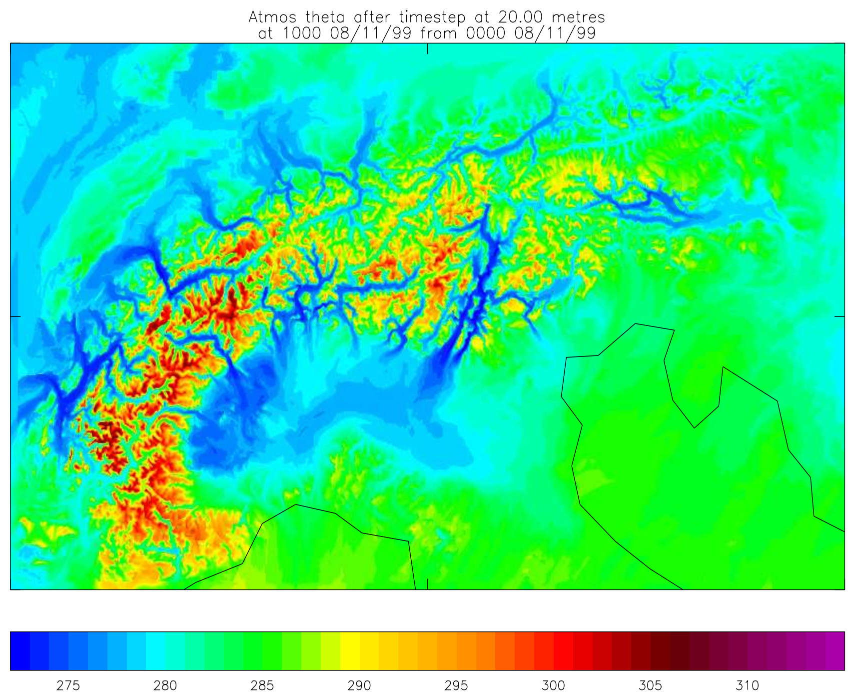
(g) **Zonally averaged W at T+2, Timestep=40s, 2-iterations**



(h) **W at 12km, Timestep=40s, 2-iterations, non-extrapolating**



(i) **Low level θ at T+6: 1-iteration, Δt=15s**



(j) **Low level θ at T+6: 2-iterations, Δt=40s**

The 2-iteration scheme with $\Delta t = 40$s gives similar solution with the standard UM 1-iteration scheme and $\Delta t = 15$s. Small differences up to 1^oK can be observed in the valleys south of the Alps.

Summary from the mesoscale case study

On this mesoscale case study the iterative SISL scheme enables increasing the timestep. With a fully interpolating scheme for θ-advection, the maximum timestep which results in a stable and noise free UM forecast is:

- $\Delta t = 15s$ with 1-iteration
- $\Delta t = 40s$ with 2-iterations. This run is more CPU time efficient (gain $\approx 30\%$)

Overall:

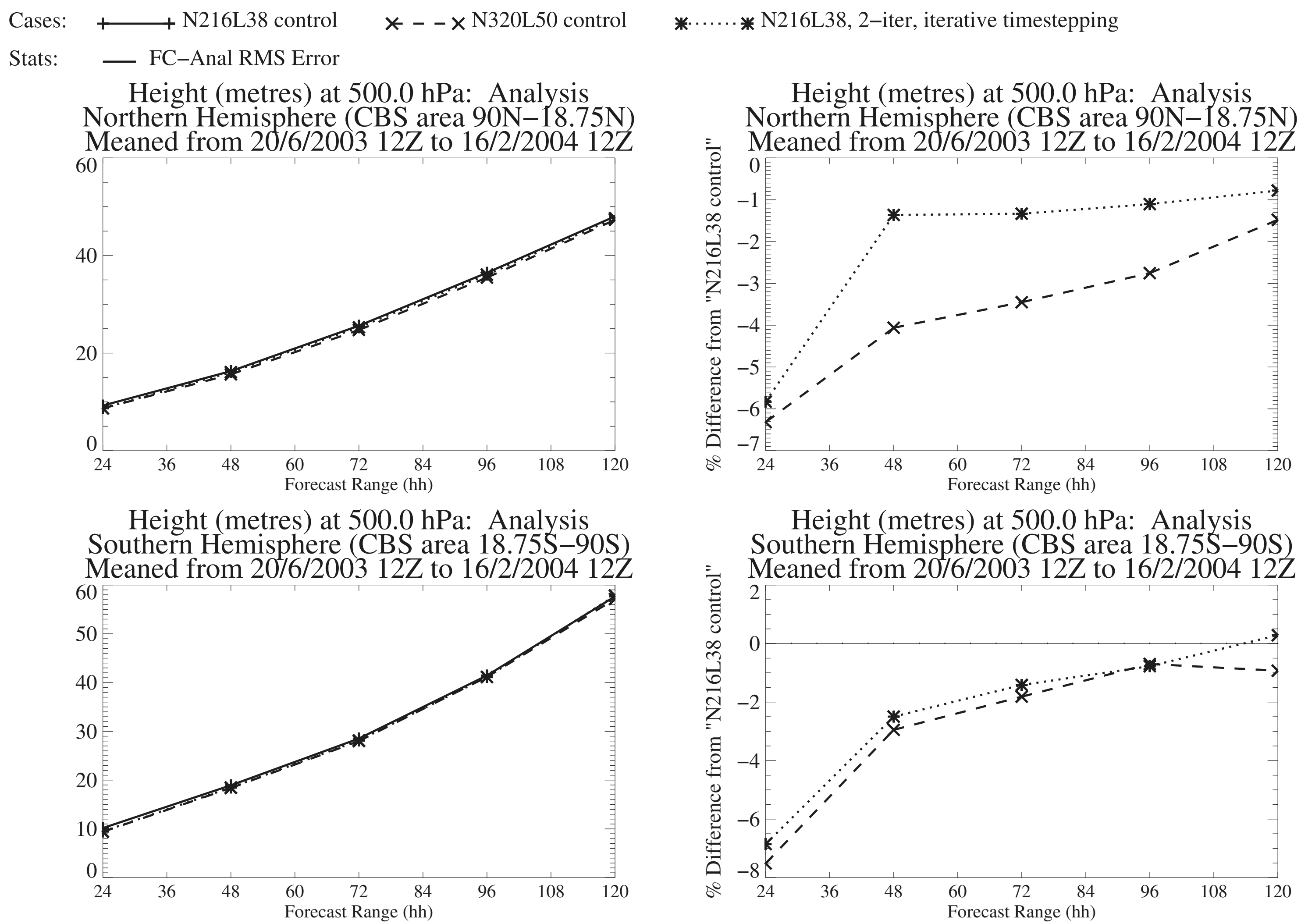
- Forecasts run stably with a small amount of de-centring in the semi-implicit time discretization
- Good accuracy and stability
- No noticeable difference in the solution when in the first iteration the standard 2nd order extrapolating scheme or the 1st order non-extrapolating scheme for the departure point calculation is used

Global model case studies

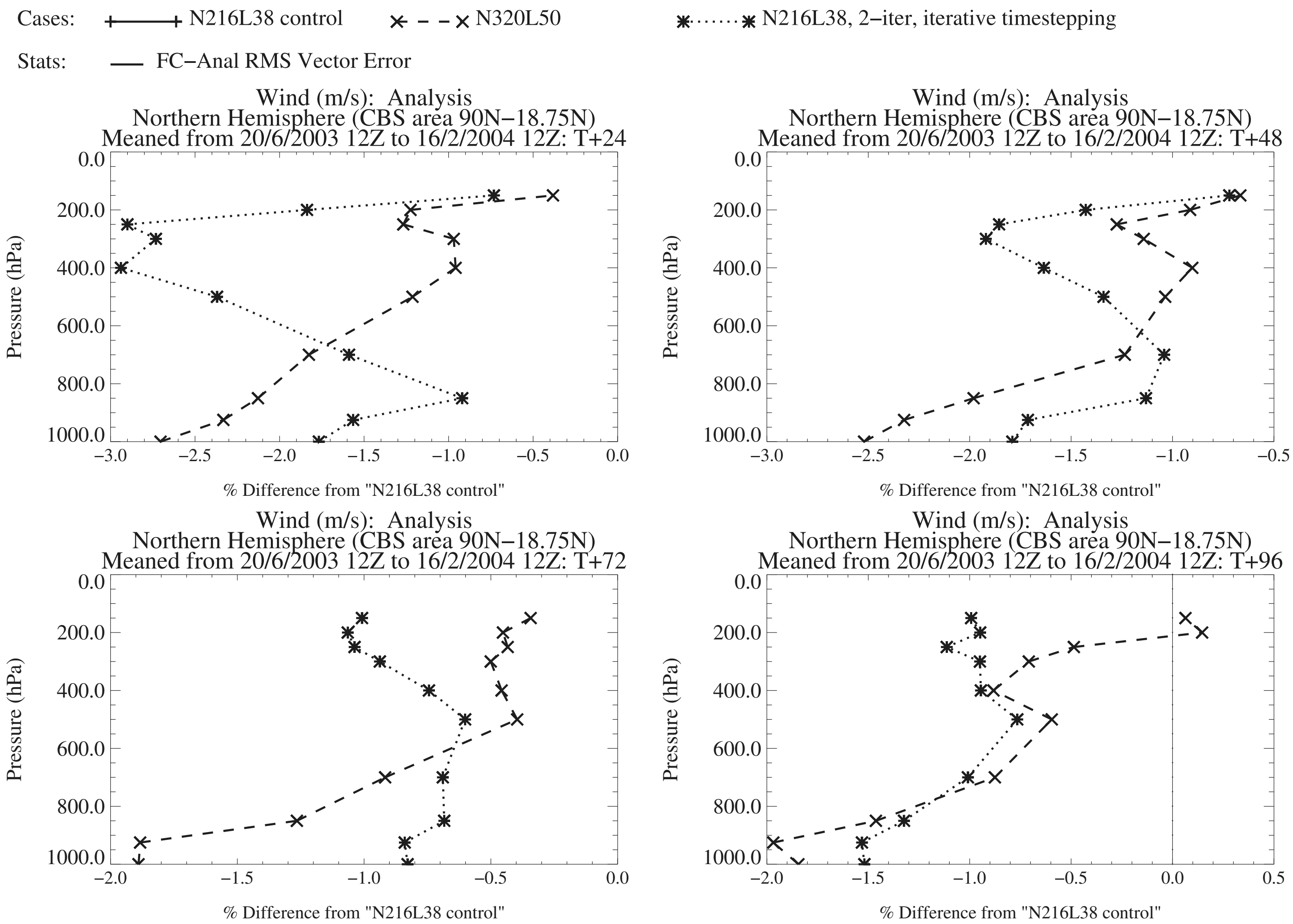
- Suite of 10 forecasts only (no data assimilation) case studies: 5 winter, 5 summer
- Case studies have run on global model at:
  ▷ old resolution (before December '05) ~ 60km at mid latitudes and 38 levels in the vertical, 20minutes timestep
  ▷ new enhanced resolution (December '05) ~ 40km at mid latitudes and 50 levels in the vertical, 15 minutes timestep. Extra levels in the stratosphere as model lid was raised from 40 to 65 km.
- Starting from ECMWF data and verified against ECMWF analyses. Root Mean Square Error (RMSE) measures displayed:

$$RMSE_{scalar} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(F_i - A_i)^2}, \quad RMSE_{winds} = \sqrt{RMSE_U^2 + RMSE_V^2},$$

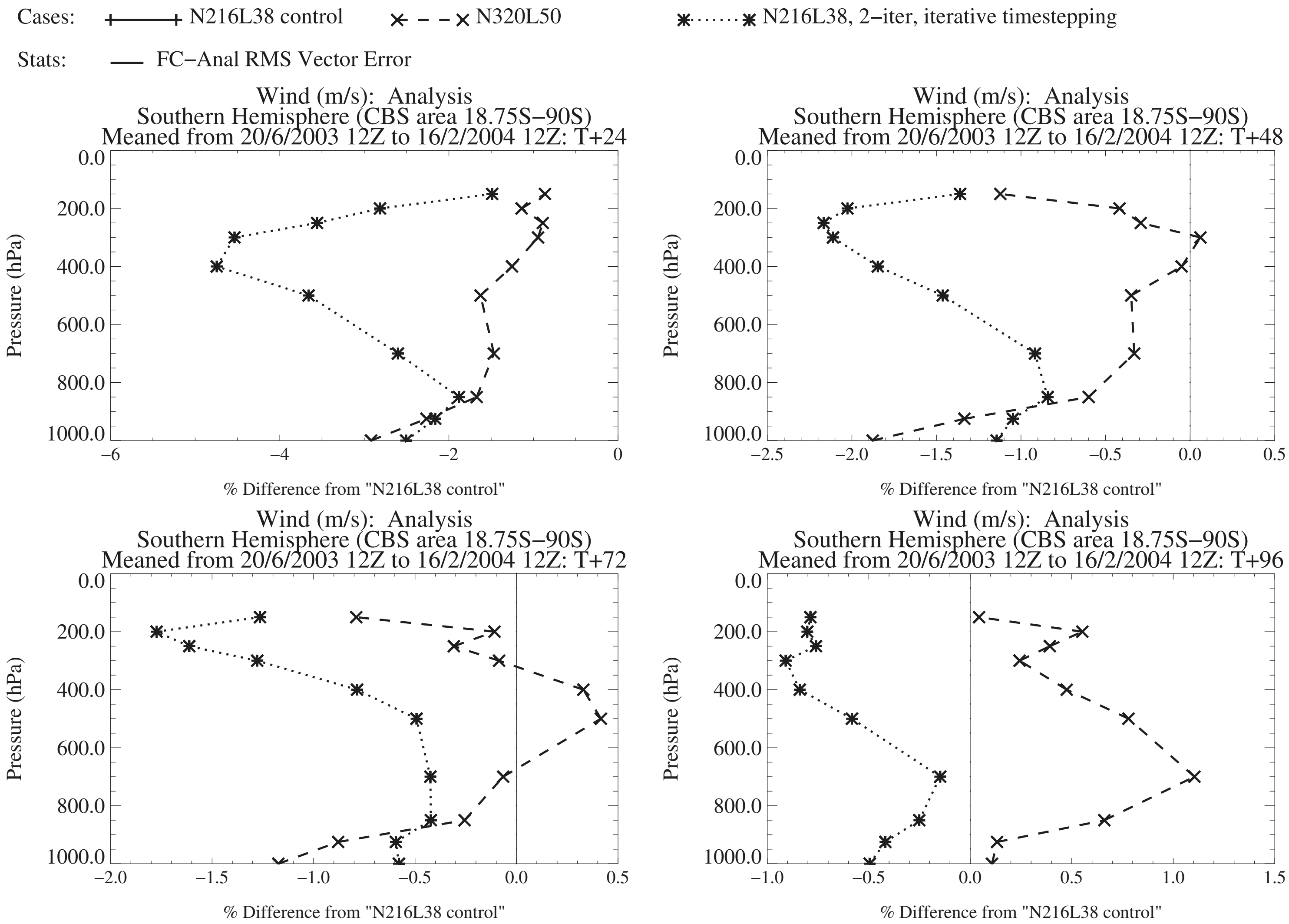
where F_i, A_i forecast and analysis values at i^{th} gridpoint.



(k) **Actual RMSE and difference in RMSE (against the 60 km res 38 level control) for the extratropical H500 hPa**



(l) **RMSE difference, against the 60km res 38 level control run RMSE, for the northern hemisphere winds**



(m) **RMSE difference, against the 60km res 38 level control run RMSE, for the southern hemisphere winds**

Summary of global model results and conclusions

- The iterative scheme improves forecasting accuracy as the reduction in RMSE suggests
- The iterative scheme is expensive: a 2-iteration global run costs an extra 60%. However, the test presented suggests that accuracy improvements comparable to those achieved using a more expensive higher resolution set up can be achieved

Overall summary:

- Applying an iterative approach in the UM improves both stability and accuracy. Although some design choices in the UM differ from other SISL models, notably the timestepping, our results are consistent with results reported in the literature by other centres.